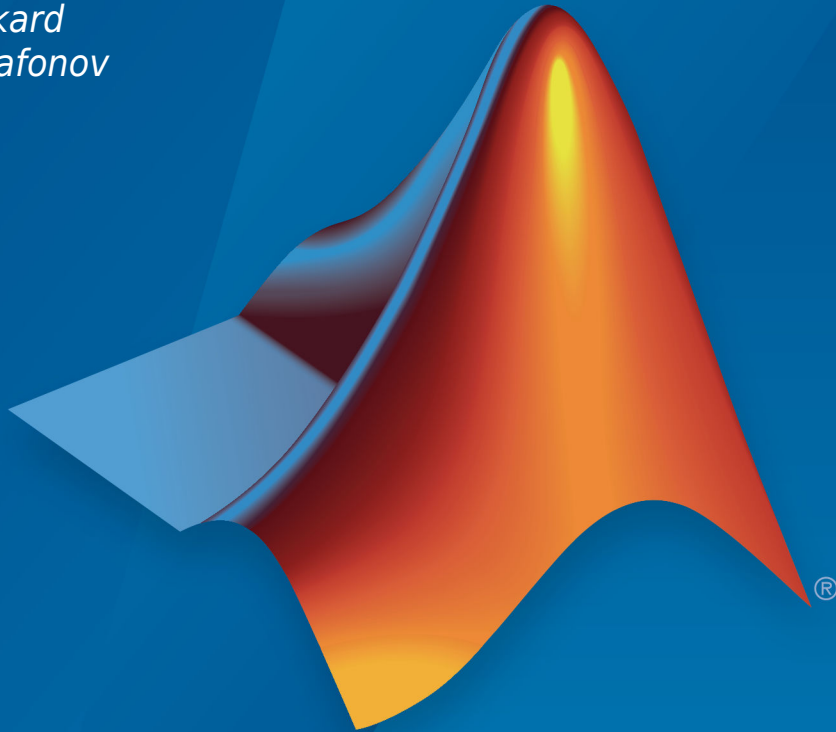


# Robust Control Toolbox™

## Reference

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| September 2017 | Online only    | Revised for Version 6.4 (Release 2017b)   |
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**1** | Alphabetical List

**2** | Block Reference



# Alphabetical List

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## actual2normalized

Transform actual values to normalized values

### Syntax

```
NV = actual2normalized(uElement,AV)  
[NV,ndist] = actual2normalized(uElement,AV)
```

### Description

`NV = actual2normalized(uElement,AV)` transforms the values `AV` of the uncertain element `uElement` into normalized values `NV`. If `AV` is the nominal value of `uElement`, `NV` is 0. Otherwise, `AV` values inside the uncertainty range of `uElement` map to the unit ball  $||NV|| \leq 1$ , and values outside the uncertainty range map to  $||NV|| > 1$ . The argument `AV` can contain a single value or an array of values. `NV` has the same dimensions as `AV`.

`[NV,ndist] = actual2normalized(uElement,AV)` also returns the normalized distance `ndist` between the values `AV` and the nominal value of `uElement`. This distance is the norm of `NV`. Therefore, `ndist`  $\leq 1$  for values inside the uncertainty range of `uElement`, and `ndist`  $> 1$  for values outside the range. If `AV` is an array of values, then `ndist` is an array of normalized distances.

The robustness margins computed by `robstab` and `robgain` serve as bounds for the normalized distances in `ndist`. For example, if an uncertain system has a stability margin of 1.4, this system is stable for all uncertain element values whose normalized distance from the nominal is less than 1.4.

### Examples



## Uncertain Real Parameter with Symmetric Range

For uncertain real parameters whose range is symmetric about their nominal value, the normalized distance is intuitive, scaling linearly with the numerical difference from the uncertain real parameter's nominal value.

Create uncertain real parameters with a range that is symmetric about the nominal value, where each end point is 1 unit from the nominal. Points that lie inside the range are less than 1 unit from the nominal, while points that lie outside the range are greater than 1 unit from the nominal.

```
a = ureal('a',3,'range',[1 5]);
NV = actual2normalized(a,[1 3 5])
```

```
NV = 1×3
    -1.0000         0     1.0000
```

```
NV = actual2normalized(a,[2 4])
```

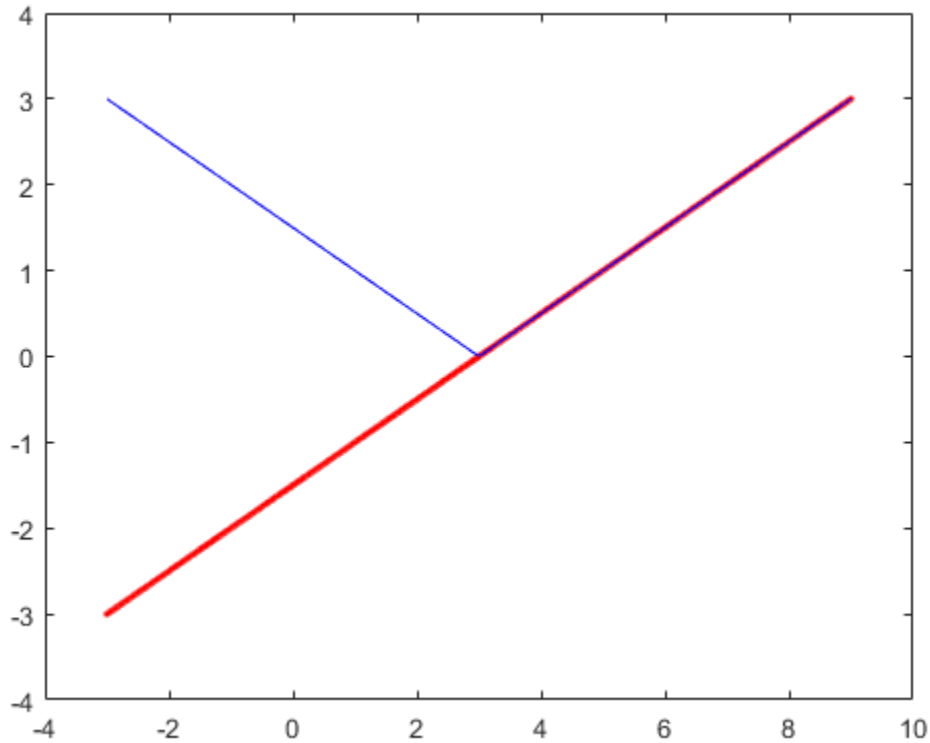
```
NV = 1×2
    -0.5000     0.5000
```

```
NV = actual2normalized(a,[0 6])
```

```
NV = 1×2
    -1.5000     1.5000
```

Plot the normalized values and normalized distance for several values.

```
values = linspace(-3,9,250);
[nv,ndist] = actual2normalized(a,values);
plot(values,nv,'r.',values,ndist,'b-')
```



## Uncertain Real Parameter with Nonsymmetric Range

Create a nonsymmetric parameter. The end points are 1 normalized unit from nominal, and the nominal is 0 normalized units from nominal. Moreover, points inside the range are less than 1 unit from nominal, and points outside the range are greater than 1 unit from nominal. However, the relationship between the normalized distance and numerical difference is nonlinear.

```
au = ureal('ua',4,'range',[1 5]);  
NV = actual2normalized(au,[1 4 5])
```

```
NV = 1×3
```

```
    -1     0     1
```

```
NV = actual2normalized(au,[2 4.5])
```

```
NV = 1×2
```

```
   -0.8000    0.4000
```

```
NV = actual2normalized(au,[0 6])
```

```
NV = 1×2
```

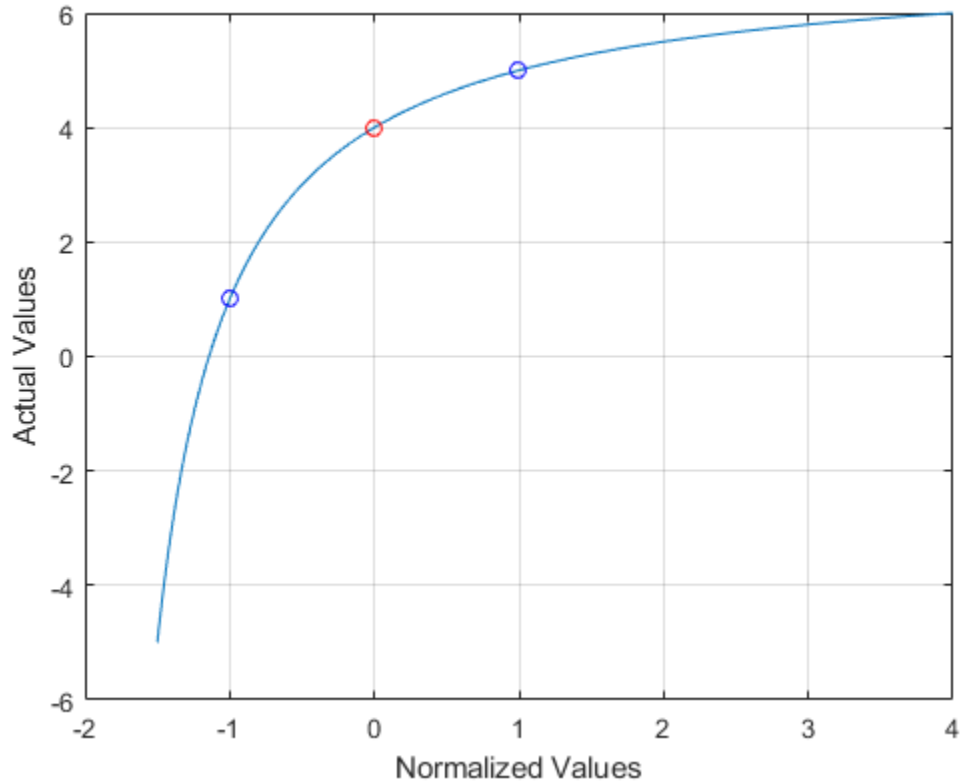
```
   -1.1429    4.0000
```

Graph the relationship between actual and normalized values. The relationship is very nonlinear.

```
AV = linspace(-5,6,250);
```

```
NV = actual2normalized(au,AV);
```

```
plot(NV,AV,0,au.NominalValue,'ro',-1,au.Range(1),'bo',1,au.Range(2),'bo')  
grid, xlabel('Normalized Values'), ylabel('Actual Values')
```



The red circle shows the nominal value (normalized value = 0). The blue circles show the values at the edges of the uncertainty range (normalized values = -1, 1).

## Algorithms

For details on the normalize distance, see “Normalizing Functions for Uncertain Elements”.

## See Also

`getLimits` | `normalized2actual` | `robgain` | `robstab`

**Introduced before R2006a**

## aff2pol

Convert affine parameter-dependent models to polytopic models

### Syntax

```
polysys = aff2pol(affsys)
```

### Description

`aff2pol` derives a polytopic representation `polysys` of the *affine* parameter-dependent system

$$E(p)\dot{x} = A(p)x + B(p)u$$

$$y = C(p)x + D(p)u$$

where  $p = (p_1, \dots, p_n)$  is a vector of uncertain or time-varying real parameters taking values in a box or a polytope. The description `affsys` of this system should be specified with `psys`.

The vertex systems of `polysys` are the instances of “Equation 1-1” on page 1-8 and “Equation 1-2” on page 1-8 at the vertices  $p_{ex}$  of the parameter range, i.e., the SYSTEM matrices

$$\begin{pmatrix} A(p_{ex}) + jE(p_{ex}) & B(p_{ex}) \\ C(p_{ex}) & D(p_{ex}) \end{pmatrix}$$

for all corners  $p_{ex}$  of the parameter box or all vertices  $p_{ex}$  of the polytope of parameter values.

### See Also

`psys` | `pvec` | `uss`

**Introduced before R2006a**

## augw

State-space or transfer function plant augmentation for use in weighted mixed-sensitivity  $H_\infty$  and  $H_2$  loopshaping design

## Syntax

$P = \text{AUGW}(G, W1, W2, W3)$

## Description

$P = \text{AUGW}(G, W1, W2, W3)$  computes a state-space model of an augmented LTI plant  $P(s)$  with weighting functions  $W_1(s)$ ,  $W_2(s)$ , and  $W_3(s)$  penalizing the error signal, control signal and output signal respectively (see block diagram) so that the closed-loop transfer function matrix is the weighted mixed sensitivity

$$Ty_1u_1 \triangleq \begin{bmatrix} W_1S \\ W_2R \\ W_3T \end{bmatrix}$$

where  $S$ ,  $R$  and  $T$  are given by

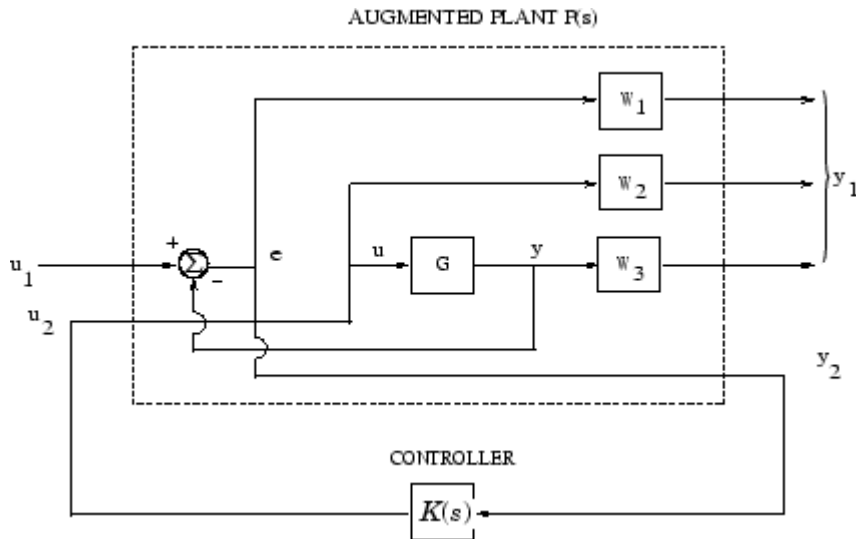
$$S = (I + GK)^{-1}$$

$$R = K(I + GK)^{-1}$$

$$T = GK(I + GK)^{-1}$$

The LTI systems  $S$  and  $T$  are called the *sensitivity* and *complementary sensitivity*, respectively.





## Plant Augmentation

For dimensional compatibility, each of the three weights  $W_1$ ,  $W_2$  and  $W_3$  must be either empty, a scalar (SISO) or have respective input dimensions  $N_y$ ,  $N_u$ , and  $N_y$  where  $G$  is  $N_y$ -by- $N_u$ . If one of the weights is not needed, you may simply assign an empty matrix  $[]$ ; e.g.,  $P = \text{augw}(G, W_1, [], W_3)$  is  $P(s)$  as in the “Algorithms” on page 1-13 section below, but without the second row (without the row containing  $W_2$ ).

## Examples

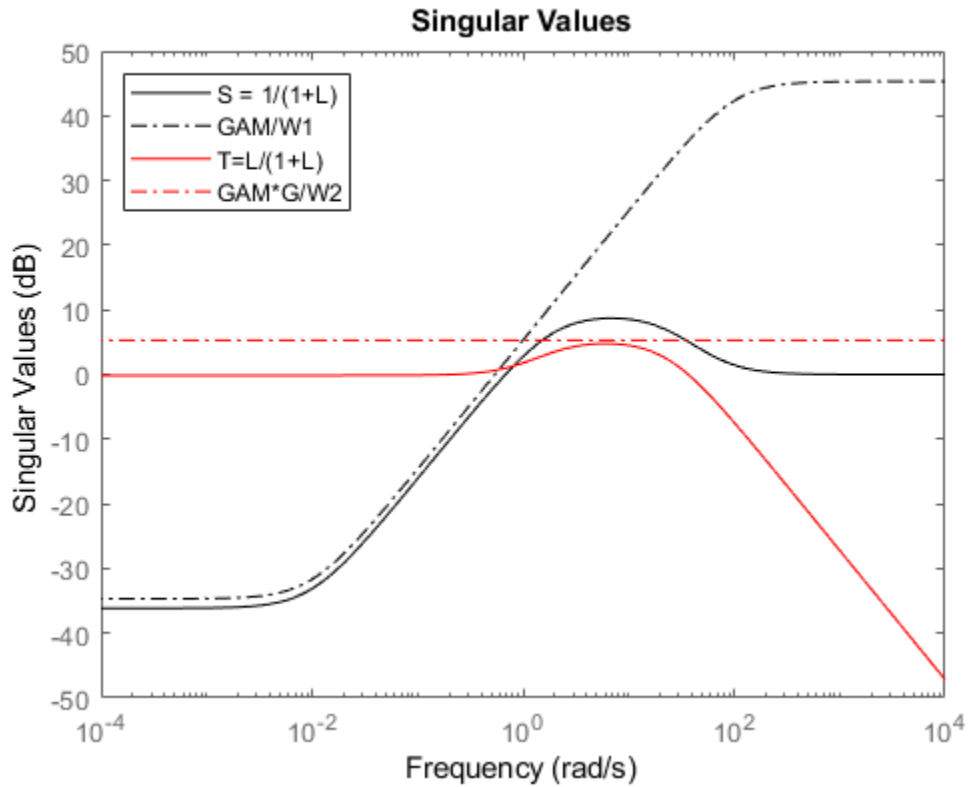
### Create Augmented Plant for H-Infinity Synthesis

```
s = zpk('s');
G = (s-1)/(s+1);
W1 = 0.1*(s+100)/(100*s+1);
W2 = 0.1;
W3 = [];
P = augw(G,W1,W2,W3);
```

```
[K,CL,GAM] = hinfsyn(P);
[K2,CL2,GAM2] = h2syn(P);
```

```
L = G*K;  
S = inv(1+L);  
T = 1-S;
```

```
sigma(S, 'k', GAM/W1, 'k-.', T, 'r', GAM*G/W2, 'r-.')  
legend('S = 1/(1+L)', 'GAM/W1', 'T=L/(1+L)', 'GAM*G/W2', 'Location', 'Northwest')
```



## Limitations

The transfer functions  $G$ ,  $W_1$ ,  $W_2$  and  $W_3$  must be proper, i.e., bounded as  $s \rightarrow \infty$  or, in the discrete-time case, as  $z \rightarrow \infty$ . Additionally,  $W_1$ ,  $W_2$  and  $W_3$  should be stable. The plant  $G$  should be stabilizable and detectable; else,  $P$  will not be stabilizable by any  $K$ .

## Algorithms

The augmented plant  $P(s)$  produced by is

$$P(s) = \left[ \begin{array}{c|c} W_1 & -W_1G \\ 0 & W_2 \\ 0 & W_3G \\ \hline I & -G \end{array} \right]$$

Partitioning is embedded via `P=mktito(P,NY,NU)`, which sets the `InputGroup` and `OutputGroup` properties of  $P$  as follows

```
[r,c]=size(P);
P.InputGroup = struct('U1',1:c-NU,'U2',c-NU+1:c);
P.OutputGroup = struct('Y1',1:r-NY,'Y2',r-NY+1:r);
```

## See Also

`h2syn` | `hinfo` | `mixsyn` | `mktito`

**Introduced before R2006a**

## balancmr

Balanced model truncation via square root method

### Syntax

```
GRED = balancmr(G)
```

```
GRED = balancmr(G,order)
```

```
[GRED,redinfo] = balancmr(G,key1,value1,...)
```

```
[GRED,redinfo] = balancmr(G,order,key1,value1,...)
```

### Description

`balancmr` returns a reduced order model `GRED` of `G` and a struct array `redinfo` containing the error bound of the reduced model and Hankel singular values of the original system.

The error bound is computed based on Hankel singular values of `G`. For a stable system these values indicate the respective state energy of the system. Hence, reduced order can be directly determined by examining the system Hankel singular values,  $\sigma_i$ .

With only one input argument `G`, the function will show a Hankel singular value plot of the original model and prompt for model order number to reduce.

This method guarantees an error bound on the infinity norm of the *additive error*  $\|G - GRED\|_\infty$  for well-conditioned model reduced problems [1]:

$$\|G - Gred\|_\infty \leq 2 \sum_{k=1}^n \sigma_k$$

This table describes input arguments for `balancmr`.

| Argument | Description   |
|----------|---|
| G        | LTI model to be reduced. Without any other inputs, <code>balancmr</code> will plot the Hankel singular values of G and prompt for reduced order |
| ORDER    | (Optional) Integer for the desired order of the reduced model, or optionally a vector packed with desired orders for batch runs                 |

A batch run of a serial of different reduced order models can be generated by specifying `order = x:y`, or a vector of positive integers. By default, all the anti-stable part of a system is kept, because from control stability point of view, getting rid of unstable state(s) is dangerous to model a system.

'*MaxError*' can be specified in the same fashion as an alternative for 'Order'. In this case, reduced order will be determined when the sum of the tails of the Hankel singular values reaches the '*MaxError*'.

This table lists the input arguments 'key' and its 'value'.

| Argument   | Value                                     | Description  |
|------------|---|--|
| 'MaxError' | Real number or vector of different errors | Reduce to achieve $H_\infty$ error. When present, ' <i>MaxError</i> ' overrides ORDER input. |

| Argument  | Value                         | Description  |
|-----------|-------------------------------|--|
| 'Weights' | {Wout,Win} cell array         | <p>Optional 1-by-2 cell array of LTI weights Wout (output) and Win (input). The weights must be stable, minimum phase and invertible. When you supply these weights, <code>balancmr</code> finds the reduced model that minimizes the Hankel norm of</p> $W_{out}^{-1} (G - G_{red}) W_{in}^{-1}.$ <p>You can use weighting functions to make the model reduction algorithm focus on frequency bands of interest. See:</p> <ul style="list-style-type: none"> <li>• “Reduction with Focus on Particular Frequency Band” on page 1-23</li> <li>• “Model Reduction With Frequency-Dependent Error Profile” on page 1-25</li> </ul> <p>As an alternative, you can use <code>balred</code> to focus model reduction on a particular frequency band without defining a weighting function. Using <code>balancmr</code> and providing your own weighting functions allows more precise control over the error profile.</p> <p>Default weights are both identity.</p> |
| 'Display' | 'on' or 'off'                 | Display Hankel singular plots (default 'off').   |
| 'Order'   | Integer, vector or cell array | Order of reduced model. Use only if not specified as 2nd argument.   |

This table describes output arguments.

| Argument | Description  |
|----------|--|
| GRED     | LTI reduced order model. Becomes multidimensional array when input is a serial of different model order array  |
| REDINFO  | A STRUCT array with three fields: <ul style="list-style-type: none"> <li>• REDINFO.ErrorBound (bound on <math>\ G-GRED\ _{\infty}</math>)</li> <li>• REDINFO.StabSV (Hankel SV of stable part of G)</li> <li>• REDINFO.UnstabSV (Hankel SV of unstable part of G)</li> </ul> |

G can be stable or unstable, continuous or discrete.

## Examples

### Choose Order of Reduced Model

If you do not specify any target order for the reduced model, `balancmr` displays the Hankel singular values of the model and prompts you to choose a reduced-model order.

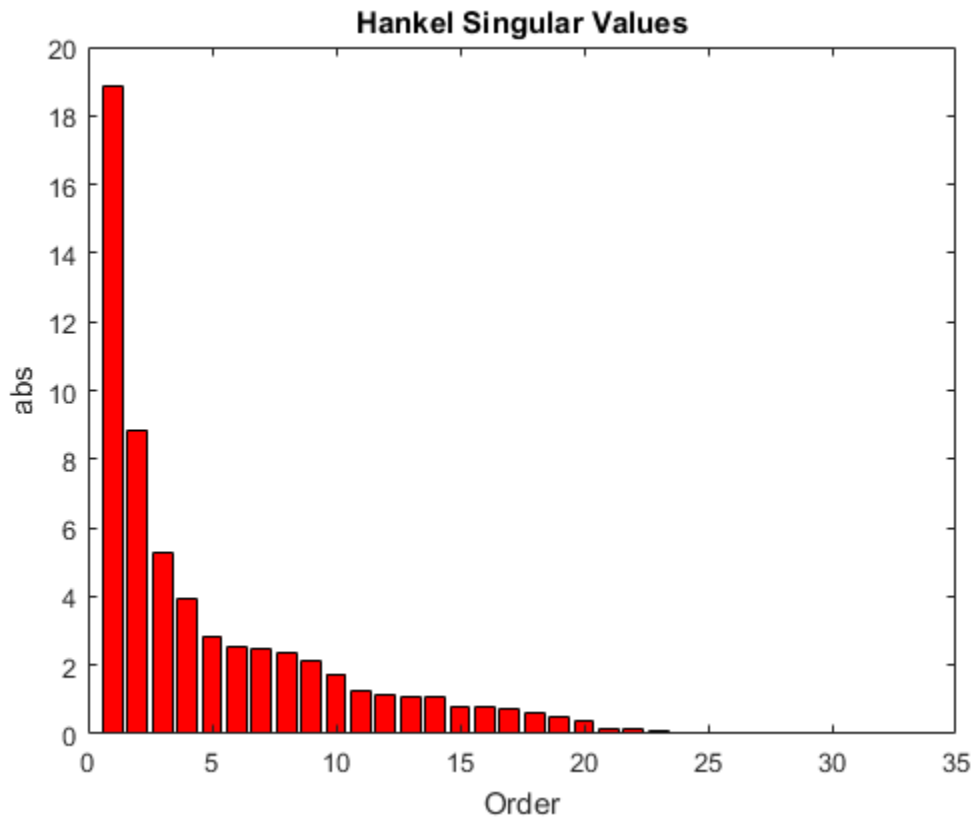
For this example, use a random 30th-order state-space model.

```
rng(1234, 'twister');    % fix random seed for example repeatability
G = rss(30,5,4);
```

```
G1 = balancmr(G)
```

```
Please enter the desired order: (>=0)
```

Examine the Hankel singular value plot.

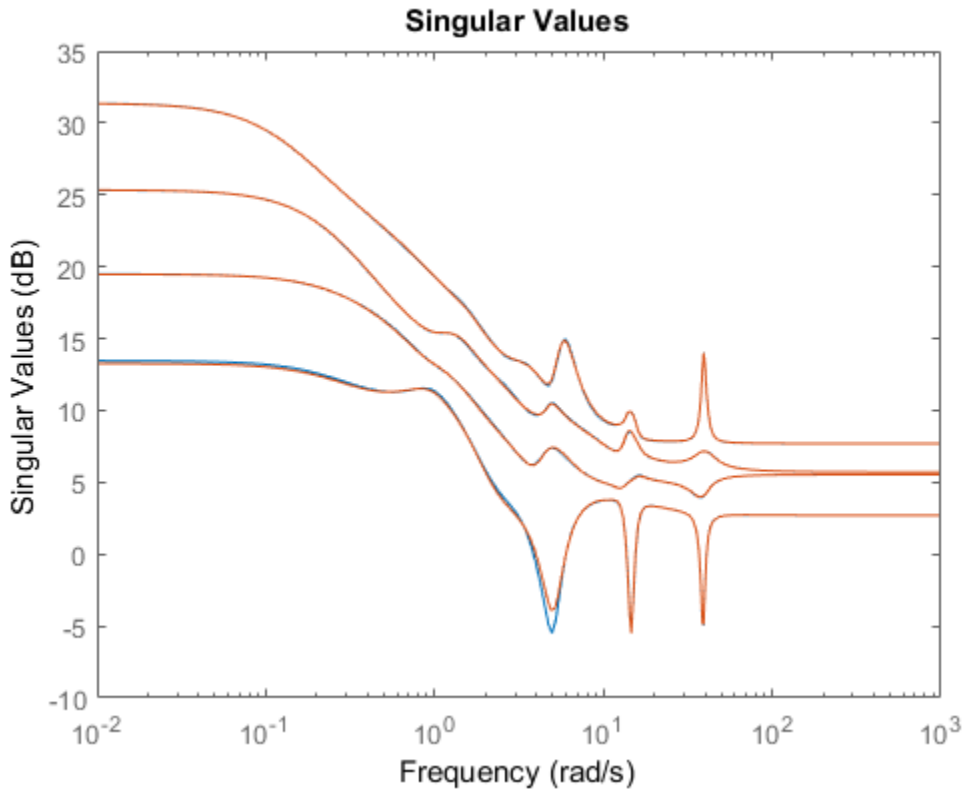


The plot shows that most of the energy of the system can be captured in a 20th-order approximation. In the command window, enter `20. balancmr` returns `G1`.

Examine the response of the original and reduced models.

```
sigma(G,G1)
```





The 20th-order approximation matches the dynamics of the original 30th-order model fairly well.

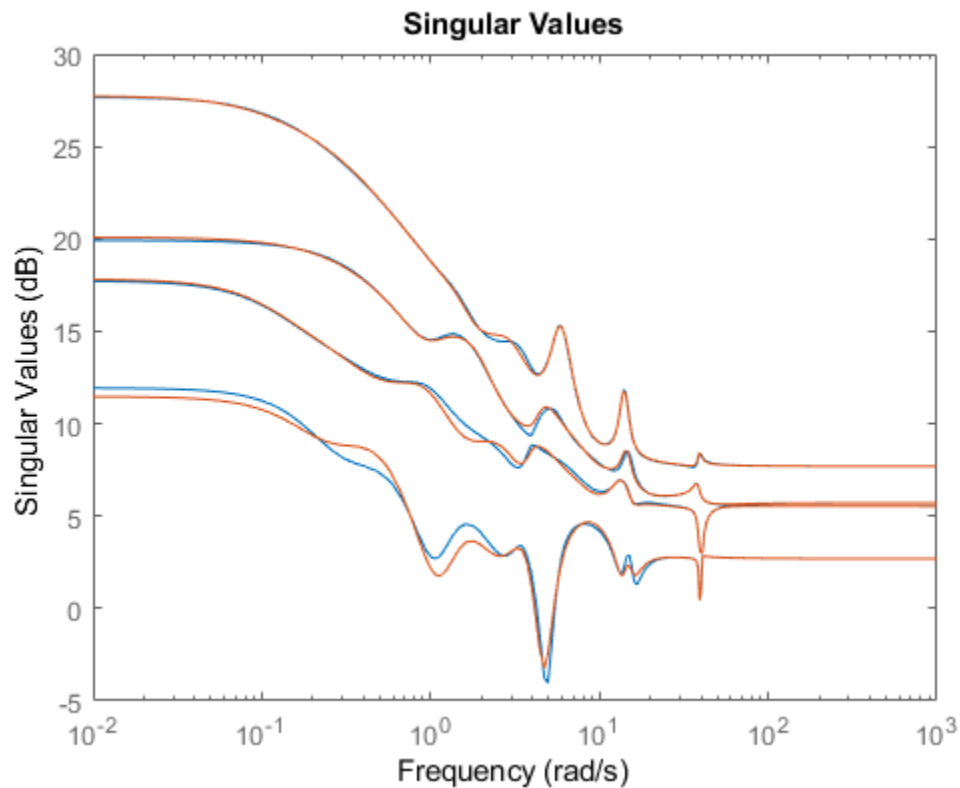
### Model Reduction to Specified Order

When you have particular target order or orders in mind, you can use `balancmr` to reduce a high-order model to those orders. For this example, use a random 30th-order state-space model.

```
rng(1234, 'twister');    % fix random seed for example repeatability
G = rss(30,5,4);
```

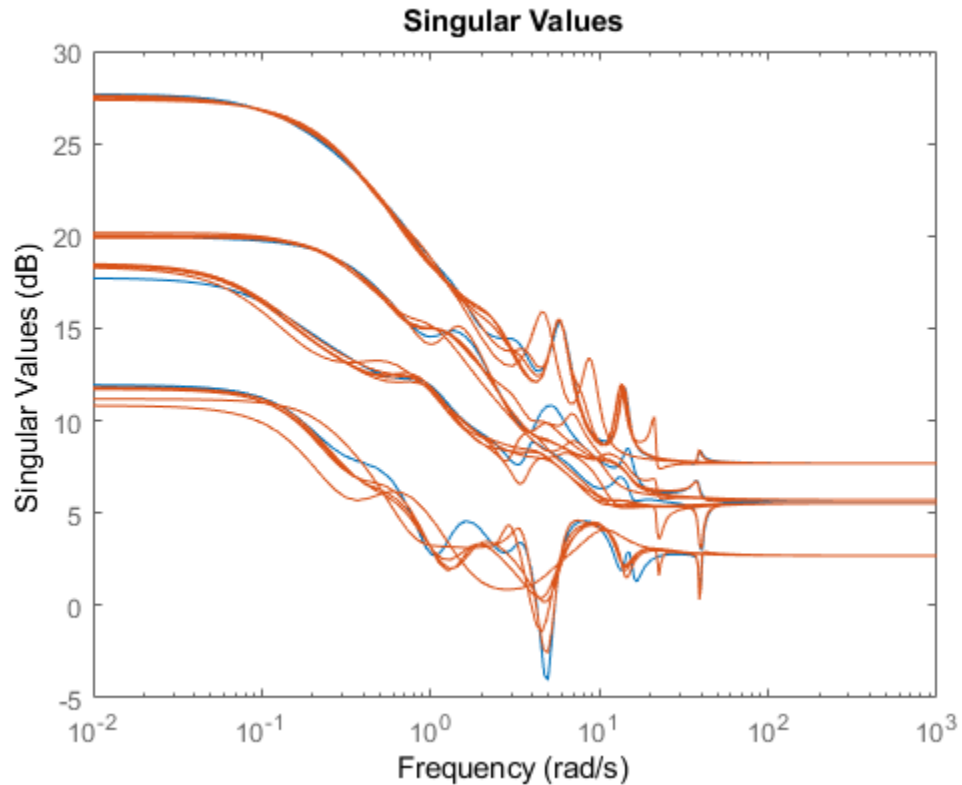
Use a scalar input argument to reduce the model to a single order. For example, compute a 20th-order approximation.

```
[G1,info1] = balancmr(G,20);  
sigma(G,G1)
```



Use a vector to generate several approximations. The following command returns an array of models of even orders from 10 to 18.

```
[G2,info2] = balancmr(G,[10:2:18]);  
sigma(G,G2)
```



### Model Reduction to Specified Maximum Error

Obtain the lowest-order approximation such that the sum of the Hankel singular values of the truncated states does not exceed a specified value. For this example, use a random 30th-order state-space model.

```
rng(1234, 'twister');    % fix random seed for example repeatability
G = rss(30,5,4);
```

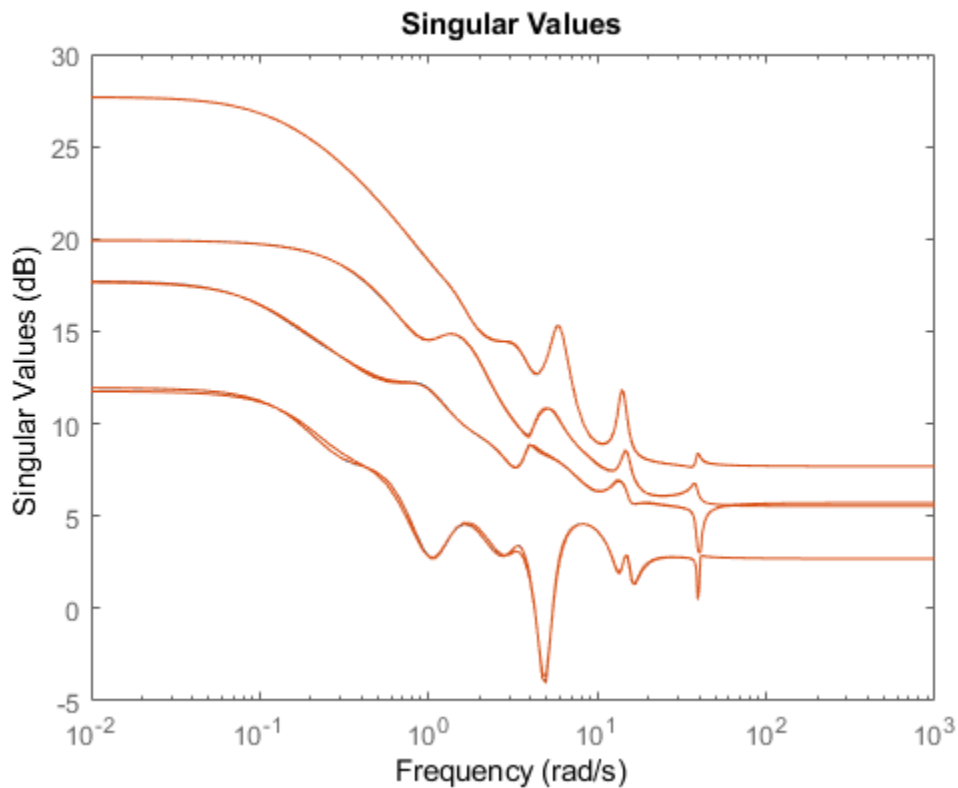
Compute two approximate models, one for which the error does not exceed 0.1, and a second for which the error does not exceed 0.5. To do so, provide these values in an array. `balancmr` returns an array of approximate models.

```
Gr = balancmr(G, 'MaxError', [0.1 0.5]);  
size(Gr)
```

2x1 array of state-space models.  
Each model has 5 outputs, 4 inputs, and between 24 and 26 states.

Examine the results.

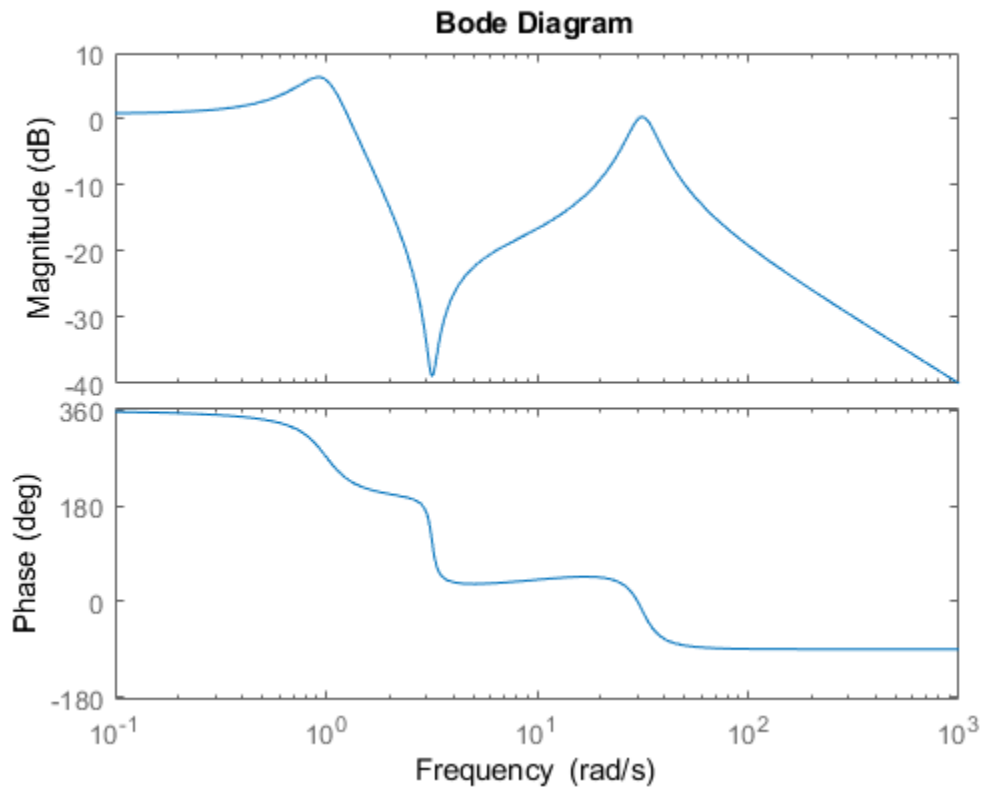
```
sigma(G,Gr)
```



### Reduction with Focus on Particular Frequency Band

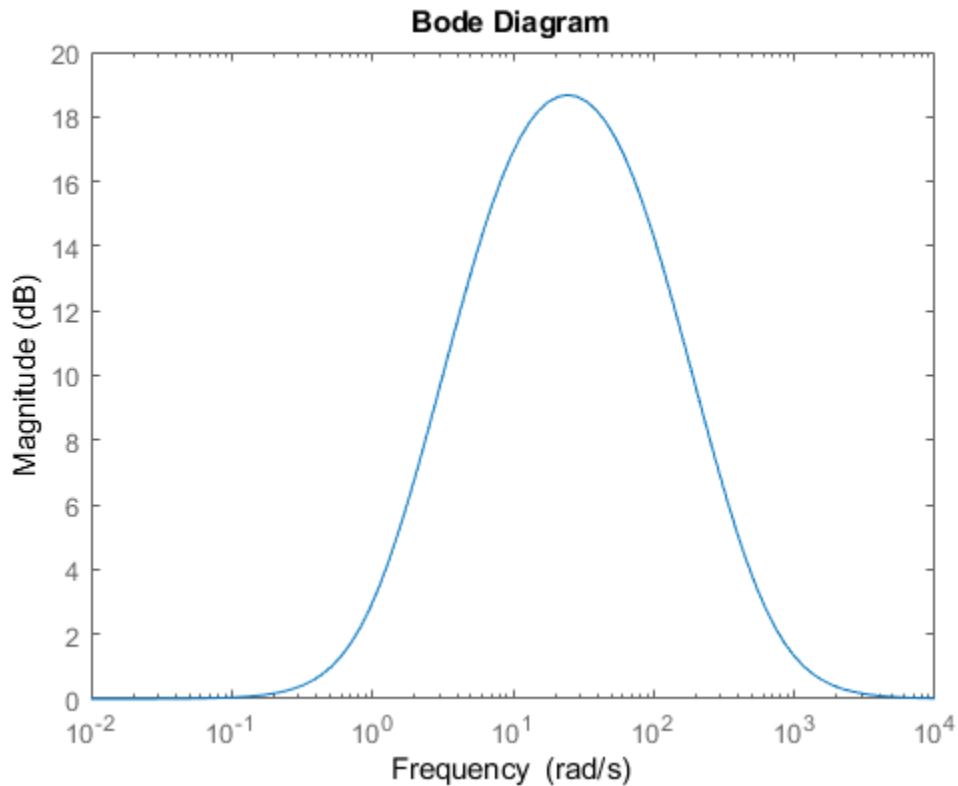
Reduce a 4th-order system to a second-order approximation with emphasis on the frequency band 10 rad/s - 100 rad/s. Consider the following system.

```
sys = tf(1,[1 0.5 1]) + tf(100*[1/10 1],[1 10 1000]);  
bode(sys)
```



To focus the model-reduction algorithm on the higher-frequency dynamics, specify a function with a bandpass profile.

```
s = tf('s');  
w1 = (s+1)/(s/10+1)/(s/60+1)*(s/600+1);  
bodemag(w1)
```



The plot confirms that the weighting function  $w1$  has the desired profile, peaking between 10 rad/s and 100 rad/s. To perform the reduction, specify the inverse of this profile as the output weight, using the 'Weights' option of `balancmr`.

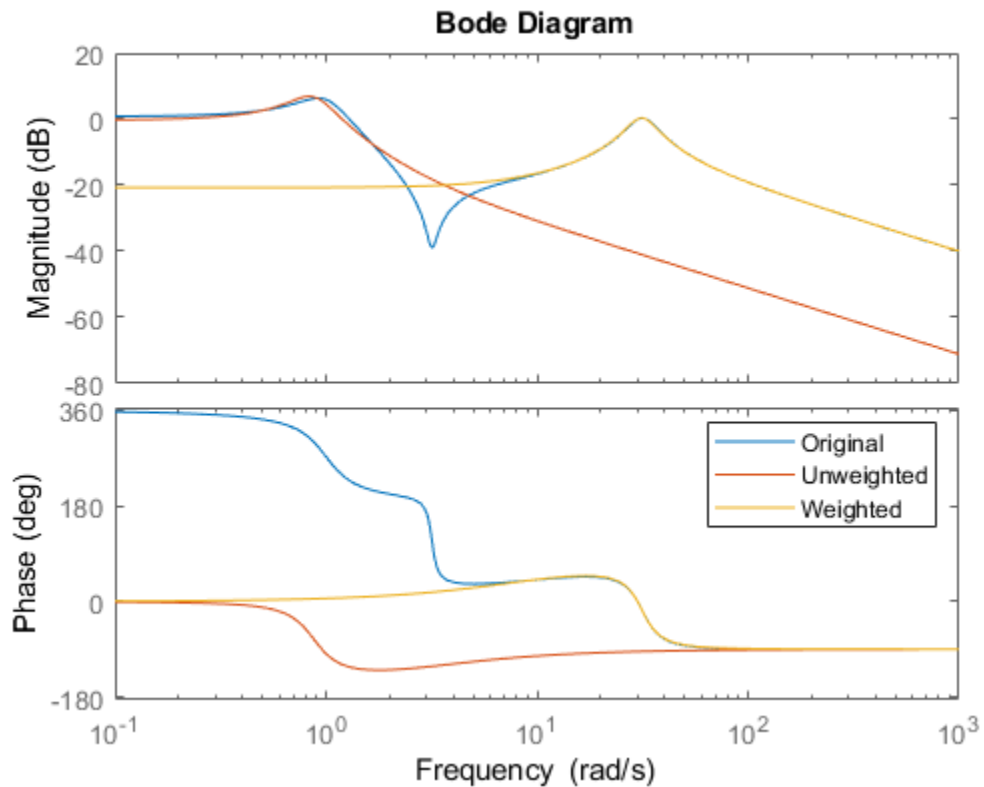
```
weight = {1/w1,1};  
wrsys = balancmr(sys,2,'Weights',weight);
```

Compare the result with a second-order model obtained without the weighting.

```

rsys = balancmr(sys,2);
bode(sys,rsys,wrsys)
legend('Original','Unweighted','Weighted')

```



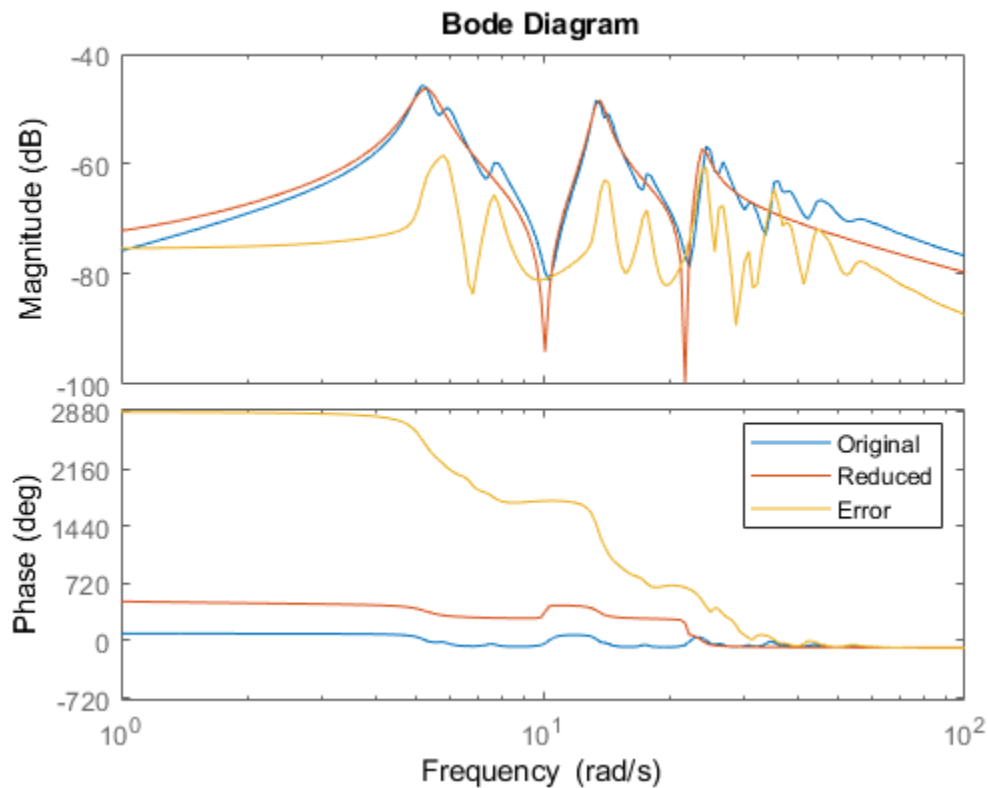
The model obtained with the weighting function provides a better match for the dynamics in the frequency band 10 rad/s - 100 rad/s.

### Model Reduction With Frequency-Dependent Error Profile

Use a weighting function to control the frequency dependence of the error between the original and reduced models.

For this example, load a 48-state SISO model and reduce it to 6th order.

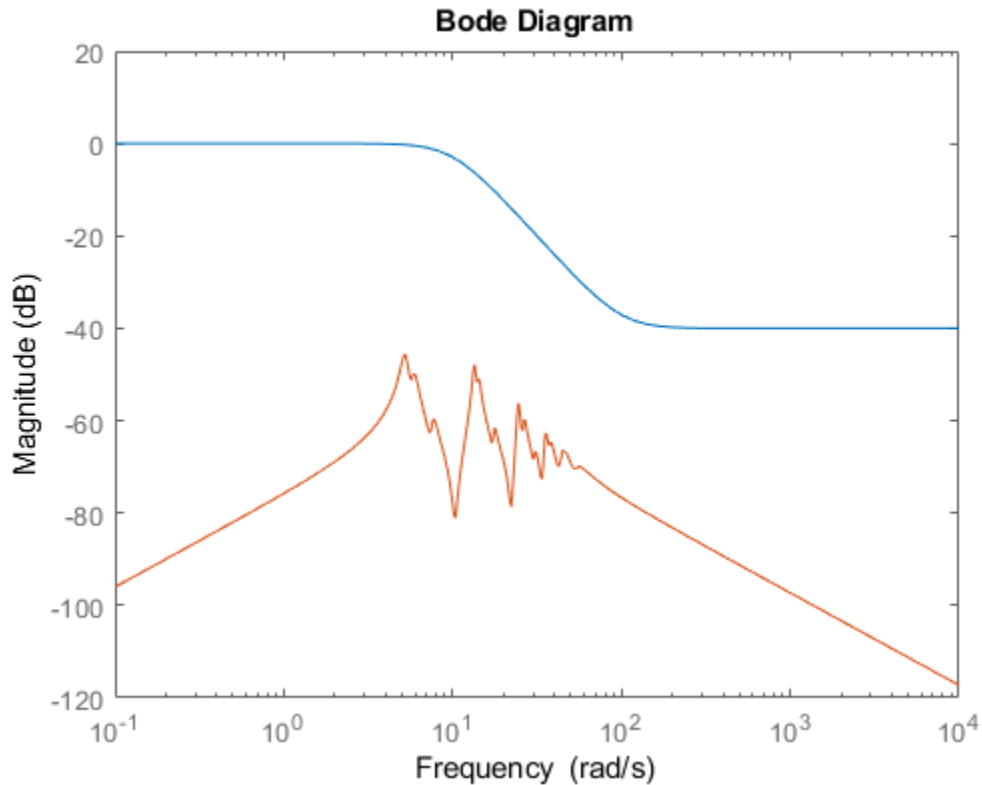
```
load(fullfile(matlabroot,'examples','robust','balancmrData.mat'),'bplant')
bplant6 = balancmr(bplant,6);
bode(bplant,bplant6,bplant-bplant6,logspace(0,2,200))
legend('Original','Reduced','Error')
```



The error is fairly uniformly distributed in frequency. Create a weighting function that allows for a larger error at frequencies below 100 rad/s. In this case, use a biproper function that has unit gain at low frequency, but drops to -40 dB at higher frequencies.

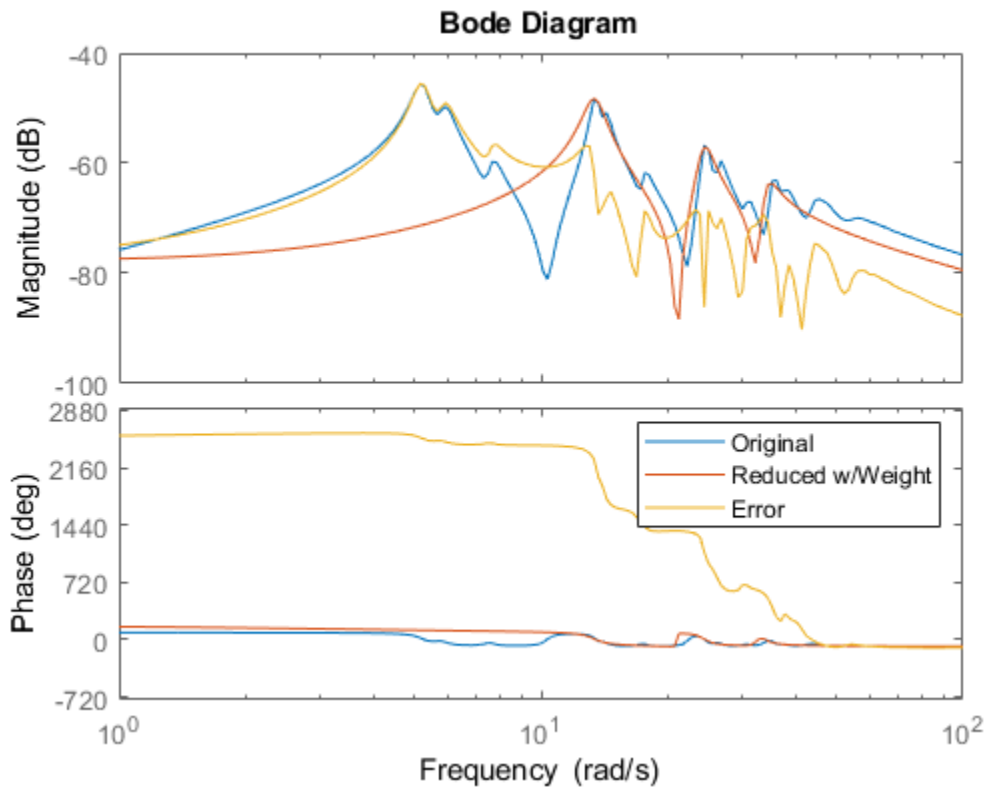
```
W = tf(0.01*[1 1.4e2 1e4],[1 14 100]);
bodemag(W,bplant)
```





Reduce the model to 6th order again, using this weighting function. Because `bplant` is a SISO model, you can use the function as either input or output weight.

```
bplant6W = balancmr(bplant,6,'Weights',{W,1});
bode(bplant,bplant6W,bplant-bplant6W,logspace(0,2,200))
legend('Original','Reduced w/Weight','Error')
```



The error is now larger at low frequencies, with a correspondingly better match at high frequencies between the original and reduced models.

## Algorithms

Given a state space  $(A,B,C,D)$  of a system and  $k$ , the desired reduced order, the following steps will produce a similarity transformation to truncate the original state-space system to the  $k^{\text{th}}$  order reduced model.

- 1 Find the SVD of the controllability and observability grammians

$$P = U_p \Sigma_p V_p^T$$

$$Q = U_q \Sigma_q V_q^T$$

- 2 Find the square root of the grammians (left/right eigenvectors)

$$L_p = U_p \Sigma_p^{1/2}$$

$$L_o = U_q \Sigma_q^{1/2}$$

- 3 Find the SVD of  $(L_o^T L_p)$

$$L_o^T L_p = U \Sigma V^T$$

- 4 Then the left and right transformation for the final  $k^{\text{th}}$  order reduced model is

$$S_{L,BIG} = L_o U(:,1:k) \Sigma(1;k,1:k)^{-1/2}$$

$$S_{R,BIG} = L_p V(:,1:k) \Sigma(1;k,1:k)^{-1/2}$$

- 5 Finally,

$$\begin{bmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{bmatrix} = \begin{bmatrix} S_{L,BIG}^T A S_{R,BIG} & S_{L,BIG}^T B \\ C S_{R,BIG} & D \end{bmatrix}$$

The proof of the square root balance truncation algorithm can be found in [2].

## References

- [1] Glover, K., "All Optimal Hankel Norm Approximation of Linear Multivariable Systems, and Their  $L_\mu$ -error Bounds," *Int. J. Control*, Vol. 39, No. 6, 1984, p. 1145-1193
- [2] Safonov, M.G., and R.Y. Chiang, "A Schur Method for Balanced Model Reduction," *IEEE Trans. on Automat. Contr.*, Vol. 34, No. 7, July 1989, p. 729-733

## See Also

balred | bstmr | hankelmr | hankelsv | ncfmr | reduce | schurmr

Introduced before R2006a

## **bilin**

Multivariable bilinear transform of frequency ( $s$  or  $z$ )

## **Syntax**

`GT = bilin(G,VERS,METHOD,AUG)`

## **Description**

`bilin` computes the effect on a system of the frequency-variable substitution,

$$s = \frac{\alpha z + \delta}{\gamma z + \beta}$$

The variable `VERS` denotes the transformation direction:

`VERS= 1`, forward transform ( $s \rightarrow z$ ) or ( $s \rightarrow \tilde{s}$ ).

`VERS=-1`, reverse transform ( $z \rightarrow s$ ) or ( $\tilde{s} \rightarrow s$ ).

This transformation maps lines and circles to circles and lines in the complex plane. People often use this transformation to do sampled-data control system design [1] or, in general, to do shifting of  $j\omega$  modes [2], [3], [4].

`Bilin` computes several state-space bilinear transformations such as backward rectangular, etc., based on the `METHOD` you select

### Bilinear Transform Types

| Method    | Type of bilinear transform   |
|-----------|--|
| 'BwdRec ' | backward rectangular:<br><br>$s = \frac{z-1}{Tz}$ AUG = T, the sample time.  |
| 'FwdRec ' | forward rectangular:<br><br>$s = \frac{z-1}{T}$ AUG = T, the sample time.  |
| 'S_Tust ' | shifted Tustin:<br><br>$s = \frac{2}{T} \left( \frac{z-1}{\frac{z}{h} + 1} \right)$ AUG = [T h], is the "shift" coefficient.   |
| 'S_ftjw'  | shifted $j\omega$ -axis, bilinear pole-shifting, continuous-time to continuous-time:<br><br>$s = \frac{\tilde{s} + p_1}{1 + \tilde{s} / p_2}$ AUG = [ $p_2$ $p_1$ ]. |

| Method   | Type of bilinear transform   |
|----------|--|
| 'G_Bili' | <p>METHOD = 'G_Bili', general bilinear, continuous-time to continuous-time:</p> $s = \frac{\alpha\tilde{s} + \delta}{\gamma\tilde{s} + \beta}$ <p>AUG = <math>[\alpha \ \beta \ \gamma \ \delta]</math>.</p> |

## Examples

### Tustin Continuous s-Plane to Discrete z-Plane Transforms

Consider the following continuous-time plant (sampled at 20 Hz):

$$A = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}; T_s = 0.05$$

Following is an example of four common “continuous to discrete” bilinear transformations for the sampled plant:

```

A = [-1 1; 0 -2];
B = [1 0; 1 1];
C = [1 0; 0 1];
D = [0 0; 0 0];
sys = ss(A,B,C,D); % ANALOG
Ts = 0.05; % sample time
syst = c2d(sys,Ts,'tustin'); % Tustin
sysp = c2d(sys,Ts,'prewarp',40); % Pre-warped Tustin
sysb = bilin(sys,1,'BwdRec',Ts); % Backward Rectangular
sysf = bilin(sys,1,'FwdRec',Ts); % Forward Rectangular

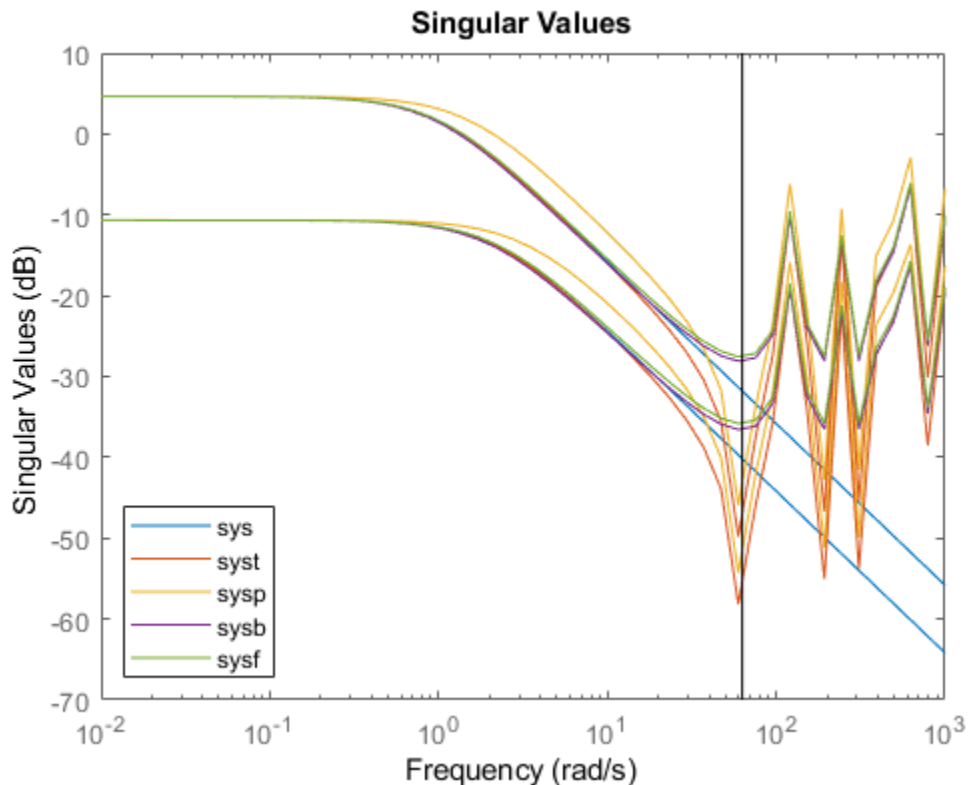
```

Plot the response of the continuous-time plant and the transformed discrete-time plants.

```

w = logspace(-2,3,50); % frequencies to plot
sigma(sys,syst,sysp,sysb,sysf,w);
legend('sys','syst','sysp','sysb','sysf','Location','SouthWest')

```



## Bilinear continuous to continuous pole-shifting

Design an H mixed-sensitivity controller for the ACC Benchmark plant

$$G(s) = \frac{1}{s^2(s^2 + 2)}$$

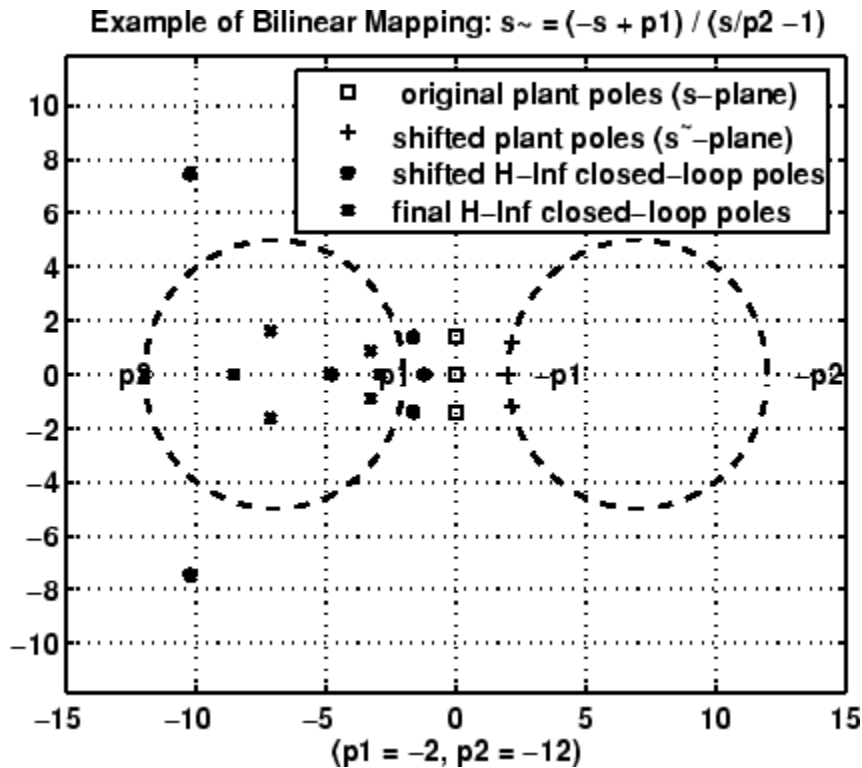
such that all closed-loop poles lie inside a circle in the left half of the s-plane whose diameter lies on between points [p1,p2]=[-12,-2]:

```
p1=-12; p2=-2; s=zpk('s');
G=ss(1/(s^2*(s^2+2)));           % original unshifted plant
```

```
Gt=bilin(G,1,'Sft_jw',[p1 p2]); % bilinear pole shifted plant Gt
Kt=mixsyn(Gt,1,[],1); % bilinear pole shifted controller
K =bilin(Kt,-1,'Sft_jw',[p1 p2]); % final controller K
```

As shown in the following figure, closed-loop poles are placed in the left circle [p1 p2]. The shifted plant, which has its non-stable poles shifted to the inside the right circle, is

$$G_t(s) = 4.765 \times 10^{-5} \frac{(s-12)^4}{(s-2)^2(s^2 - 4.274s + 5.918)}$$



'S\_ftjw' final closed-loop poles are inside the left [p1,p2] circle



## Algorithms

bilin employs the state-space formulae in [3]:

$$\left[ \begin{array}{c|c} A_b & B_b \\ \hline C_b & D_b \end{array} \right] = \left[ \begin{array}{c|c} (\beta A - \delta I)(\alpha I + \gamma A)^{-1} & (\alpha\beta - \gamma\delta)(\alpha I - \gamma A)^{-1} B \\ \hline C(\alpha I - \gamma A)^{-1} & D + \gamma C(\alpha I - \gamma A)^{-1} B \end{array} \right]$$

## References

- [1] Franklin, G.F., and J.D. Powell, *Digital Control of Dynamics System*, Addison-Wesley, 1980.
- [2] Safonov, M.G., R.Y. Chiang, and H. Flashner, " $H_\infty$  Control Synthesis for a Large Space Structure," *AIAA J. Guidance, Control and Dynamics*, 14, 3, p. 513-520, May/June 1991.
- [3] Safonov, M.G., "Imaginary-Axis Zeros in Multivariable  $H_\infty$  Optimal Control", in R.F. Curtain (editor), *Modelling, Robustness and Sensitivity Reduction in Control Systems*, p. 71-81, Springer-Varlet, Berlin, 1987.
- [4] Chiang, R.Y., and M.G. Safonov, " $H_\infty$  Synthesis using a Bilinear Pole Shifting Transform," *AIAA, J. Guidance, Control and Dynamics*, vol. 15, no. 5, p. 1111-1117, September-October 1992.

## See Also

c2d | d2c | sectf

**Introduced before R2006a**

## bstmr

Balanced stochastic model truncation (BST) via Schur method

### Syntax

```
GRED = bstmr(G)
```

```
GRED = bstmr(G,order)
```

```
[GRED,redinfo] = bstmr(G,key1,value1,...)
```

```
[GRED,redinfo] = bstmr(G,order,key1,value1,...)
```

### Description

`bstmr` returns a reduced order model `GRED` of `G` and a struct array `redinfo` containing the error bound of the reduced model and Hankel singular values of the *phase matrix* of the original system [2].

The error bound is computed based on Hankel singular values of the phase matrix of `G`. For a stable system these values indicate the respective state energy of the system. Hence, reduced order can be directly determined by examining these values.

With only one input argument `G`, the function will show a Hankel singular value plot of the phase matrix of `G` and prompt for model order number to reduce.

This method guarantees an error bound on the infinity norm of the *multiplicative*  $\|GRED-1(G-GRED)\|_\infty$  or *relative error*  $\|G^{-1}(G-GRED)\|_\infty$  for well-conditioned model reduction problems [1]:

$$\|G^{-1}(G-Gred)\|_\infty \leq \prod_{k=1}^n \left(1 + 2\sigma_k(\sqrt{1 + \sigma_k^2} + \sigma_k)\right) - 1$$

This table describes input arguments for `bstmr`.

| Argument | Description  |
|----------|--|
| G        | LTI model to be reduced (without any other inputs will plot its Hankel singular values and prompt for reduced order) |
| ORDER    | (Optional) an integer for the desired order of the reduced model, or a vector of desired orders for batch runs       |

A batch run of a serial of different reduced order models can be generated by specifying `order = x:y`, or a vector of integers. By default, all the anti-stable part of a system is kept, because from control stability point of view, getting rid of unstable state(s) is dangerous to model a system.

'*MaxError*' can be specified in the same fashion as an alternative for 'ORDER'. In this case, reduced order will be determined when the accumulated product of Hankel singular values shown in the above equation reaches the '*MaxError*'.

| Argument            | Value                                     | Description   |
|---------------------|---|---|
| ' <i>MaxError</i> ' | Real number or vector of different errors | Reduce to achieve $H_\infty$ error.<br>When present, ' <i>MaxError</i> ' overrides ORDER input. |
| ' <i>Display</i> '  | 'on' or 'off'                             | Display Hankel singular plots (default 'off').  |
| ' <i>Order</i> '    | Integer, vector or cell array             | Order of reduced model. Use only if not specified as 2nd argument.                              |

This table describes output arguments.

| Argument | Description  |
|----------|--|
| GRED     | LTI reduced order model. Become multi-dimension array when input is a serial of different model order array.   |
| REDINFO  | A STRUCT array with three fields: <ul style="list-style-type: none"> <li>REDINFO.ErrorBound (bound on <math>\ G^{-1}(G-GRED)\ _\infty</math>)</li> <li>REDINFO.StabSV (Hankel SV of stable part of G)</li> <li>REDINFO.UnstabSV (Hankel SV of unstable part of G)</li> </ul> |

G can be stable or unstable, continuous or discrete.

## Examples

Given a continuous or discrete, stable or unstable system,  $G$ , the following commands can get a set of reduced order models based on your selections:

```
rng(1234, 'twister');
G = rss(30,5,4);
G.D = zeros(5,4);
[g1, redinfo1] = bstmr(G); % display Hankel SV plot
                        % and prompt for order (try 15:20)
[g2, redinfo2] = bstmr(G,20);
[g3, redinfo3] = bstmr(G,[10:2:18]);
[g4, redinfo4] = bstmr(G,'MaxError',[0.01, 0.05]);
for i = 1:4
    figure(i)
    eval(['sigma(G,g' num2str(i) ');']);
end
```

## Algorithms

Given a state space  $(A,B,C,D)$  of a system and  $k$ , the desired reduced order, the following steps will produce a similarity transformation to truncate the original state-space system to the  $k^{\text{th}}$  order reduced model.

- 1 Find the controllability grammian  $P$  and observability grammian  $Q$  of the left *spectral factor*  $\Phi = \Gamma(\sigma)\Gamma^*(-\sigma) = \Omega^*(-\sigma)\Omega(\sigma)$  by solving the following Lyapunov and Riccati equations

$$AP + PA^T + BB^T = 0$$

$$B_W = PC^T + BD^T$$

$$QA + A^T Q + (QB_W - C^T)(-DD^T)(QB_W - C^T)^T = 0$$

- 2 Find the Schur decomposition for  $PQ$  in both ascending and descending order, respectively,

$$V_A^T P Q V_A = \begin{bmatrix} \lambda_1 & \cdots & \cdots \\ 0 & \cdots & \cdots \\ 0 & 0 & \lambda_n \end{bmatrix}$$

$$V_D^T P Q V_D = \begin{bmatrix} \lambda_n & \cdots & \cdots \\ 0 & \cdots & \cdots \\ 0 & 0 & \lambda_1 \end{bmatrix}$$

- 3 Find the left/right orthonormal eigen-bases of  $PQ$  associated with the  $k^{\text{th}}$  big Hankel singular values of the all-pass phase matrix  $(W^*(s))^{-1}G(s)$ .

$k$

$$V_A = [V_{R,SMALL}, \overbrace{V_{L,BIG}}^k]$$

$$V_D = [\overbrace{V_{R,BIG}}^k, V_{L,SMALL}]$$

- 4 Find the SVD of  $(V_{L,BIG}^T V_{R,BIG}) = U \Sigma \zeta^T$   
 5 Form the left/right transformation for the final  $k^{\text{th}}$  order reduced model

$$S_{L,BIG} = V_{L,BIG} U \Sigma(1:k, 1:k)^{-1/2}$$

$$S_{R,BIG} = V_{R,BIG} \zeta \Sigma(1:k, 1:k)^{-1/2}$$

- 6 Finally,

$$\left[ \begin{array}{c|c} \hat{A} & \hat{B} \\ \hline \hat{C} & \hat{D} \end{array} \right] = \left[ \begin{array}{c|c} S_{L,BIG}^T A S_{R,BIG} & S_{L,BIG}^T B \\ \hline C S_{R,BIG} & D \end{array} \right]$$

The proof of the Schur BST algorithm can be found in [1].

---

**Note** The BST model reduction theory requires that the original model  $D$  matrix be full rank, for otherwise the Riccati solver fails. For any problem with strictly proper model, you can shift the  $j\omega$ -axis via `bilin` such that BST/REM approximation can be achieved up to a particular frequency range of interests. Alternatively, you can attach a small but full rank  $D$  matrix to the original problem but remove the  $D$  matrix of the reduced order model afterwards. As long as the size of  $D$  matrix is insignificant inside the control

bandwidth, the reduced order model should be fairly close to the true model. By default, the `bstmr` program will assign a full rank  $D$  matrix scaled by 0.001 of the minimum eigenvalue of the original model, if its  $D$  matrix is not full rank to begin with. This serves the purpose for most problems if user does not want to go through the trouble of model pretransformation.

---

## References

- [1] Zhou, K., "Frequency-weighted model reduction with  $L^\infty$  error bounds," *Syst. Contr. Lett.*, Vol. 21, 115-125, 1993.
- [2] Safonov, M.G., and R.Y. Chiang, "Model Reduction for Robust Control: A Schur Relative Error Method," *International J. of Adaptive Control and Signal Processing*, Vol. 2, p. 259-272, 1988.

## See Also

`balancmr` | `hanke_lmr` | `hanke_lsv` | `ncfmr` | `reduce` | `schurmr`

**Introduced before R2006a**

# complexify

Replace `ureal` atoms by summations of `ureal` and `ucomplex` (or `ultidyn`) atoms

## Syntax

```
MC = complexify(M,alpha)
```

```
MC = complexify(M,alpha,'ultidyn')
```

## Description

The command `complexify` replaces `ureal` atoms with sums of `ureal` and `ucomplex` atoms using `usubs`. Optionally, the sum can consist of a `ureal` and `ultidyn` atom.

`complexify` is used to improve the conditioning of robust stability calculations (`robstab`) for situations when there are predominantly `ureal` uncertain elements.

`MC = complexify(M,alpha)` results in each `ureal` atom in `MC` having the same `Name` and `NominalValue` as the corresponding `ureal` atom in `M`. If `Range` is the range of one `ureal` atom from `M`, then the range of the corresponding `ureal` atom in `MC` is

$$[\text{Range}(1)+\alpha*\text{diff}(\text{Range})/2 \quad \text{Range}(2)-\alpha*\text{diff}(\text{Range})/2]$$

The net effect is that the same real range is covered with a real and complex uncertainty. The real parameter range is reduced by equal amounts at each end, and `alpha` represents (in a relative sense) the reduction in the total range. The `ucomplex` atom will add this reduction in range back into `MC`, but as a ball with real and imaginary parts.

The `ucomplex` atom has `NominalValue` of 0, and `Radius` equal to `alpha*diff(Range)`. Its name is the name of the original `ureal` atom, appended with the characters `'_cmpxfy'`.

`MC = complexify(M,alpha,'ultidyn')` is the same, except that gain-bounded `ultidyn` atoms are used instead of `ucomplex` atoms. The `ultidyn` atom has its `Bound` equal to `alpha*diff(Range)`.

## Examples

### Complexified Uncertain Parameter

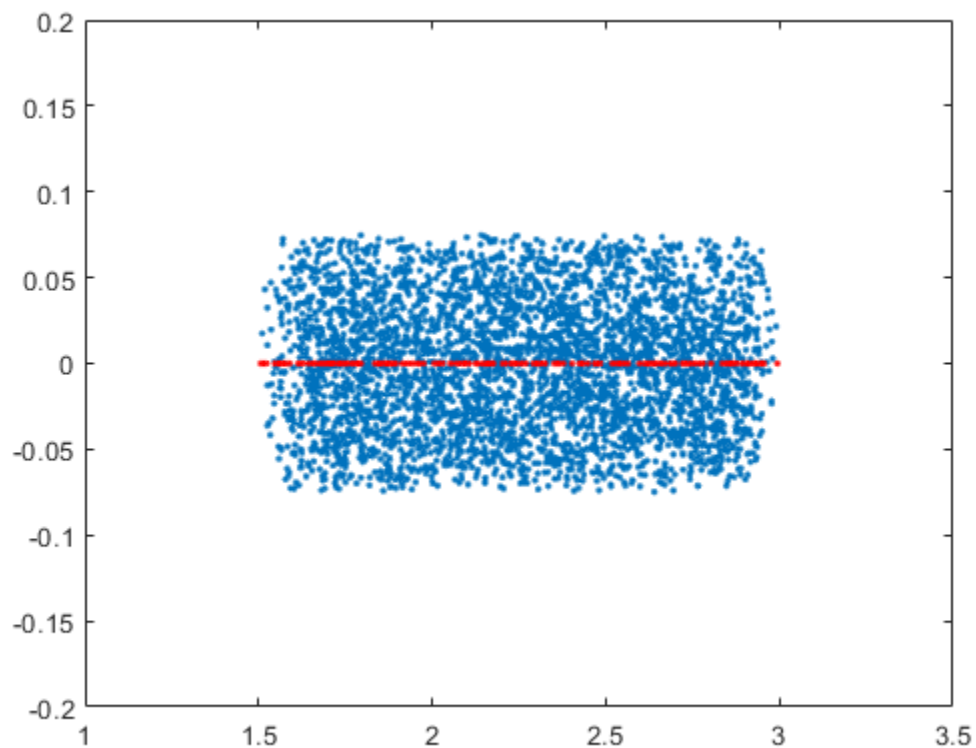
To illustrate complexification, create an uncertain real parameter, cast it to an uncertain matrix, and apply a 10% complexification.

```
a = umat(ureal('a',2.25,'Range',[1.5 3]));  
b = complexify(a,.1);  
as = usample(a,200);  
bs = usample(b,4000);
```

Make a scatter plot of the values that the complexified matrix (scalar) can take, as well as the values of the original uncertain real parameter.

```
plot(real(bs(:)),imag(bs(:)),'.',real(as(:)),imag(as(:)),'.r.')  
axis([1 3.5 -0.2 0.2])
```





- Getting Reliable Estimates of Robustness Margins

## See Also

`icomplexify` | `robstab`

## Topics

Getting Reliable Estimates of Robustness Margins

**Introduced in R2007a**

## cmsclsyn

Approximately solve constant-matrix, upper bound  $\mu$ -synthesis problem

### Syntax

```
[QOPT,BND] = cmsclsyn(R,U,V,BlockStructure);
```

```
[QOPT,BND] = cmsclsyn(R,U,V,BlockStructure,opt);
```

```
[QOPT,BND] = cmsclsyn(R,U,V,BlockStructure,opt,qinit);
```

```
[QOPT,BND] = cmsclsyn(R,U,V,BlockStructure,opt,'random',N)
```

### Description

`cmsclsyn` approximately solves the constant-matrix, upper bound  $\mu$ -synthesis problem by minimization,

$$\min_{Q \in \mathbf{C}^{r \times t}} \mu_{\Delta}(R + UQV)$$

for given matrices  $R \in \mathbf{C}^{n \times m}$ ,  $U \in \mathbf{C}^{n \times r}$ ,  $V \in \mathbf{C}^{t \times m}$ , and a set  $\Delta \subset \mathbf{C}^{m \times n}$ . This applies to constant matrix data in  $R$ ,  $U$ , and  $V$ .

`[QOPT,BND] = cmsclsyn(R,U,V,BlockStructure)` minimizes, by choice of  $Q$ . `QOPT` is the optimum value of  $Q$ , the upper bound of `mussv(R+U*Q*V,BLK)`, `BND`. The matrices  $R$ ,  $U$  and  $V$  are constant matrices of the appropriate dimension. `BlockStructure` is a matrix specifying the perturbation blockstructure as defined for `mussv`.

`[QOPT,BND] = cmsclsyn(R,U,V,BlockStructure,OPT)` uses the options specified by `OPT` in the calls to `mussv`. See `mussv` for more information. The default value for `OPT` is `'cUsw'`.

`[QOPT,BND] = cmsclsyn(R,U,V,BlockStructure,OPT,QINIT)` initializes the iterative computation from  $Q = QINIT$ . Because of the nonconvexity of the overall problem, different starting points often yield different final answers. If `QINIT` is an N-D

array, then the iterative computation is performed multiple times - the  $i$ 'th optimization is initialized at  $Q = QINIT(:, :, i)$ . The output arguments are associated with the best solution obtained in this brute force approach.

`[QOPT,BND] = cmsclsyn(R,U,V,BlockStructure,OPT,'random',N)` initializes the iterative computation from  $N$  random instances of `QINIT`. If  $NCU$  is the number of columns of  $U$ , and  $NRV$  is the number of rows of  $V$ , then the approximation to solving the constant matrix  $\mu$  synthesis problem is two-fold: only the upper bound for  $\mu$  is minimized, and the minimization is not convex, hence the optimum is generally not found. If  $U$  is full column rank, or  $V$  is full row rank, then the problem can (and is) cast as a convex problem, [Packard, Zhou, Pandey and Becker], and the global optimizer (for the upper bound for  $\mu$ ) is calculated.

## Algorithms

The `cmsclsyn` algorithm is iterative, alternatively holding  $Q$  fixed, and computing the `mussv` upper bound, followed by holding the upper bound multipliers fixed, and minimizing the bound implied by choice of  $Q$ . If  $U$  or  $V$  is square and invertible, then the optimization is reformulated (exactly) as an linear matrix inequality, and solved directly, without resorting to the iteration.

## References

Packard, A.K., K. Zhou, P. Pandey, and G. Becker, "A collection of robust control problems leading to LMI's," *30th IEEE Conference on Decision and Control*, Brighton, UK, 1991, p. 1245-1250.

## See Also

`dksyn` | `hinfsv` | `mussv` | `robgain` | `robstab`

**Introduced before R2006a**

## dcgainmr

Reduced order model

### Syntax

```
[sysr, syse, gain] = dcgainmr(sys, ord)
```

### Description

`[sysr, syse, gain] = dcgainmr(sys, ord)` returns a reduced order model of a continuous-time LTI system `SYS` by truncating modes with least DC gain.

Specify your LTI continuous-time system in `sys`. The order is specified in `ord`.

This function returns:

- `sysr`—The reduced order models (a multidimensional array if `sys` is an LTI array)
- `syse`—The difference between `sys` and `sysr` (`syse=sys - sysr`)
- `gain`—The g-factors (dc-gains)

The DC gain of a complex mode

$$(1/(s+p)) * c * b'$$

is defined as

$$\text{norm}(b) * \text{norm}(c) / \text{abs}(p)$$

### See Also

`reduce`

**Introduced in R2008a**

## decay

Quadratic decay rate of polytopic or affine P-systems

### Syntax

`[drate,P] = decay(ps,options)`

### Description

For affine parameter-dependent systems

$$E(p)\dot{x} = A(p)x, \quad p(t) = (p_1(t), \dots, p_n(t)),$$

or polytopic systems

$$E(t)\dot{x} = A(t)x, \quad (A, E) \in \text{Co}\{(A_1, E_1), \dots, (A_n, E_n)\},$$

`decay` returns the quadratic decay rate `drate`, i.e., the smallest  $\alpha \in R$  such that

$$A^T Q E + E Q A^T < \alpha Q$$

holds for some Lyapunov matrix  $Q > 0$  and all possible values of  $(A, E)$ . Two control parameters can be reset via `options(1)` and `options(2)`:

- If `options(1)=0` (default), `decay` runs in fast mode, using the least expensive sufficient conditions. Set `options(1)=1` to use the least conservative conditions.
- `options(2)` is a bound on the condition number of the Lyapunov matrix  $P$ . The default is 109.

### See Also

`pdlstab` | `psys` | `quadstab`

**Introduced before R2006a**

## decinfo

Describe how entries of matrix variable  $X$  relate to decision variables

### Syntax

```
decinfo(lmisys)
```

```
decX = decinfo(lmisys,X)
```

### Description

The function `decinfo` expresses the entries of a matrix variable  $X$  in terms of the decision variables  $x_1, \dots, x_N$ . Recall that the decision variables are the free scalar variables of the problem, or equivalently, the free entries of all matrix variables described in `lmisys`. Each entry of  $X$  is either a hard zero, some decision variable  $x_n$ , or its opposite  $-x_n$ .

If  $X$  is the identifier of  $X$  supplied by `lmivar`, the command

```
decX = decinfo(lmisys,X)
```

returns an integer matrix `decX` of the same dimensions as  $X$  whose  $(i, j)$  entry is

- 0 if  $X(i, j)$  is a hard zero
- $n$  if  $X(i, j) = x_n$  (the  $n$ -th decision variable)
- $-n$  if  $X(i, j) = -x_n$

`decX` clarifies the structure of  $X$  as well as its entry-wise dependence on  $x_1, \dots, x_N$ . This is useful to specify matrix variables with atypical structures (see `lmivar`).

`decinfo` can also be used in interactive mode by invoking it with a single argument. It then prompts the user for a matrix variable and displays in return the decision variable content of this variable.

## Examples

### Example 1

Consider an LMI with two matrix variables  $X$  and  $Y$  with structure:

- $X = x I_3$  with  $x$  scalar
- $Y$  rectangular of size 2-by-1

If these variables are defined by

```
setlmis([])
X = lmivar(1,[3 0])
Y = lmivar(2,[2 1])
  :
  :
lmis = getlmis
```

the decision variables in  $X$  and  $Y$  are given by

```
dX = decinfo(lmis,X)
```

```
dX =
     1     0     0
     0     1     0
     0     0     1
```

```
dY = decinfo(lmis,Y)
```

```
dY =
     2
     3
```

This indicates a total of three decision variables  $x_1, x_2, x_3$  that are related to the entries of  $X$  and  $Y$  by

$$X = \begin{pmatrix} x_1 & 0 & 0 \\ 0 & x_1 & 0 \\ 0 & 0 & x_1 \end{pmatrix}, Y = \begin{pmatrix} x_2 \\ x_3 \end{pmatrix}$$

Note that the number of decision variables corresponds to the number of free entries in  $X$  and  $Y$  when taking structure into account.



## Example 2

Suppose that the matrix variable  $X$  is symmetric block diagonal with one 2-by-2 full block and one 2-by-2 scalar block, and is declared by

```
setlmis([])
X = lmivar(1,[2 1;2 0])
      :
lmis = getlmis
```

The decision variable distribution in  $X$  can be visualized interactively as follows:

```
decinfo(lmis)
```

There are 4 decision variables labeled x1 to x4 in this problem.

Matrix variable Xk of interest (enter k between 1 and 1, or 0 to quit):

```
?> 1
```

The decision variables involved in X1 are among {-x1,...,x4}.  
Their entry-wise distribution in X1 is as follows  
(0,j>0,-j<0 stand for 0,xj,-xj, respectively):

X1 :

|   |   |   |   |
|---|---|---|---|
| 1 | 2 | 0 | 0 |
| 2 | 3 | 0 | 0 |
| 0 | 0 | 4 | 0 |
| 0 | 0 | 0 | 4 |

\*\*\*\*\*

Matrix variable Xk of interest (enter k between 1 and 1, or 0 to quit):

```
?> 0
```

## See Also

[dec2mat](#) | [lmivar](#) | [mat2dec](#)

**Introduced before R2006a**

## **decnbr**

Total number of decision variables in system of LMIs

### **Syntax**

```
ndec = decnbr(lmisys)
```

### **Description**

The function `decnbr` returns the number `ndec` of decision variables (free scalar variables) in the LMI problem described in `lmisys`. In other words, `ndec` is the length of the vector of decision variables.

### **Examples**

For an LMI system `lmis` with two matrix variables  $X$  and  $Y$  such that

- $X$  is symmetric block diagonal with one 2-by-2 full block, and one 2-by-2 scalar block
- $Y$  is 2-by-3 rectangular,

the number of decision variables is

```
ndec = decnbr(LMIs)
```

```
ndec =  
    10
```

This is exactly the number of free entries in  $X$  and  $Y$  when taking structure into account (see `decinfo` for more details).

### **See Also**

`dec2mat` | `decinfo` | `mat2dec`

**Introduced before R2006a**

## dec2mat

Given values of decision variables, derive corresponding values of matrix variables

### Syntax

```
valX = dec2mat(lmisys,decvars,X)
```

### Description

Given a value `decvars` of the vector of decision variables, `dec2mat` computes the corresponding value `valX` of the matrix variable with identifier `X`. This identifier is returned by `lmivar` when declaring the matrix variable.

Recall that the decision variables are all free scalar variables in the LMI problem and correspond to the free entries of the matrix variables  $X_1, \dots, X_K$ . Since LMI solvers return a feasible or optimal value of the vector of decision variables, `dec2mat` is useful to derive the corresponding feasible or optimal values of the matrix variables.

### Examples

See the description of `feasp`.

### See Also

`decinfo` | `decnbr` | `mat2dec`

**Introduced before R2006a**

## defcx

Help specify  $c^T x$  objectives for mincx solver

## Syntax

```
[V1,...,Vk] = defcx(lmisys,n,X1,...,Xk)
```

## Description

defcx is useful to derive the c vector needed by mincx when the objective is expressed in terms of the matrix variables.

Given the identifiers  $X_1, \dots, X_k$  of the matrix variables involved in this objective, defcx returns the values  $V_1, \dots, V_k$  of these variables when the  $n$ -th decision variable is set to one and all others to zero.

## See Also

decinfo | mincx

**Introduced before R2006a**

## dellmi

Remove LMI from system of LMIs

### Syntax

```
newsys = dellmi(lmisys,n)
```

### Description

`dellmi` deletes the  $n$ -th LMI from the system of LMIs described in `lmisys`. The updated system is returned in `newsys`.

The ranking  $n$  is relative to the order in which the LMIs were declared and corresponds to the identifier returned by `newlmi`. Since this ranking is not modified by deletions, it is safer to refer to the remaining LMIs by their identifiers. Finally, matrix variables that only appeared in the deleted LMI are removed from the problem.

### Examples

Suppose that the three LMIs

$$A_1^T X_1 + X_1 A_1 + Q_1 < 0$$

$$A_2^T X_2 + X_2 A_2 + Q_2 < 0$$

$$A_3^T X_3 + X_3 A_3 + Q_3 < 0$$

have been declared in this order, labeled LMI1, LMI2, LMI3 with `newlmi`, and stored in `lmisys`. To delete the second LMI, type

```
lmis = dellmi(lmisys,LMI2)
```

`lmis` now describes the system of LMIs

$$A_1^T X_1 + X_1 A_1 + Q_1 < 0$$

$$A_3^T X_3 + X_3 A_3 + Q_3 < 0$$

and the second variable  $X_2$  has been removed from the problem since it no longer appears in the system.

To further delete LMI3 from the system, type

```
lmis = dellmi(lmis,LMI3)
```

or equivalently

```
lmis = dellmi(lmis,3)
```

Note that the system has retained its original ranking after the first deletion.

## See Also

`lmiedit` | `lmiinfo` | `newlmi`

**Introduced before R2006a**

## delmvar

Remove one matrix variable from LMI problem

### Syntax

```
newsys = delmvar(lmisys,X)
```

### Description

`delmvar` removes the matrix variable  $X$  with identifier  $X$  from the list of variables defined in `lmisys`. The identifier  $X$  should be the second argument returned by `lmivar` when declaring  $X$ . All terms involving  $X$  are automatically removed from the list of LMI terms. The description of the resulting system of LMIs is returned in `newsys`.

### Examples

Consider the LMI

$$0 < \begin{pmatrix} A^T Y + B^T Y A + Q & C X + D \\ X^T C^T + D^T & -(X + X^T) \end{pmatrix}$$

involving two variables  $X$  and  $Y$  with identifiers  $X$  and  $Y$ . To delete the variable  $X$ , type

```
lmisys = delmvar(lmisys,X)
```

Now `lmisys` describes the LMI

$$0 < \begin{pmatrix} A^T Y B + B^T Y A + Q & D \\ D^T & 0 \end{pmatrix}$$

with only one variable  $Y$ . Note that  $Y$  is still identified by the label  $Y$ .



## See Also

`lmiinfo` | `lmivar` | `setmvar`

**Introduced before R2006a**

## diag

Diagonalize vector of uncertain matrices and systems

### Syntax

```
v = diag(x)
```

### Description

If  $x$  is a vector of uncertain system models or matrices, `diag(x)` puts  $x$  on the main diagonal. If  $x$  is a matrix of uncertain system models or matrices, `diag(x)` is the main diagonal of  $x$ . `diag(diag(x))` is a diagonal matrix of uncertain system models or matrices.

### Examples

The statement produces a diagonal system `mxg` of size 4-by-4. Given multivariable system `xx`, a vector of the diagonal elements of `xxg` is found using `diag`.

```
x = rss(3,4,1);  
xg = frd(x,logspace(-2,2,80));  
size(xg)
```

FRD model with 4 output(s) and 1 input(s), at 80 frequency point(s).

```
mxg = diag(xg);  
size(mxg)  
FRD model with 4 output(s) and 4 input(s), at 80 frequency point(s).
```

```
xxg = [xg(1:2,1) xg(3:4,1)];  
m = diag(xxg);  
size(m)  
FRD model with 2 output(s) and 1 input(s), at 80 frequency point(s).
```

## **See Also**

append

**Introduced before R2006a**

## **dksyn**

Robust controller design using  $\mu$ -synthesis

### **Syntax**

```
[k,clp,bnd] = dksyn(p,nmeas,ncont)
```

```
[k,clp,bnd] = dksyn(p,nmeas,ncont,opt)
```

```
[k,clp,bnd,dkinfo] = dksyn(p,nmeas,ncont,...)
```

```
[k,clp,bnd,dkinfo] = dksyn(p,nmeas,ncont,prevdkinfo,opt)
```

```
[...] = dksyn(p)
```

### **Description**

`[k,clp,bnd] = dksyn(p,nmeas,ncont)` synthesizes a robust controller `k` for the uncertain open-loop plant model `p` via the D-K or D-G-K algorithm for  $\mu$ -synthesis. `p` is an uncertain state-space `uss` model. The last `nmeas` outputs and `ncont` inputs of `p` are assumed to be the measurement and control channels. `k` is the controller, `clp` is the closed-loop model and `bnd` is the robust closed-loop performance bound. `p`, `k`, `clp`, and `bnd` are related as follows:

```
clp = lft(p,k);  
bnd1 = dksynperf(clp);  
bnd = 1/bnd1.LowerBound
```

`[k,clp,bnd] = dksyn(p,nmeas,ncont,opt)` specifies user-defined options `opt` for the D-K or D-K-G algorithm. Use `dksynOptions` to create `opt`.

`[k,clp,bnd,dkinfo] = dksyn(p,nmeas,ncont,...)` returns a log of the algorithm execution in `dkinfo`. `dkinfo` is an  $N$ -by-1 cell array where  $N$  is the total number of iterations performed. The  $i$ th cell contains a structure with the following fields:

| Field     | Description  |
|-----------|--|
| K         | Controller at ith iteration, a <code>ss</code> object                      |
| Bnds      | Robust performance bound on the closed-loop system ( <code>double</code> ) |
| DL        | Left D-scale, an <code>ss</code> object                                    |
| DR        | Right D-scale, an <code>ss</code> object                                   |
| GM        | Offset G-scale, an <code>ss</code> object                                  |
| GR        | Right G-scale, an <code>ss</code> object                                   |
| GFC       | Center G-scale, an <code>ss</code> object                                  |
| MussvBnds | Upper and lower $\mu$ bounds, an <code>frd</code> object                   |
| MussvInfo | Structure returned from <code>mussv</code> at each iteration.              |

`[k,clp,bnd,dkinfo] = dksyn(p,nmeas,ncont,prevdkinfo,opt)` allows you to use information from a previous `dksyn` iteration. `prevdkinfo` is a structure from a previous attempt at designing a robust controller using `dksyn`. `prevdkinfo` is used when the `dksyn` starting iteration is not 1 (`opt.StartingIterationNumber = 1`) to determine the correct D-scalings to initiate the iteration procedure.

`[...] = dksyn(p)` takes `p` as a `uss` object that has two-input/two-output partitioning as defined by `mktito`.

## Examples

The following statements create a robust performance control design for an unstable, uncertain single-input/single-output plant model. The nominal plant model, `G`, is an

unstable first order system  $\frac{s}{s-1}$ .

```
G = tf(1,[1 -1]);
```

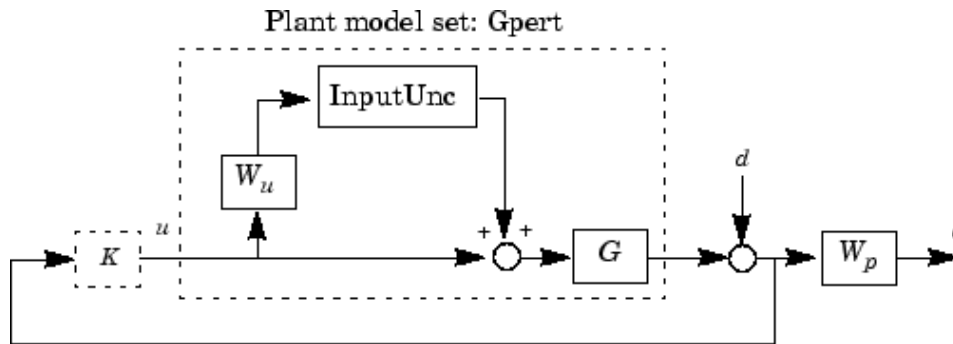
The model itself is uncertain. At low frequency, below 2 rad/s, it can vary up to 25% from its nominal value. Around 2 rad/s the percentage variation starts to increase and reaches 400% at approximately 32 rad/s. The percentage model uncertainty is represented by the weight `Wu` which corresponds to the frequency variation of the model uncertainty and the uncertain LTI dynamic object `InputUnc`.

```
Wu = 0.25*tf([1/2 1],[1/32 1]);
InputUnc = ultidyn('InputUnc',[1 1]);
```

The uncertain plant model  $G_{pert}$  represents the model of the physical system to be controlled.

```
Gpert = G*(1+InputUnc*Wu);
```

The robust stability objective is to synthesize a stabilizing LTI controller for all the plant models parameterized by the uncertain plant model,  $G_{pert}$ . The performance objective is defined as a weighted sensitivity minimization problem. The control interconnection structure is shown in the following figure.



The sensitivity function,  $S$ , is defined as

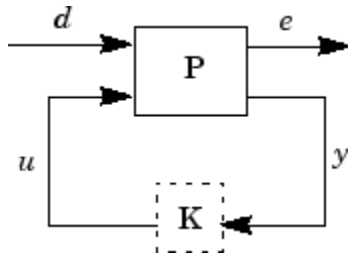
$$S = \frac{1}{1 + PK}$$

where  $P$  is the plant model and  $K$  is the controller. A weighted sensitivity minimization problem selects a weight  $W_p$ , which corresponds to the *inverse* of the desired sensitivity function of the closed-loop system as a function of frequency. Hence the product of the sensitivity weight  $W_p$  and actual closed-loop sensitivity function is less than 1 across all frequencies. The sensitivity weight  $W_p$  has a gain of 100 at low frequency, begins to decrease at 0.006 rad/s, and reaches a minimum magnitude of 0.25 after 2.4 rad/s.

```
Wp = tf([1/4 0.6],[1 0.006]);
```

The defined sensitivity weight  $W_p$  implies that the desired disturbance rejection should be at least 100:1 disturbance rejection at DC, rise slowly between 0.006 and 2.4 rad/s, and allow the disturbance rejection to increase above the open-loop level, 0.25, at high frequency.

When the plant model is uncertain, the closed-loop performance objective is to achieve the desired sensitivity function for all plant models defined by the uncertain plant model,  $G_{pert}$ . The performance objective for an uncertain system is a robust performance objective. A block diagram of this uncertain closed-loop system illustrating the performance objective (closed-loop transfer function from  $d \rightarrow e$ ) is shown.



From the definition of the robust performance control objective, the weighted, uncertain control design interconnection model, which includes the robustness and performance objectives, can be constructed and is denoted by  $P$ . The robustness and performance weights are selected such that if the robust performance structure singular value,  $bnd$ , of the closed-loop uncertain system,  $clp$ , is less than 1 then the performance objectives have been achieved for all the plant models in the model set.

You can form the uncertain transfer matrix  $P$  from  $[d; u]$  to  $[e; y]$  using the following commands.

```
P = [Wp; 1] * [1 Gpert];
[K, clp, bnd] = dksyn(P, 1, 1);
bnd
```

```
bnd =
    0.6819
```

The controller  $K$  achieves a robust performance  $\mu$  value  $bnd$  of 0.6819. Therefore you have achieved the robust performance objectives for the given problem.

You can use the `robgain` command to analyze the closed-loop robust performance of  $clp$ .

```
[rpmarg, rpmargunc] = robgain(clp, 1);
```

## Limitations

There are two shortcomings of the D-K iteration control design procedure:

- Calculation of the structured singular value  $\mu\Delta(\cdot)$  is approximated by its upper bound. This is not a serious problem because the value of  $\mu$  and its upper bound are often close.
- The D-K iteration is not guaranteed to converge to a global, or even local minimum. This is a serious problem, and represents the biggest limitation of the design procedure.

In spite of these drawbacks, the D-K iteration control design technique appears to work well on many engineering problems. It has been applied to a number of real-world applications with success. These applications include vibration suppression for flexible structures, flight control, chemical process control problems, and acoustic reverberation suppression in enclosures.

## Tutorials

Control of Spring-Mass-Damper Using Mixed  $\mu$ -Synthesis

## Algorithms

`dksyn` synthesizes a robust controller via D-K iteration. The D-K iteration procedure is an approximation to  $\mu$ -synthesis control design. The objective of  $\mu$ -synthesis is to minimize the structure singular value  $\mu$  of the corresponding robust performance problem associated with the uncertain system  $\mathbf{p}$ . The uncertain system  $\mathbf{p}$  is an open-loop interconnection containing known components including the nominal plant model, uncertain parameters, `ucomplex`, and unmodeled LTI dynamics, `ultidyn`, and performance and uncertainty weighting functions. You use weighting functions to include magnitude and frequency shaping information in the optimization. The control objective is to synthesize a stabilizing controller  $\mathbf{k}$  that minimizes the robust performance  $\mu$  value, which corresponds to `bnd`.

The D-K iteration procedure involves a sequence of minimizations, first over the controller variable  $K$  (holding the  $D$  variable associated with the scaled  $\mu$  upper bound fixed), and then over the  $D$  variable (holding the controller  $K$  variable fixed). The D-K iteration



procedure is not guaranteed to converge to the minimum  $\mu$  value, but often works well in practice.

dksyn automates the D-K iteration procedure and the options object `dksynOptions` allows you to customize its behavior. Internally, the algorithm works with the generalized scaled plant model  $P$ , which is extracted from a `uss` object using the command `lftdata`.

The following is a list of what occurs during a single, complete step of the D-K iteration.

- 1 (In the first iteration, this step is skipped.) The  $\mu$  calculation (from the previous step) provides a frequency-dependent scaling matrix,  $D_f$ . The fitting procedure fits these scalings with rational, stable transfer function matrices. After fitting, plots of

$$\bar{\sigma}(\hat{D}_f(j\omega)F_L(P,K)(j\omega)D_f^{-1}(j\omega))$$

and

$$\bar{\sigma}(\hat{D}_f(j\omega)F_L(P,K)(j\omega)\hat{D}_f^{-1}(j\omega))$$

are shown for comparison.

(In the first iteration, this step is skipped.) The rational  $\hat{D}$  is absorbed into the open-loop interconnection for the next controller synthesis. Using either the previous

frequency-dependent  $D$ 's or the just-fit rational  $\hat{D}$ , an estimate of an appropriate value for the  $H_\infty$  norm is made. This is simply a conservative value of the scaled closed-loop  $H_\infty$  norm, using the most recent controller and either a frequency sweep (using the frequency-dependent  $D$ 's) or a state-space calculation (with the rational  $D$ 's).

- 2 (The first iteration begins at this point.) A controller is designed using  $H_\infty$  synthesis on the scaled open-loop interconnection. If you set the `DisplayWhileAutoIter` field in `dksynOptions` to 'on', the following information is displayed:
  - a The progress of the  $\gamma$ -iteration is displayed.
  - b The singular values of the closed-loop frequency response are plotted.
  - c You are given the option to change the frequency range. If you change it, all relevant frequency responses are automatically recomputed.
  - d You are given the option to rerun the  $H_\infty$  synthesis with a set of modified parameters if you set the `AutoIter` field in `dksynOptions` to 'off'. This is

convenient if, for instance, the bisection tolerance was too large, or if maximum  $\gamma$  value was too small.

- 3 The structured singular value of the closed-loop system is calculated and plotted.
- 4 An iteration summary is displayed, showing all the controller order, as well as the peak value of  $\mu$  of the closed-loop frequency responses.
- 5 The choice of stopping or performing another iteration is given.

Subsequent iterations proceed along the same lines without the need to reenter the iteration number. A summary at the end of each iteration is updated to reflect data from all previous iterations. This often provides valuable information about the progress of the robust controller synthesis procedure.

## Interactive Fitting of D-Scalings

Setting the `AutoIter` field in `dksynOptions` to 'off' requires that you interactively fit the  $D$ -scales each iteration. During step 2 of the  $D$ -K iteration procedure, you are prompted to enter your choice of options for fitting the  $D$ -scaling data. You press return after, the following is a list of your options.

Enter Choice (return for list):

Choices:

```
nd          Move to Next D-Scaling
nb          Move to Next D-Block

i           Increment Fit Order
d           Decrement Fit Order
apf        Auto-PreFit
mx 3       Change Max-Order to 3
at 1.01    Change Auto-PreFit tol to 1.01
0          Fit with zeroth order
2          Fit with second order
n          Fit with n'th order
e          Exit with Current Fittings
s          See Status
```

- `nd` and `nb` allow you to move from one  $D$ -scale data to another. `nd` moves to the next scaling, whereas `nb` moves to the next scaling block. For scalar  $D$ -scalings, these are identical operations, but for problems with full  $D$ -scalings, (perturbations of the form  $\delta I$ ) they are different. In the (1,2) subplot window, the title displays the  $D$ -scaling block number, the row/column of the scaling that is currently being fitted, and the order of the current fit (with `d` for data when no fit exists).

- You can increment or decrement the order of the current fit (by 1) using `i` and `d`.
- `apf` automatically fits each  $D$ -scaling data. The default maximum state order of individual  $D$ -scaling is 5. The `mx` variable allows you to change the maximum  $D$ -scaling state order used in the automatic prefitting routine. `mx` must be a positive, nonzero integer. `at` allows you to define how close the rational, scaled  $\mu$  upper bound is to approximate the actual  $\mu$  upper bound in a norm sense. Setting `at` to 1 would require an exact fit of the  $D$ -scale data, and is not allowed. Allowable values for `at` are greater than 1. This setting plays a role (mildly unpredictable, unfortunately) in determining where in the  $(D,K)$  space the D-K iteration converges.
- Entering a positive integer at the prompt will fit the current  $D$ -scale data with that state order rational transfer function.
- `e` exits the  $D$ -scale fitting to continue the D-K iteration.
- The variable `s` displays a status of the current and fits.

## References

- [1] Balas, G.J., and J.C. Doyle, "Robust control of flexible modes in the controller crossover region," *AIAA Journal of Guidance, Dynamics and Control*, Vol. 17, no. 2, March-April, 1994, p. 370-377.
- [2] Balas, G.J., A.K. Packard, and J.T. Harduvel, "Application of  $\mu$ -synthesis techniques to momentum management and attitude control of the space station," *AIAA Guidance, Navigation and Control Conference*, New Orleans, August 1991.
- [3] Doyle, J.C., K. Lenz, and A. Packard, "Design examples using  $\mu$ -synthesis: Space shuttle lateral axis FCS during reentry," *NATO ASI Series, Modelling, Robustness, and Sensitivity Reduction in Control Systems*, vol. 34, Springer-Verlag, Berlin 1987.
- [4] Packard, A., J. Doyle, and G. Balas, "Linear, multivariable robust control with a  $\mu$  perspective," *ASME Journal of Dynamic Systems, Measurement and Control*, 50th Anniversary Issue, Vol. 115, no. 2b, June 1993, p. 310-319.
- [5] Stein, G., and J. Doyle, "Beyond singular values and loopshapes," *AIAA Journal of Guidance and Control*, Vol. 14, No. 1, January, 1991, p. 5-16.

## **See Also**

`dkSynOptions` | `dkSynperf` | `h2syn` | `hinfoSyn` | `mktito` | `mussv` | `robGain` | `robStab`  
| `wcGain` | `wcMargin` | `wcSens`

**Introduced before R2006a**

# dksynOptions

Set options for dksyn

## Syntax

```
opt = dksynOptions  
opt = dksynOptions(Name,Value)
```

## Description

`opt = dksynOptions` returns the default options for dksyn.

`opt = dksynOptions(Name,Value)` returns an option set with additional options specified by one or more `Name,Value` pair arguments.

## Input Arguments

### Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes ( ' '). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

`dksynOptions` takes the following `Name` arguments:

#### FrequencyVector

Frequencies for mu-analysis, specified as a vector. When empty, `dksyn` automatically chooses the frequency range and number of points.

**Default:** []

## **InitialController**

Controller for initializing first iteration, specified as a state-space (ss) model.

**Default:** []

## **AutoIter**

Automated mu-synthesis mode, specified as either 'on' or 'off'. When automated mu-synthesis mode is off, `dksyn` performs an interactive D-K iteration procedure. You are prompted to fit the D-scale data and provide input on the control design process.

**Default:** 'on'

## **DisplayWhileAutoIter**

Status of display in automated mu-synthesis mode, specified as either 'off' or 'on'. When the display is on, and automated mu-synthesis mode is active, `dksyn` displays the iteration progress during the synthesis computation.

**Default:** 'off'

## **StartingIterationNumber**

Iteration number for initiating iteration procedure, specified as a positive integer. Use this option when you provide the `prevdkinfo` argument to `dksyn` to use information from a previous `dksyn` calculation. In this case, specify the starting iteration number from which to resume the iteration procedure.

**Default:** 1

## **NumberOfAutoIterations**

Number of iterations to perform in automatic mu-synthesis mode, specified as a positive integer.

**Default:** 10

## **MixedMU**

Flag indicating whether to perform mixed real/complex mu-synthesis when real parameters are present, specified as either 'off' or 'on'. Mixed mu-synthesis accounts for uncertain real parameters directly in the synthesis process. Setting 'MixedMU' to

'on' when you have uncertain real parameters can result in improved robust performance of the synthesized controller.

**Default:** 'off'

### **AutoScalingOrder**

State order for fitting  $D$ -scaling and  $G$ -scaling data for real/complex mu-synthesis, specified as a vector of the form [dorder, gorder].

**Default:** [5 2] (5th-order  $D$ -scalings and 2nd-order  $G$ -scalings)

### **AutoIterSmartTerminate**

Automatic termination mode, specified as either 'on' or 'off'. When AutoIterSmartTerminate is 'on', the iteration procedure terminates based on the progress of the design iteration. Set the tolerance for automatic termination using AutoIterSmartTerminateTol.

In automatic termination mode, the iteration procedure terminates when a stopping criterion is satisfied. The stopping criterion involves the objective value (peak value, across frequency, of the upper bound for  $\mu$ ) in the current iteration, denoted  $v_0$ . The stopping criterion also involves the objective value in the previous two iterations, denoted  $v_{-1}$  and  $v_{-2}$ . The stopping criterion is satisfied for lack of progress if:

$$|v_0 - v_{-1}| < \text{AutoIterSmartTerminateTol} * v_0,$$

and

$$|v_{-1} - v_{-2}| < \text{AutoIterSmartTerminateTol} * v_0.$$

The stopping criteria is also satisfied for an undesirable significant increase in the objective value if:

$$v_0 > v_{-1} + 20 * \text{AutoIterSmartTerminateTol} * v_0.$$

**Default:** 'on'

### **AutoIterSmartTerminateTol**

Tolerance for AutoIterSmartTerminate mode.

**Default:** 0.005

## Output Arguments

### **options**

Option set containing the specified options for the `dksyn` command.

## Examples

### **Create Options Set for `dksyn`**

Create an options set for a `dksyn` run using a logarithmic distribution of frequency points for analysis and performing 24 iterations.

```
options = dksynOptions('FrequencyVector',logspace(-2,3,80),...  
                      'NumberOfAutoIterations',24);
```

Alternatively, use dot notation to set the values of options.

```
options = dksynOptions;  
options.FrequencyVector = logspace(-2,3,80);  
options.NumberOfAutoIterations = 24;
```

## See Also

`dksyn`

**Introduced in R2013a**



# dksynperf

Robust  $H_\infty$  performance optimized by dksyn

The robust  $H_\infty$  performance (or robust  $H_\infty$  norm) of an uncertain closed-loop system is the smallest value  $\gamma$  such that the I/O gain of the system stays below  $\gamma$  for all modeled uncertainty up to size  $1/\gamma$  (in normalized units). The `dksyn` function synthesizes a robust controller by minimizing this quantity over all possible choices of controller. `dksynperf` computes this quantity for a specified uncertain model.

## Syntax

```
[gamma,wcu] = dksynperf(clp)
[gamma,wcu] = dksynperf(clp,w)
[gamma,wcu] = dksynperf(____,opts)
[gamma,wcu,info] = dksynperf(____)
```

## Description

`[gamma,wcu] = dksynperf(clp)` calculates the robust  $H_\infty$  performance for an uncertain closed-loop system, `clp`. The robust  $H_\infty$  performance is the smallest value  $\gamma$  for which the peak I/O gain stays below  $\gamma$  for all modeled uncertainty up to  $1/\gamma$ , in normalized units. For example, a value of  $\gamma = 1.125$  implies the following:

- The I/O gain of `clp` remains less than 1.125 as long as the uncertain elements stay within 0.8 normalized units of their nominal values. In other words, for uncertain element values within 0.8 normalized units, the largest possible  $H_\infty$  norm is 1.125.
- There is a perturbation of size 0.8 normalized units that drives the peak I/O gain to 1.125.

The peak I/O gain is the maximum I/O gain over all inputs, which is also the peak of the largest singular value over all frequencies and uncertainties. In other words, if  $\Delta$  represents all possible values of the uncertain parameters in the closed-loop transfer function  $CLP(j\omega)$ , then

$$\gamma = \max_{\Delta} \max_{\omega} \sigma_{max}(CLP(j\omega)).$$

The output structure `gamma` contains upper and lower bounds on the robust  $H_\infty$  performance and the critical frequency at which the I/O gain of `clp` reaches the lower bound. The structure `wcu` contains the uncertain-element values that drive the peak I/O gain to the lower bound.

`[gamma,wcu] = dksynperf(clp,w)` computes the robust  $H_\infty$  performance at the frequencies specified by `w`.

- If `w` is a cell array of the form `{wmin,wmax}`, then `dksynperf` restricts the computation to the interval between `wmin` and `wmax`.
- If `w` is a vector of frequencies, then `dksynperf` computes the  $H_\infty$  performance at the specified frequencies only.

`[gamma,wcu] = dksynperf(____,opts)` specifies additional options for the computation. Use `robOptions` to create `opts`. You can use this syntax with any of the previous input-argument combinations.

`[gamma,wcu,info] = dksynperf(____)` returns a structure with additional information about the  $H_\infty$  performance values and the perturbations that drive the I/O gain to  $\gamma$ . See `info` for details about this structure. You can use this syntax with any of the previous input-argument combinations.

## Examples

### Reduce Synthesized Controller While Preserving Robust Performance

The example on the `dksyn` reference page synthesizes a robust controller with no restriction on controller order. To simplify the controller while preserving the robust performance of the closed-loop system, use `dksynperf`.

Synthesize a controller for the system described in the `dksyn` example.

```
G = tf(1,[1 -1]);  
Wu = 0.25*tf([1/2 1],[1/32 1]);  
InputUnc = ultidyn('InputUnc',[1 1]);  
Gpert = G*(1+InputUnc*Wu);  
Wp = tf([1/4 0.6],[1 0.006]);  
P = [Wp; 1]*[1 Gpert];
```

```
[K,clp,clperf] = dksyn(P,1,1);
N = order(K)
```

```
N = 7
```

dksyn returns a 7th-order controller, the closed-loop system with that controller, clp, and the robust  $H_\infty$  performance of that system, clperf. Compute reduced-order controllers for orders ranging from 1 full order.

```
Kred = reduce(K,1:N);
```

Find the lowest-order controller Klow with performance no worse than 1.05\*clperf, or 5% degradation compared to the full-order controller.

```
for k=1:N
    Klow = Kred(:,:,k);
    clp = lft(P,Klow);
    [gamma,~] = dksynperf(clp);
    if gamma.UpperBound < 1.05*clperf
        break
    end
end
order(Klow)

ans = 4
```

To validate the reduced-order controller, compare the robust  $H_\infty$  performance of the system using the simplified controller with that of the system using the full-order controller. The latter value is clperf, returned by dksyn when synthesizing the controller.

```
clplow = lft(P,Klow);
dksynperf(clplow)

ans = struct with fields:
    LowerBound: 0.6939
    UpperBound: 0.6953
    CriticalFrequency: 0.4659
```

```
clperf
```

`clperf = 0.6806`

The third-order controller achieves almost the same robust  $H_\infty$  performance as the 7th-order controller returned by `dksyn`.

## Input Arguments

### **clp** — Closed-loop uncertain system

`uss` | `ufrd` | `genss` | `genfrd`

Closed-loop uncertain system, specified as a `uss`, `ufrd`, `genss`, or `genfrd` model that contains uncertain elements. For `genss` or `genfrd` models, `dksynperf` uses the current value of any tunable blocks and folds them into the known (not uncertain) part of the model.

### **w** — Frequencies

`{wmin,wmax}` | vector

Frequencies at which to compute robust  $H_\infty$  performance, specified as the cell array `{wmin,wmax}` or as a vector of frequency values.

- If `w` is a cell array of the form `{wmin,wmax}`, then the function computes the  $H_\infty$  performance at frequencies ranging between `wmin` and `wmax`.
- If `w` is a vector of frequencies, then the function computes the  $H_\infty$  performance at each specified frequency. For example, use `logspace` to generate a row vector with logarithmically spaced frequency values.

Specify frequencies in units of `rad/TimeUnit`, where `TimeUnit` is the `TimeUnit` property of the model.

### **opts** — Options for $H_\infty$ performance computation

`robOptions` object

Options for computation of robust  $H_\infty$  performance, specified as an object you create with `robOptions`. The available options include settings that let you:

- Extract frequency-dependent  $H_\infty$  performance values.
- Examine the sensitivity of the  $H_\infty$  performance to each uncertain element.

- Improve the results of the calculation by setting certain options for the underlying `mussv` calculation. In particular, setting the option `'MussvOptions'` to `'mN'` can reduce the gap between the lower bound and upper bound. `N` is the number of restarts.

For more information about all available options, see `robOptions`.

Example: `robOptions('Sensitivity','on','MussvOptions','m3')`

## Output Arguments

### **gamma** — Robust $H_\infty$ performance and critical frequency

structure

Robust  $H_\infty$  performance and critical frequency, returned as a structure containing the following fields:

| Field             | Description   |
|-------------------|---|
| LowerBound        | Lower bound on the actual robust $H_\infty$ performance $\gamma$ , returned as a scalar value. The exact value of $\gamma$ is guaranteed to be no smaller than LowerBound. In other words, there exist some uncertain-element values of magnitude $1/\text{LowerBound}$ for which the I/O gain of <code>clp</code> reaches LowerBound. The function returns one such instance in <code>wcu</code> . |
| UpperBound        | Upper bound on the actual robust $H_\infty$ performance, returned as a scalar value. The exact value is guaranteed to be no larger than UpperBound. In other words, for all modeled uncertainty with normalized magnitude up to $1/\text{UpperBound}$ , the peak I/O gain of <code>clp</code> is less than UpperBound.  |
| CriticalFrequency | Frequency at which the I/O gain reaches LowerBound, in rad/TimeUnit, where TimeUnit is the TimeUnit property of <code>clp</code> .  |

Use `normalized2actual` to convert the normalized uncertainty values  $1/\text{LowerBound}$  or  $1/\text{UpperBound}$  to actual deviations from nominal values.

## **wcu — Perturbations driving I/O gain to `gamma.LowerBound`**

structure

Perturbations driving I/O gain to `gamma.LowerBound`, returned as a structure whose fields are the names of the uncertain elements of `clp`. Each field contains the actual value of the corresponding uncertain element. For example, if `clp` includes an uncertain matrix `M` and SISO uncertain dynamics `delta`, then `wcu.M` is a numeric matrix and `wcu.delta` is a SISO state-space model.

Use `usubs(clp,wcu)` to substitute these values for the uncertain elements in `clp` and obtain the corresponding dynamic system. This system has peak gain `gamma.LowerBound`.

Use `actual2normalized` to convert these actual uncertainty values to the normalized units in which `1/gamma.LowerBound` or `1/gamma.UpperBound` are expressed.

## **info — Additional information about $\gamma$ values**

structure

Additional information about the  $\gamma$  values, returned as a structure with the following fields:

| Field     | Description  |
|-----------|--|
| Frequency | <p>Frequency points at which dksynperf returns <math>\gamma</math> values, returned as a vector.</p> <ul style="list-style-type: none"> <li>• If the 'VaryFrequency' option of robOptions is 'off', then info.Frequency is the critical frequency, the frequency at which the I/O gain reaches gamma.LowerBound. If the smallest lower bound and the smallest upper bound on <math>\gamma</math> occur at different frequencies, then info.Frequency is a vector containing these two frequencies.</li> <li>• If the 'VaryFrequency' option of robOptions is 'on', then info.Frequency contains the frequencies selected by dksynperf. These frequencies are guaranteed to include the frequency at which the peak gain occurs.</li> <li>• If you specify a vector of frequencies w at which to compute <math>\gamma</math>, then info.Frequency = w. When you specify a frequency vector, these frequencies are not guaranteed to include the frequency at which the peak gain occurs.</li> </ul> <p>The 'VaryFrequency' option is meaningful only for uss and genss models. dksynperf ignores the option for ufrd and genfrd models.</p> |
| Bounds    | <p>Lower and upper bounds on the actual <math>\gamma</math> values, returned as an array. info.Bounds(:,1) contains the lower bound at each corresponding frequency in info.Frequency, and info.Bounds(:,2) contains the corresponding upper bounds.</p>   |

| <b>Field</b>      | <b>Description</b>   |
|-------------------|--|
| WorstPerturbation | Smallest perturbations at each frequency point in <code>info.Frequency</code> , returned as a structure array. The fields of <code>info.WorstPerturbation</code> are the names of the uncertain elements in <code>clp</code> . Each field contains the value of the corresponding element that drives the I/O gain to the corresponding lower bound at each frequency. For example, if <code>clp</code> includes an uncertain parameter <code>p</code> and SISO uncertain dynamics <code>delta</code> , then <code>info.WorstPerturbation.p</code> is a collection of numeric values and <code>info.WorstPerturbation.delta</code> is a collection of SISO state-space models.                             |
| Sensitivity       | Sensitivity of $\gamma$ to each uncertain element, returned as a structure when the 'Sensitivity' option of <code>robOptions</code> is 'on'. The fields of <code>info.Sensitivity</code> are the names of the uncertain elements in <code>clp</code> . Each field contains a percentage that measures how much the uncertainty in the corresponding element affects $\gamma$ . For example, if <code>info.Sensitivity.p</code> is 50, then a given fractional change in the uncertainty range of <code>p</code> causes half as much fractional change in $\gamma$ .<br><br>If the 'Sensitivity' option of <code>robOptions</code> is off (the default setting), then <code>info.Sensitivity</code> is NaN. |

## See Also

`actual2normalized` | `dksyn` | `normalized2actual` | `robOptions` | `robgain` | `robstab` | `wcgain`

**Introduced in R2017a**



# dmpplot

Interpret disk gain and phase margins

## Syntax

```
dmpplot
```

```
dmpplot(diskgm)
```

```
[dgm,dpm] = dmpplot
```

## Description

`dmpplot` brings up a plot that illustrates the disk gain margin (`dgm`) and disk phase margin (`dpm`) for a sample system. Both margins are derived from the largest disk that

- Contains the critical point  $(-1,0)$
- Does not intersect the Nyquist plot of the open-loop response  $L$

`diskgm` is the radius of this disk and a lower bound on the classical gain margin.

`dmpplot(diskgm)` plots the maximum allowable phase variation as a function of the actual gain variation for a given disk gain margin `diskgm` (the maximum gain variation being `diskgm`). The closed-loop system is guaranteed to remain stable for all combined gain/phase variations inside the plotted ellipse.

`[dgm,dpm] = dmpplot` returns the data used to plot the gain/phase variation ellipse.

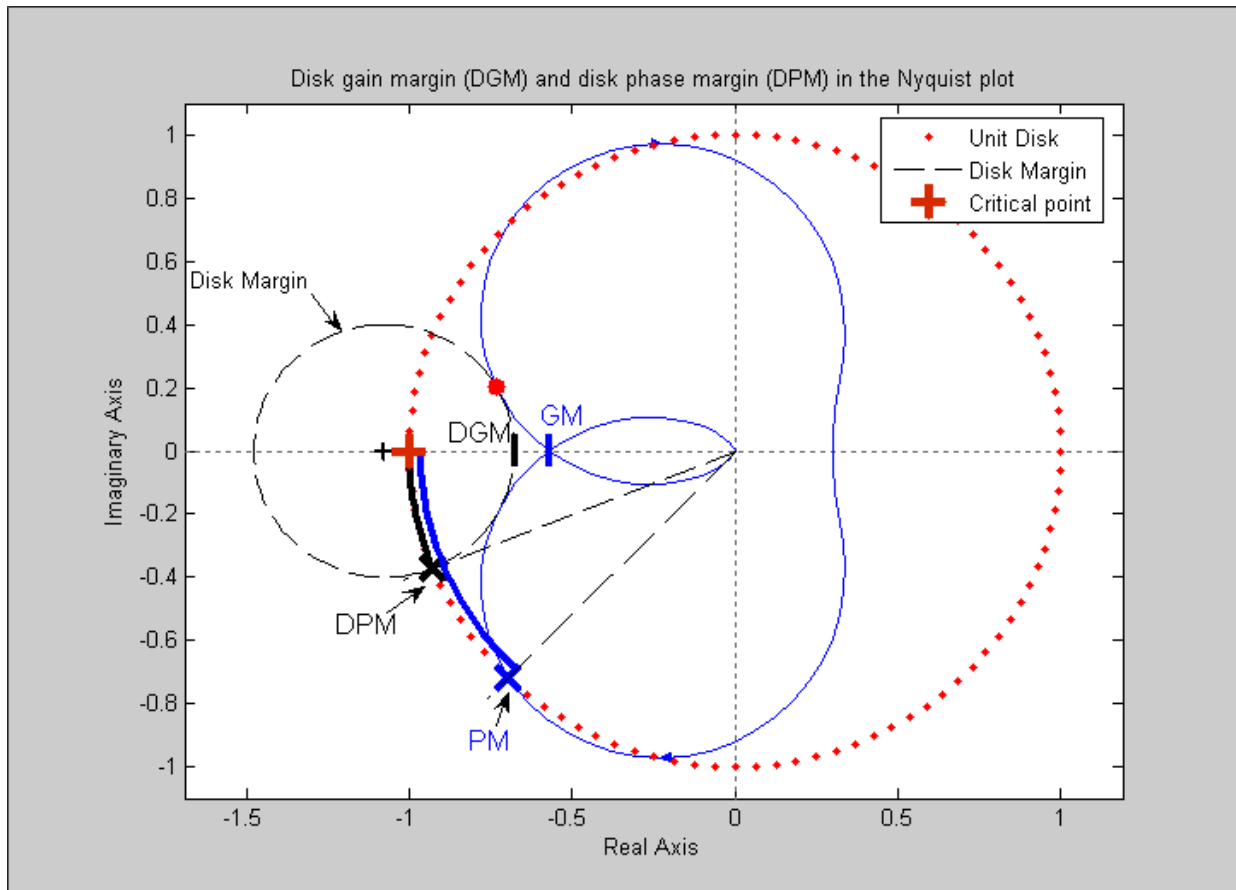
## Examples

## Disk Gain and Phase Margins

When you call `dmpLot` without an argument, the resulting plot and text shows a comparison of a disk margin analysis with the classical notations of gain and phase margins. The Nyquist plot is of the loop transfer function  $L(s)$ :

$$L(s) = \frac{\frac{s}{30} + 1}{(s + 1)(s^2 + 1.6s + 16)}$$

`dmpLot`



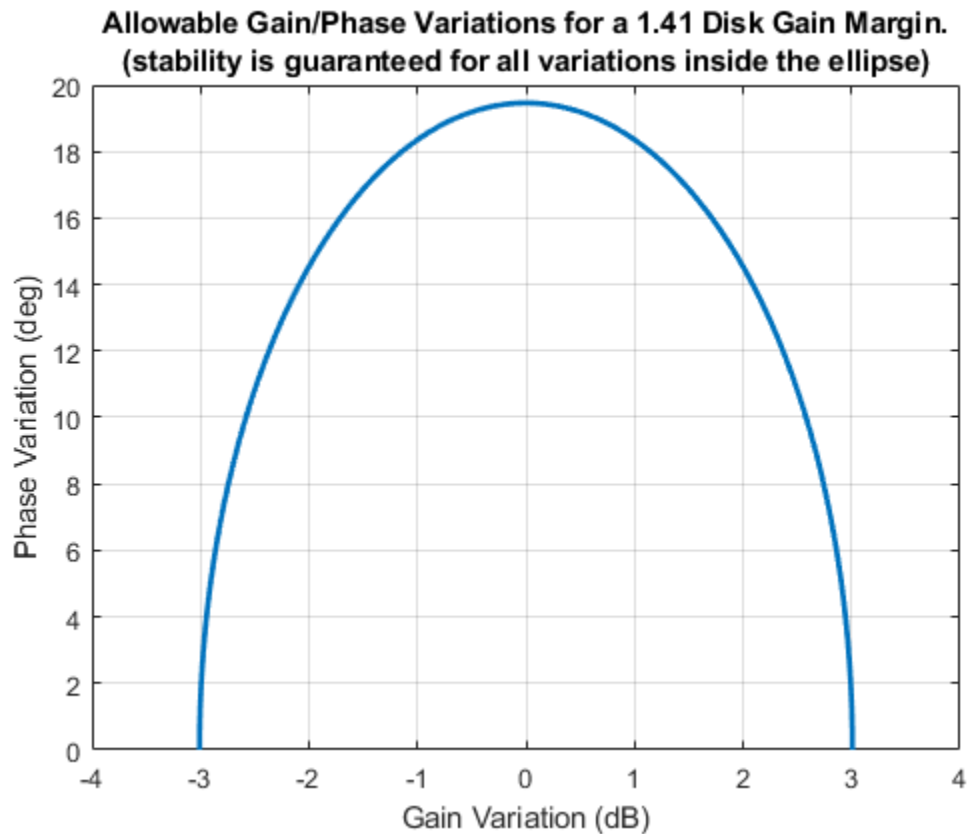
This figure shows a comparison of a disk margin analysis with the classical notations of gain and phase margins. The Nyquist plot is of the loop transfer function

$$L = 4(s/30 + 1)/((s+1)*(s^2 + 1.6s + 16))$$

- The Nyquist plot of L corresponds to the blue line
- The unit disk corresponds to the dotted red line
- GM and PM indicate the location of the classical gain and phase margins for the system L.
- DGM and DPM correspond to the disk gain and phase margins. The disk margins provide a lower bound on classical gain and phase margins.
- The disk margin circle corresponds to the dashed black line. The disk margin corresponds to the largest disk centered at  $(GMD + 1/GMD)/2$  that just touches the loop transfer function L. This location is indicated by the red dot.

The x-axis corresponds to the gain variation in dB and the y-axis corresponds to the allowable phase variation in degrees. For a disk gain margin corresponding to 3 dB (1.414), the closed-loop system is stable for all phase and gain variations inside the blue ellipse. For example, the closed-loop system can simultaneously tolerate +/- 2 dB gain variation and +/- 14 deg phase variations. To see the allowable variations for a given disk gain margin, use the given value as an input to dmp1ot.

```
figure          % new figure window
dmp1ot(1.414)
```



## References

Barrett, M.F., Conservatism with robustness tests for linear feedback control systems, Ph.D. Thesis. Control Science and Dynamical Systems, University of Minnesota, 1980.

Blight, J.D., R.L. Dailey, and Gangsass, D., "Practical control law design for aircraft using multivariable techniques," *International Journal of Control*, Vol. 59, No. 1, 1994, 93-137.

Bates, D., and I. Postlethwaite, *Robust Multivariable Control of Aerospace Systems*, Delft University Press, Delft, The Netherlands, ISBN: 90-407-2317-6, 2002.

## **See Also**

wcmargin

**Introduced before R2006a**

## drawmag

Mouse-based tool for sketching and fitting

### Syntax

```
[sysout,pts] = drawmag(data)
```

```
[sysout,pts] = drawmag(data,init_pts)
```

### Description

drawmag interactively uses the mouse in the plot window to create `pts` (the `frd` object) and `sysout` (a stable minimum-phase `ss` object), which approximately fits the frequency response (magnitude) in `pts`.

Input arguments:

|                       |   |
|-----------------------|---|
| <code>data</code>     | Either a frequency response object that is plotted as a reference, or a constant matrix of the form $[x_{min} \ x_{max} \ y_{min} \ y_{max}]$ specifying the plot window on the data. |
| <code>init_pts</code> | Optional <code>frd</code> objects of initial set of points  |

Output arguments:

|                     |  |
|---------------------|--|
| <code>sysout</code> | Stable, minimum-phase <code>ss</code> object that approximately fits, in magnitude, the <code>pts</code> data. |
| <code>pts</code>    | Frequency response of points.  |

While `drawmag` is running, all interaction with the program is through the mouse and/or the keyboard. The mouse, if there is one, must be in the plot window. The program recognizes several commands:

- Clicking the mouse button adds a point at the cross-hairs. If the cross-hairs are outside the plotting window, the points are plotted when the fitting, windowing, or replotting mode is invoked. Typing `a` is the same as clicking the mouse button.

- Typing `r` removes the point with frequency nearest that of the cross-hairs.
- Typing any integer between `0` and `9` fits the existing points with a transfer function of that order. The fitting routine approximately minimizes the maximum error in a log sense. The new fit is displayed along with the points, and the most recent previous fit, if it exists.
- Typing `w` uses the cross-hair location as the initial point in creating a window. Moving the cross-hairs and clicking the mouse or pressing any key then gives a second point at the new cross-hair location. These two points define a new window on the data, which is immediately replotted. This is useful in fine tuning parts of the data. You can call windowing repeatedly.
- Typing `p` simply replots the data using a window that covers all the current data points as well as whatever was specified in `in`. Typically used after windowing to view all the data.
- Typing `k` invokes the keyboard using the `keyboard` command. Be cautious when using this option to avoid unintended changes to variables.

## See Also

`ginput` | `loglog`

**Introduced before R2006a**

## evallmi

Given particular instance of decision variables, evaluate all variable terms in system of LMIs

### Syntax

```
evalsys = evallmi(lmisys,decvars)
```

### Description

`evallmi` evaluates all LMI constraints for a particular instance `decvars` of the vector of decision variables. Recall that `decvars` fully determines the values of the matrix variables  $X_1, \dots, X_K$ . The “evaluation” consists of replacing all terms involving  $X_1, \dots, X_K$  by their matrix value. The output `evalsys` is an LMI system containing only constant terms.

The function `evallmi` is useful for validation of the LMI solvers' output. The vector returned by these solvers can be fed directly to `evallmi` to evaluate all variable terms. The matrix values of the left and right sides of each LMI are then returned by `showlmi`.

### Observation

`evallmi` is meant to operate on the output of the LMI solvers. To evaluate all LMIs for particular instances of the matrix variables  $X_1, \dots, X_K$ , first form the corresponding decision vector `x` with `mat2dec` and then call `evallmi` with `x` as input.

### Examples

Consider the feasibility problem of finding  $X > 0$  such that

$$A^T X A - X + I < 0$$

where



$$A = \begin{pmatrix} 0.5 & -0.2 \\ 0.1 & -0.7 \end{pmatrix}.$$

This LMI system is defined by:

```
setlmiis([])
X = lmivar(1,[2 1])    % full symmetric X

lmiterm([1 1 1 X],A',A)    % LMI #1: A'*X*A
lmiterm([1 1 1 X],[-1,1])    % LMI #1: -X
lmiterm([1 1 1 0],1)    % LMI #1: I
lmiterm([-2 1 1 X],1,1)    % LMI #2: X
lmiis = getlmiis
```

To compute a solution `xfeas`, call `feasp` by

```
[tmin,xfeas] = feasp(lmiis)
```

The result is

```
tmin =
    -4.7117e+00

xfeas' =
    1.1029e+02    -1.1519e+01    1.1942e+02
```

The LMI constraints are therefore feasible since `tmin < 0`. The solution `X` corresponding to the feasible decision vector `xfeas` would be given by `X = dec2mat(lmiis,xfeas,X)`.

To check that `xfeas` is indeed feasible, evaluate all LMI constraints by typing

```
evals = evallmi(lmiis,xfeas)
```

The left and right sides of the first and second LMIs are then given by

```
[lhs1,rhs1] = showlmi(evals,1)
[lhs2,rhs2] = showlmi(evals,2)
```

and the test

```
eig(lhs1-rhs1)
ans =
    -8.2229e+01
    -5.8163e+01
```

confirms that the first LMI constraint is satisfied by `xfeas`.

## **See Also**

`dec2mat` | `mat2dec` | `setmvar` | `showlmi`

**Introduced before R2006a**

## feasp

Compute solution to given system of LMIs

### Syntax

```
[tmin,xfеas] = feasp(lmisys,options,target)
```

### Description

The function `feasp` computes a solution `xfеas` (if any) of the system of LMIs described by `lmisys`. The vector `xfеas` is a particular value of the decision variables for which all LMIs are satisfied.

Given the LMI system

$$N^T L x N \leq M^T R(x) M,$$

`xfеas` is computed by solving the auxiliary convex program:

Minimize  $t$  subject to  $N^T L(x) N - M^T R(x) M \leq t I$ .

The global minimum of this program is the scalar value `tmin` returned as first output argument by `feasp`. The LMI constraints are feasible if `tmin`  $\leq 0$  and strictly feasible if `tmin`  $< 0$ . If the problem is feasible but not strictly feasible, `tmin` is positive and very small. Some post-analysis may then be required to decide whether `xfеas` is close enough to feasible.

The optional argument `target` sets a target value for `tmin`. The optimization code terminates as soon as a value of  $t$  below this target is reached. The default value is `target = 0`.

Note that `xfеas` is a solution in terms of the decision variables and not in terms of the matrix variables of the problem. Use `dec2mat` to derive feasible values of the matrix variables from `xfеas`.

## Control Parameters

The optional argument `options` gives access to certain control parameters for the optimization algorithm. This five-entry vector is organized as follows:

- `options(1)` is not used.
- `options(2)` sets the maximum number of iterations allowed to be performed by the optimization procedure (100 by default).
- `options(3)` resets the *feasibility radius*. Setting `options(3)` to a value  $R > 0$  further constrains the decision vector  $x = (x_1, \dots, x_N)$  to lie within the ball

$$\sum_{i=1}^N x_i^2 < R^2$$

In other words, the Euclidean norm of `xfeas` should not exceed  $R$ . The feasibility radius is a simple means of controlling the magnitude of solutions. Upon termination, `feasp` displays the *f-radius saturation*, that is, the norm of the solution as a percentage of the feasibility radius  $R$ .

The default value is  $R = 109$ . Setting `options(3)` to a negative value activates the “flexible bound” mode. In this mode, the feasibility radius is initially set to 108, and increased if necessary during the course of optimization

- `options(4)` helps speed up termination. When set to an integer value  $J > 0$ , the code terminates if  $t$  did not decrease by more than one percent in relative terms during the last  $J$  iterations. The default value is 10. This parameter trades off speed vs. accuracy. If set to a small value ( $< 10$ ), the code terminates quickly but without guarantee of accuracy. On the contrary, a large value results in natural convergence at the expense of a possibly large number of iterations.
- `options(5) = 1` turns off the trace of execution of the optimization procedure. Resetting `options(5)` to zero (default value) turns it back on.

Setting `option(i)` to zero is equivalent to setting the corresponding control parameter to its default value. Consequently, there is no need to redefine the entire vector when changing just one control parameter. To set the maximum number of iterations to 10, for instance, it suffices to type

```
options=zeros(1,5)      % default value for all parameters
options(2)=10
```

## Memory Problems

When the least-squares problem solved at each iteration becomes ill conditioned, the feasp solver switches from Cholesky-based to QR-based linear algebra (see “Memory Problems” on page 1-266 for details). Since the QR mode typically requires much more memory, MATLAB® may run out of memory and display the message

```
??? Error using ==> feaslsv
Out of memory. Type HELP MEMORY for your options.
```

You should then ask your system manager to increase your swap space or, if no additional swap space is available, set `options(4) = 1`. This will prevent switching to QR and feasp will terminate when Cholesky fails due to numerical instabilities.

## Examples

### Solve System of LMIs

Consider the problem of finding  $P > I$  such that:

$$A_1^T P + P A_1 < 0,$$

$$A_2^T P + P A_2 < 0,$$

$$A_3^T P + P A_3 < 0,$$

with data

$$A_1 = \begin{pmatrix} -1 & 2 \\ 1 & -3 \end{pmatrix}, A_2 = \begin{pmatrix} -0.8 & 1.5 \\ 1.3 & -2.7 \end{pmatrix}, A_3 = \begin{pmatrix} -1.4 & 0.9 \\ 0.7 & -2.0 \end{pmatrix}.$$

This problem arises when studying the quadratic stability of the polytope of the matrices,  $\text{Co}\{A_1, A_2, A_3\}$ .

To assess feasibility using feasp, first enter the LMIs.

```

setlmis([])
p = lmivar(1,[2 1]);

A1 = [-1 2;1 -3];
A2 = [-0.8 1.5; 1.3 -2.7];
A3 = [-1.4 0.9;0.7 -2.0];

lmiterm([1 1 1 p],1,A1,'s'); % LMI #1
lmiterm([2 1 1 p],1,A2,'s'); % LMI #2
lmiterm([3 1 1 p],1,A3,'s'); % LMI #3
lmiterm([-4 1 1 p],1,1); % LMI #4: P
lmiterm([4 1 1 0],1,1); % LMI #4: I
lmis = getlmis;

```

Call `feasp` to find a feasible decision vector.

```
[tmin,xfeas] = feasp(lmis);
```

Solver for LMI feasibility problems  $L(x) < R(x)$   
 This solver minimizes  $t$  subject to  $L(x) < R(x) + t*I$   
 The best value of  $t$  should be negative for feasibility

Iteration : Best value of t so far

|   |           |
|---|-----------|
| 1 | 0.972718  |
| 2 | 0.870460  |
| 3 | -3.136305 |

Result: best value of t: -3.136305  
 f-radius saturation: 0.000% of R = 1.00e+09

The result  $t_{min} = -3.1363$  means that the problem is feasible. Therefore, the dynamical system  $\dot{x} = A(t)x$  is quadratically stable for  $A(t) \in \text{Co}\{A_1, A_2, A_3\}$ .

To obtain a Lyapunov matrix  $P$  proving the quadratic stability, use `dec2mat`.

```
P = dec2mat(lmis,xfeas,p)
```

P = 2x2

|          |          |
|----------|----------|
| 270.8553 | 126.3999 |
| 126.3999 | 155.1336 |

It is possible to add further constraints on this feasibility problem. For instance, the following command bounds the Frobenius norm of P by 10 while asking tmin to be less than or equal to -1.

```
options = [0,0,10,0,0];
[tmin,xfeas] = feasp(lmis,options,-1);
```

```
Solver for LMI feasibility problems L(x) < R(x)
  This solver minimizes t subject to L(x) < R(x) + t*I
  The best value of t should be negative for feasibility
```

```
Iteration   :   Best value of t so far

           1           0.988505
           2           0.872239
           3          -0.476638
           4          -0.920574
           5          -0.920574
***                new lower bound:   -3.726964
           6          -1.011130
***                new lower bound:   -1.602398
```

```
Result: best value of t:   -1.011130
        f-radius saturation: 91.385% of R = 1.00e+01
```

The third entry of `options` sets the feasibility radius to 10 while the third argument to `feasp`, -1, sets the target value for `tmin`. This constraint yields `tmin = -1.011` and a matrix P with largest eigenvalue  $\lambda_{\max}(P) = 8.4653$ .

```
P = dec2mat(lmis,xfeas,p);
e = eig(P)
```

```
e = 2x1

    3.8875
    8.4653
```

## References

The feasibility solver `feasp` is based on Nesterov and Nemirovski's Projective Method described in:

Nesterov, Y., and A. Nemirovski, *Interior Point Polynomial Methods in Convex Programming: Theory and Applications*, SIAM, Philadelphia, 1994.

Nemirovski, A., and P. Gahinet, "The Projective Method for Solving Linear Matrix Inequalities," *Proc. Amer. Contr. Conf.*, 1994, Baltimore, Maryland, p. 840-844.

The optimization is performed by the C-MEX file `feaslv.mex`.

## See Also

`dec2mat` | `gevp` | `mincx`

**Introduced before R2006a**



## fitfrd

Fit frequency response data with state-space model

### Syntax

```
B = fitfrd(A,N)
```

```
B = fitfrd(A,N,RD)
```

```
B = fitfrd(A,N,RD,WT)
```

### Description

`B = fitfrd(A,N)` is a state-space object with state dimension `N`, where `A` is an `frd` object and `N` is a nonnegative integer. The frequency response of `B` closely matches the  $D$ -scale frequency response data in `A`.

`A` must have either 1 row or 1 column, although it need not be 1-by-1. `B` will be the same size as `A`. In all cases, `N` should be a nonnegative scalar.

`B = fitfrd(A,N,RD)` forces the relative degree of `B` to be `RD`. `RD` must be a nonnegative integer. The default value for `RD` is 0. If `A` is a row (or column) then `RD` can be a vector of the same size as well, specifying the relative degree of each entry of `B`. If `RD` is a scalar, then it specifies the relative degree for all entries of `B`. You can specify the default value for `RD` by setting `RD` to an empty matrix.

`B = fitfrd(A,N,RD,WT)` uses the magnitude of `WT` to weight the optimization fit criteria. `WT` can be a `double`, `ss` or `frd`. If `WT` is a scalar, then it is used to weight all entries of the error criteria (`A-B`). If `WT` is a vector, it must be the same size as `A`, and each individual entry of `WT` acts as a weighting function on the corresponding entry of (`A-B`).

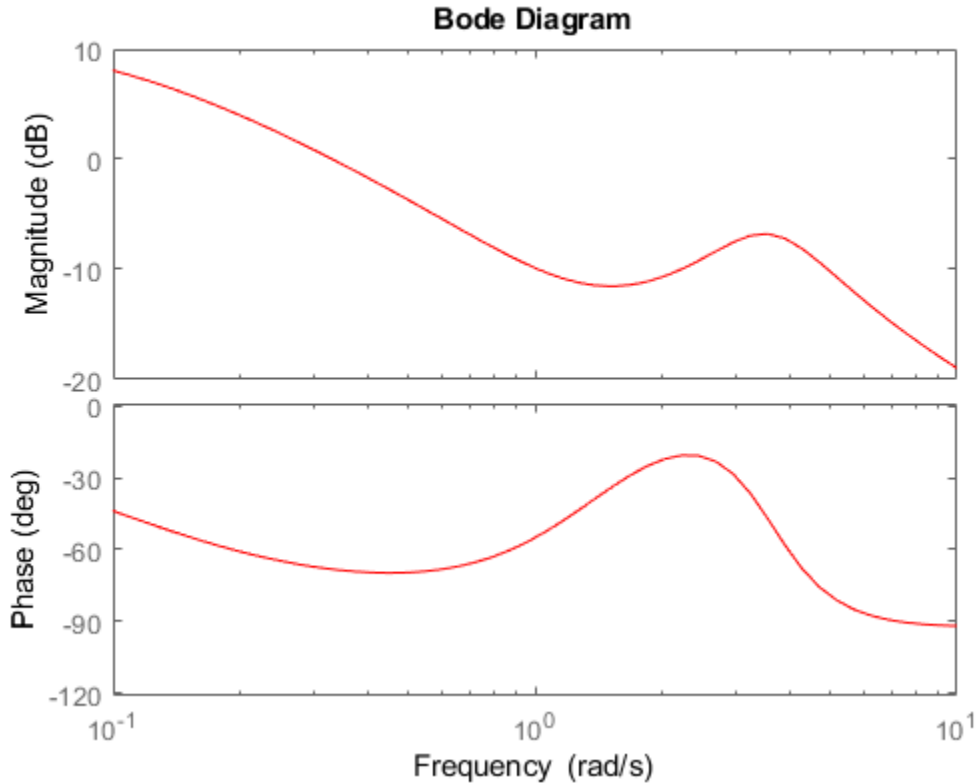
### Examples

### Fit D-scale Data

Use the `fitfrd` command to fit *D*-scale data.

Create *D*-scale frequency response data from a fifth-order system.

```
sys = tf([1 2 2],[1 2.5 1.5])*tf(1,[1 0.1]);  
sys = sys*tf([1 3.75 3.5],[1 2.5 13]);  
omeg = logspace(-1,1);  
sysg = frd(sys,omeg);  
bode(sysg, 'r-');
```

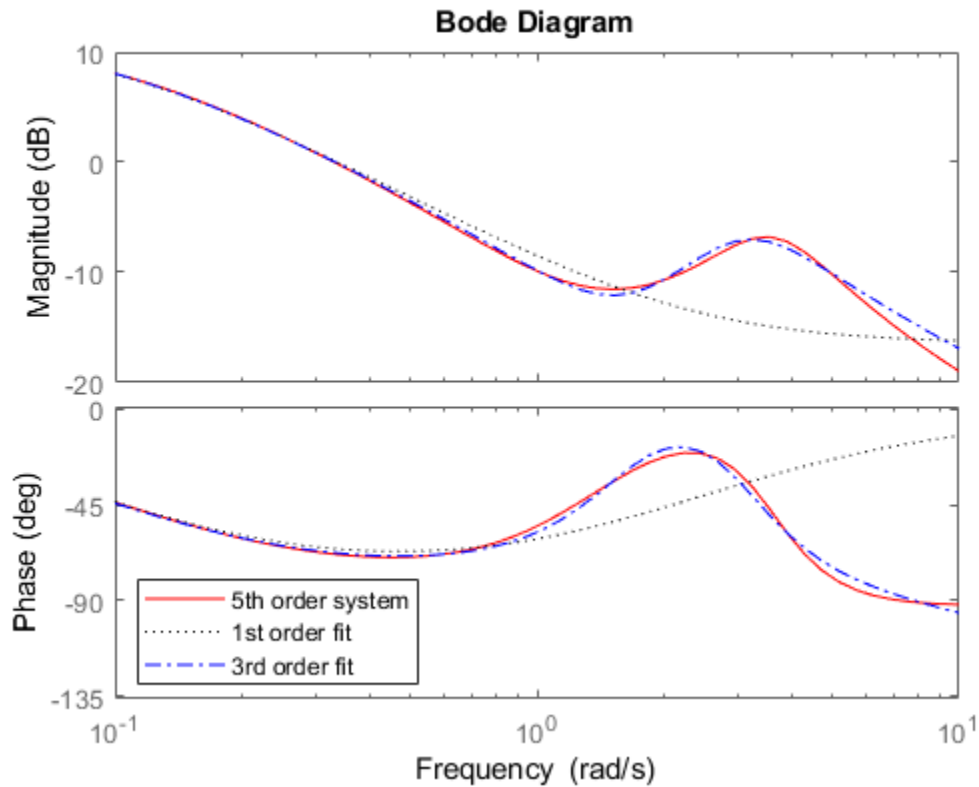


You can try to fit the frequency response *D*-scale data `sysg` with a first-order system, `b1`. Similarly, you can fit the *D*-scale data with a third-order system, `b3`.

```
b1 = fitfrd(sysg,1);  
b3 = fitfrd(sysg,3);
```

Compare the original *D*-scale data *sysg* with the frequency responses of the first and third-order models calculated by *fitfrd*.

```
b1g = frd(b1,omeg);  
b3g = frd(b3,omeg);  
bode(sysg, 'r-', b1g, 'k:', b3g, 'b-.')  
legend('5th order system', '1st order fit', '3rd order fit', 'Location', 'Southwest')
```



## **Limitations**

Numerical conditioning problems arise if the state order of the fit  $N$  is selected to be higher than required by the dynamics of  $A$ .

## **See Also**

fitmagfrd

**Introduced before R2006a**

# fitmagfrd

Fit frequency response magnitude data with minimum-phase state-space model using log-Chebyshev magnitude design

## Syntax

```
B = fitmagfrd(A,N)
```

```
B = fitmagfrd(A,N,RD)
```

```
B = fitmagfrd(A,N,RD,WT)
```

```
B = fitmagfrd(A,N,RD,WT,C)
```

## Description

`B = fitmagfrd(A,N)` is a stable, minimum-phase `ss` object, with state-dimension `N`, whose frequency response magnitude closely matches the magnitude data in `A`. `A` is a 1-by-1 `frd` object, and `N` is a nonnegative integer.

`B = fitmagfrd(A,N,RD)` forces the relative degree of `B` to be `RD`. `RD` must be a nonnegative integer whose default value is 0. You can specify the default value for `RD` by setting `RD` to an empty matrix.

`B = fitmagfrd(A,N,RD,WT)` uses the magnitude of `WT` to weight the optimization fit criteria. `WT` can be a `double`, `ss` or `frd`. If `WT` is a scalar, then it is used to weight all entries of the error criteria (`A-B`). If `WT` is a vector, it must be the same size as `A`, and each individual entry of `WT` acts as a weighting function on the corresponding entry of (`A-B`). The default value for `WT` is 1, and you can specify it by setting `WT` to an empty matrix.

`B = fitmagfrd(A,N,RD,WT,C)` enforces additional magnitude constraints on `B`, specified by the values of `C.LowerBound` and `C.UpperBound`. These can be empty, `double` or `frd` (with `C.Frequency` equal to `A.Frequency`). If `C.LowerBound` is non-empty, then the magnitude of `B` is constrained to lie above `C.LowerBound`. No lower bound is enforced at frequencies where `C.LowerBound` is equal to `-inf`. Similarly, the `UpperBound` field can be used to specify an upper bound on the magnitude of `B`. If `C` is a

double or frd (with C.Frequency equal to A.Frequency), then the upper and lower bound constraints on B are taken directly from A as:

- if  $C(w) == -1$ , then enforce  $\text{abs}(B(w)) \leq \text{abs}(A(w))$
- if  $C(w) == 1$ , then enforce  $\text{abs}(B(w)) \geq \text{abs}(A(w))$
- if  $C(w) == 0$ , then no additional constraint

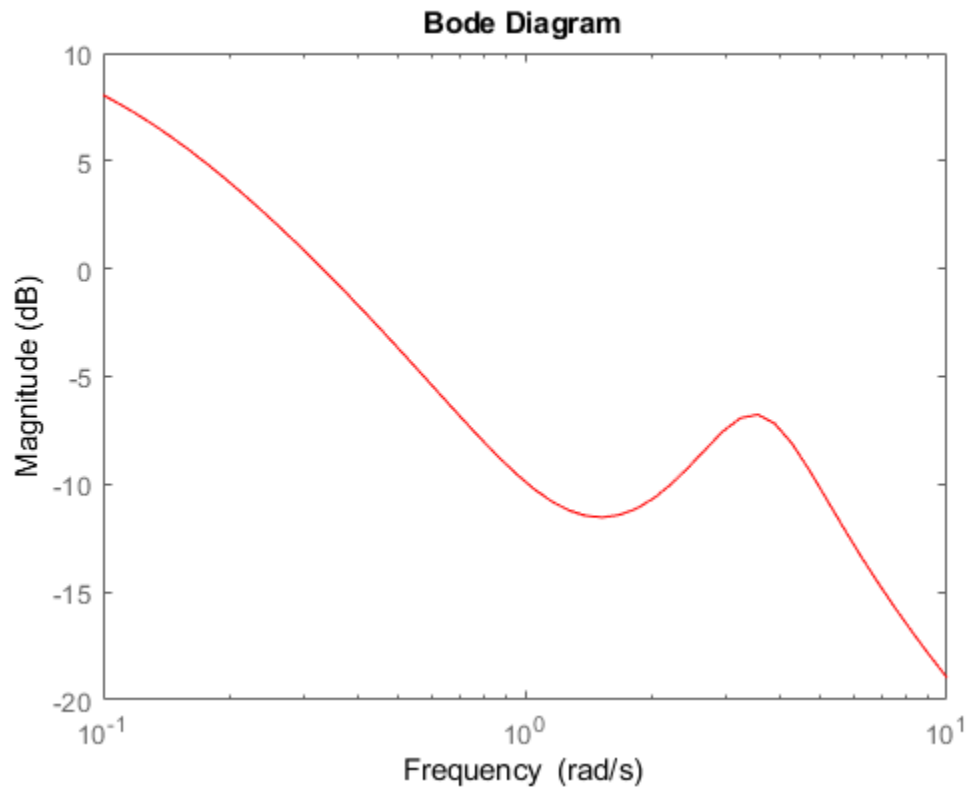
where  $w$  denotes the frequency.

## Examples

### Fit Frequency Response Data With Stable Minimum-Phase State-Space Model

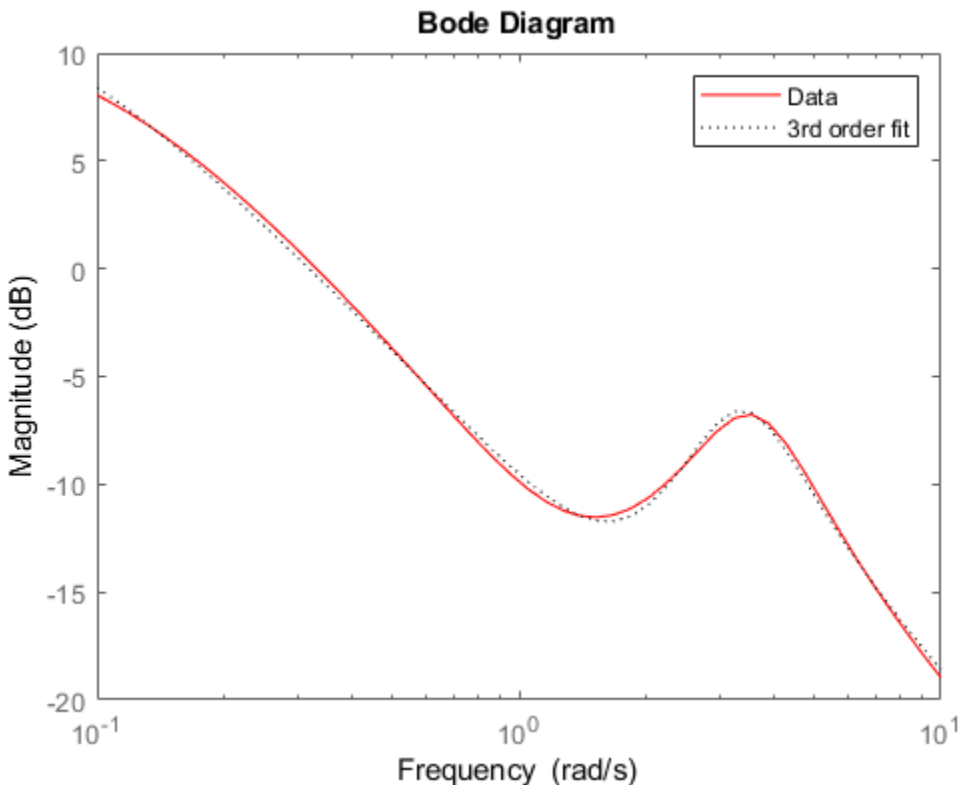
Create frequency response magnitude data from a fifth-order system.

```
sys = tf([1 2 2],[1 2.5 1.5])*tf(1,[1 0.1]);  
sys = sys*tf([1 3.75 3.5],[1 2.5 13]);  
omega = logspace(-1,1);  
sysg = abs(frd(sys,omega));  
bodemag(sysg, 'r');
```



Fit the magnitude data with a minimum-phase, stable third-order system.

```
ord = 3;  
b1 = fitmagfrd(sysg,ord);  
b1g = frd(b1,omega);  
bodemag(sysg,'r',b1g,'k:');  
legend('Data','3rd order fit');
```



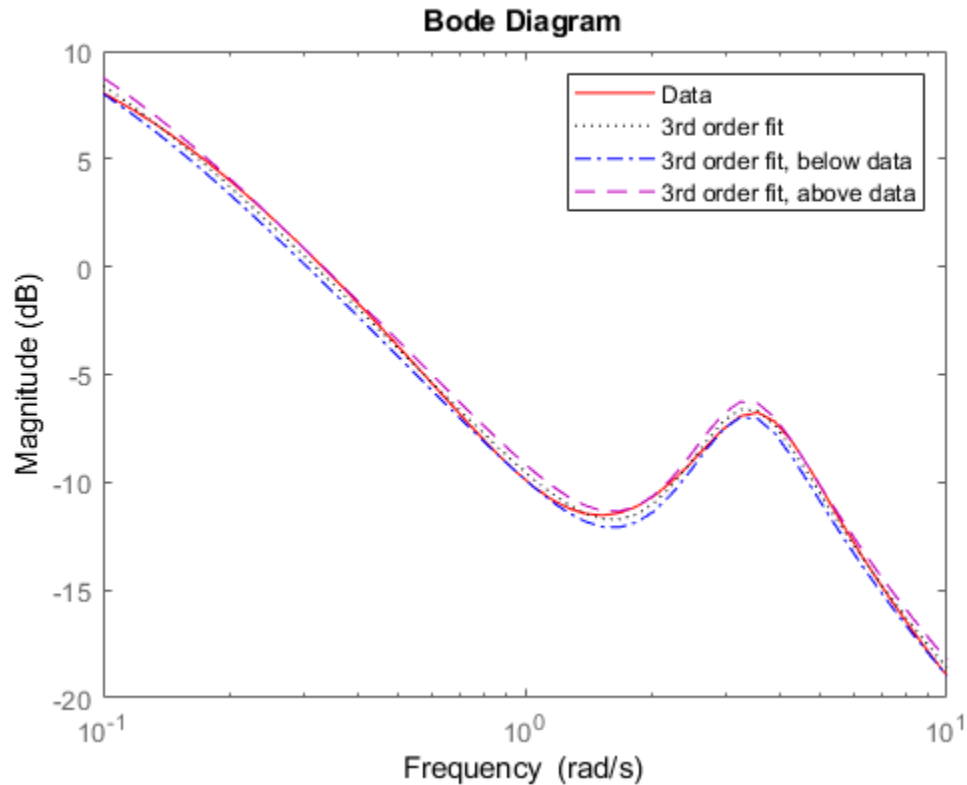
Fit the magnitude data with a third-order system constrained to lie below and above the given data.

```

C2.UpperBound = sysg;
C2.LowerBound = [];
b2 = fitmagfrd(sysg,ord,[],[],C2);
b2g = frd(b2,omega);
C3.UpperBound = [];
C3.LowerBound = sysg;
b3 = fitmagfrd(sysg,ord,[],[],C3);
b3g = frd(b3,omega);
bodemag(sysg,'r',b1g,'k:',b2g,'b-.',b3g,'m--')
legend('Data','3rd order fit','3rd order fit, below data',...
      '3rd order fit, above data')

```

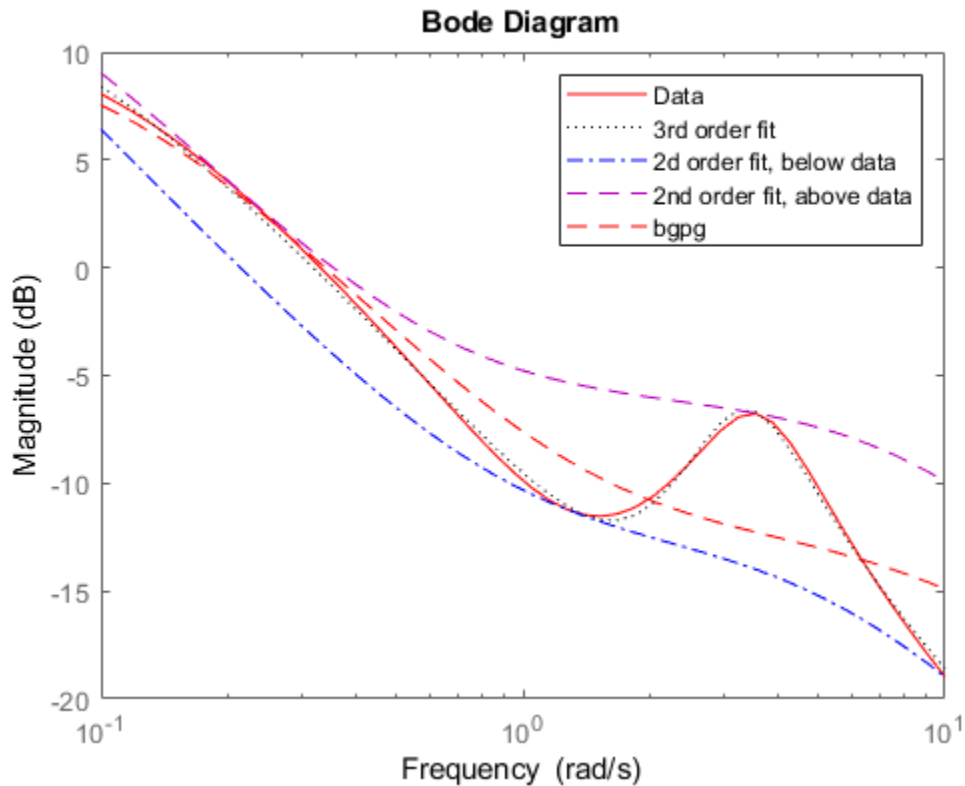




Fit the magnitude data with a second-order system constrained to lie below and above the given data.

```
ord = 2;
C2.UpperBound = sysg;
C2.LowerBound = [];
b2 = fitmagfrd(sysg,ord,[],sysg,C2);
b2g = frd(b2,omega);
C3.UpperBound = [];
C3.LowerBound = sysg;
b3 = fitmagfrd(sysg,ord,[],sysg,C3);
b3g = frd(b3,omega);
bpg = fitfrd(genphase(sysg),ord);
bpgg = frd(bpg,omega);
```

```
bodemag(sysg, 'r', b1g, 'k:', b2g, 'b-.', b3g, 'm--', bpgg, 'r--')  
legend('Data', '3rd order fit', '2d order fit, below data', ...  
       '2nd order fit, above data', 'bpgg')
```



## Limitations

This input `frd` object must be either a scalar 1-by-1 object or, a row, or column vector.

## Algorithms

fitmagfrd uses a version of log-Chebyshev magnitude design, solving

$$\min f \quad \text{subject to (at every frequency point in A):}$$

$$|d|^2 / (1 + f/WT) < |n|^2 / A^2 < |d|^2 (1 + f/WT)$$

plus additional constraints imposed with C. n, d denote the numerator and denominator, respectively, and B = n/d. n and d have orders (N-RD) and N, respectively. The problem is solved using linear programming for fixed f and bisection to minimize f. An alternate approximate method, which cannot enforce the constraints defined by C, is B = fitfrd(genphase(A), N, RD, WT).

## References

Oppenheim, A.V., and R.W. Schaffer, *Digital Signal Processing*, Prentice Hall, New Jersey, 1975, p. 513.

Boyd, S. and Vandenberghe, L., *Convex Optimization*, Cambridge University Press, 2004.

## See Also

fitfrd

**Introduced before R2006a**

## gapmetric

Compute upper bounds on Vinnicombe gap and nugap distances between two systems

### Syntax

```
[gap,nugap] = gapmetric(p0,p1)
```

```
[gap,nugap] = gapmetric(p0,p1,tol)
```

### Description

`[gap,nugap] = gapmetric(p0,p1)` calculates upper bounds on the gap and nugap (Vinnicombe) metric between systems `p0` and `p1`. The gap and nugap values lie between 0 and 1. A small value (relative to 1) implies that any controller that stabilizes `p0` will likely stabilize `p1`, and, moreover, that the closed-loop gains of the two closed-loop systems will be similar. A gap or nugap of 0 implies that `p0` equals `p1`, and a value of 1 implies that the plants are far apart. The input and output dimensions of `p0` and `p1` must be the same.

`[gap,nugap] = gapmetric(p0,p1,tol)` specifies a relative accuracy for calculating the gap metric and nugap metric. The default value for `tol` is 0.001. The computed answers are guaranteed to satisfy

```
gap-tol < gapexact(p0,p1) <= gap
```

### Examples

#### Compute gap and nugap Metrics for Stable and Unstable Plant Models

Create two plant models. One plant is unstable, first-order, with transfer function  $1/(s - 0.001)$ . The other plant is stable and first-order with transfer function  $1/(s + 0.001)$ .

```
p1 = tf(1,[1 -0.001]);  
p2 = tf(1,[1 0.001]);
```

Despite the fact that one plant is unstable and the other is stable, these plants are close in the gap and nugap metrics.

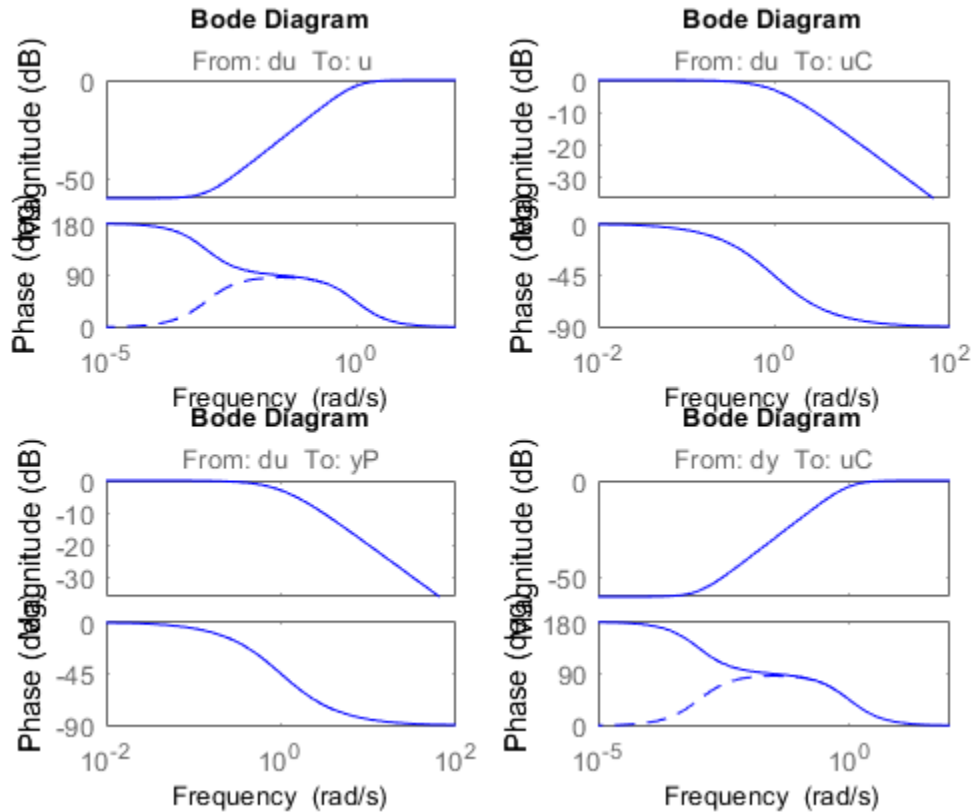
```
[g,ng] = gapmetric(p1,p2)
```

```
g = 0.0029
```

```
ng = 0.0020
```

Intuitively, this result is obvious, because, for instance, the feedback controller  $K = 1$  stabilizes both plants and renders the closed-loop systems nearly identical.

```
K = 1;  
H1 = loopsens(p1,K);  
H2 = loopsens(p2,K);  
subplot(2,2,1); bode(H1.Si, '-',H2.Si, '--');  
subplot(2,2,2); bode(H1.Ti, '-',H2.Ti, '--');  
subplot(2,2,3); bode(H1.PSi, '-',H2.PSi, '--');  
subplot(2,2,4); bode(H1.CSo, '-',H2.CSo, '--');
```



Next, consider two stable plant models that differ by a first-order system. One plant is the transfer function  $50/(s + 50)$  and the other plant is the transfer function  $50/(s + 50) * 8/(s + 8)$ .

```
p3 = tf([50],[1 50]);
p4 = tf([8],[1 8])*p3;
```

Although the two systems have similar high-frequency dynamics and the same unity gain at low frequency, the plants are modestly far apart in the gap and nugap metrics.

```
[g,ng] = gapmetric(p3,p4)
g = 0.6156
ng = 0.6147
```

## Algorithms

gap and nugap compute the gap and  $\nu$  gap metrics between two LTI objects. Both quantities give a numerical value  $\delta(p\theta, p1)$  between 0 and 1 for the distance between a nominal system  $p\theta$  ( $G_0$ ) and a perturbed system  $p1$  ( $G_1$ ). The gap metric was introduced into the control literature by Zames and El-Sakkary 1980, and exploited by Georgiou and Smith 1990. The  $\nu$  gap metric was derived by Vinnicombe 1993. For both of these metrics the following robust performance result holds from Qui and Davidson 1992, and Vinnicombe 1993

$$\arcsin b(G_1, K_1) \geq \arcsin b(G_0, K_0) - \arcsin \delta(G_0, G_1) - \arcsin \delta(K_0, K_1)$$

where

$$b(G, K) = \left\| \begin{bmatrix} I \\ K \end{bmatrix} (I - GK)^{-1} \begin{bmatrix} G & I \end{bmatrix} \right\|_{\infty}^{-1}$$

The interpretation of this result is that if a nominal plant  $G_0$  is stabilized by controller  $K_0$ , with “stability margin”  $b(G_0, K_0)$ , then the stability margin when  $G_0$  is perturbed to  $G_1$  and  $K_0$  is perturbed to  $K_1$  is degraded by no more than the above formula. Note that  $1/b(G, K)$  is also the signal gain from disturbances on the plant input and output to the input and output of the controller. The  $\nu$  gap is always less than or equal to the gap, so its predictions using the above robustness result are tighter.

To make use of the gap metrics in robust design, weighting functions need to be introduced. In the above robustness result,  $G$  needs to be replaced by  $W_2GW_1$  and  $K$  by

$W_1^{-1}KW_2^{-1}$  (similarly for  $G_0$ ,  $G_1$ ,  $K_0$  and  $K_1$ ). This makes the weighting functions compatible with the weighting structure in the  $H_{\infty}$  loop shaping control design procedure (see loopsyn and ncfsyn for more details).

The computation of the gap amounts to solving 2-block  $H_{\infty}$  problems (Georgiou, Smith 1988). The particular method used here for solving the  $H_{\infty}$  problems is based on Green et al., 1990. The computation of the nugap uses the method of Vinnicombe, 1993.

## References

Georgiou, T.T., "On the computation of the gap metric, " *Systems Control Letters*, Vol. 11, 1988, p. 253-257

Georgiou, T.T., and M. Smith, "Optimal robustness in the gap metric," *IEEE Transactions on Automatic Control*, Vol. 35, 1990, p. 673-686

Green, M., K. Glover, D. Limebeer, and J.C. Doyle, "A J-spectral factorization approach to  $H_\infty$  control," *SIAM J. of Control and Opt.*, 28(6), 1990, p. 1350-1371

Qiu, L., and E.J. Davison, "Feedback stability under simultaneous gap metric uncertainties in plant and controller," *Systems Control Letters*, Vol. 18-1, 1992 p. 9-22

Vinnicombe, G., "Measuring Robustness of Feedback Systems," PhD Dissertation, Department of Engineering, University of Cambridge, 1993.

Zames, G., and El-Sakkary, "Unstable systems and feedback: The gap metric," *Proceedings of the Allerton Conference*, October 1980, p. 380-385

## See Also

loopsyn | ncfsyn | robstab | wcmargin | wcsens

**Introduced before R2006a**



# genphase

Fit single-input/single-output magnitude data with real, rational, minimum-phase transfer function

## Syntax

```
resp = genphase(d)
```

## Description

genphase uses the complex-cepstrum algorithm to generate a complex frequency response `resp` whose magnitude is equal to the real, positive response `d`, but whose phase corresponds to a stable, minimum-phase function. The input, `d`, and output, `resp`, are `frd` objects.

## References

Oppenheim, A.V., and R.W. Schaffer, *Digital Signal Processing*, Prentice Hall, New Jersey, 1975, p. 513.

## See Also

`fitfrd` | `fitmagfrd`

**Introduced before R2006a**

## getLimits

Validity range for uncertain real (ureal) parameters

When the uncertainty range of a ureal parameter is not centered at its nominal value, there are restrictions on the range of values the parameter can take. For robust stability analysis, these restrictions mean that the smallest destabilizing perturbation of the parameter may be out of the reach of the specified ureal model. Use `getLimits` to find out the range of actual and normalized values that a ureal parameter can take.

## Syntax

```
[ActLims, NormLims] = getLimits(ublk)
```

## Description

`[ActLims, NormLims] = getLimits(ublk)` computes the intervals of actual and normalized values that an uncertain real parameter can take. For meaningful analysis results, the actual and normalized values of `ublk` must remain in these intervals. Values outside these intervals are essentially meaningless. In other words, `ActLims` and `NormLims` are the ranges of validity of the uncertainty model for real parameters.

## Examples

### Validity Range for Uncertain Parameters

Create a ureal uncertain parameter with range centered at the nominal value.

```
ublk = ureal('a', 1, 'range', [-1 3])
```

```
ublk =
```

```
    Uncertain real parameter "a" with nominal value 1 and range [-1,3].
```

For such a parameter,  $b = 0$  (see “Algorithms” on page 1-118), so there is no constraint on the values that the actual uncertainty (`ublk`) and the normalized uncertainty ( $\Delta$ ) can take. Use `getLimits` to confirm the ranges of the actual and normalized uncertainty.

```
[ActLims, NormLims] = getLimits(ublk)
```

```
ActLims = 1×2
```

```
  -Inf   Inf
```

```
NormLims = 1×2
```

```
  -Inf   Inf
```

Skew the uncertainty range to the right of the nominal value ( $DL < DR$ ).

```
ublk.PlusMinus = [-1 2]
```

```
ublk =
```

```
  Uncertain real parameter "a" with nominal value 1 and range [0,3].
```

Now, the values that `ublk` and  $\Delta$  can take for analysis purposes are limited.

```
[ActLims, NormLims] = getLimits(ublk)
```

```
ActLims = 1×2
```

```
  -3.0000   Inf
```

```
NormLims = 1×2
```

```
  -Inf     3
```

## Input Arguments

**ublk** — Uncertain real parameter

ureal

Uncertain real parameter, specified as a `ureal` object.

## Output Arguments

### **ActLims** — Limits on actual uncertainty

2-element row vector

Limits on the actual uncertainty range taken by `ublk` for analysis purposes, returned as a 2-element vector of the form `[min,max]`. When the uncertainty range specified in `ublk` is centered on the nominal value, `ActLims = -Inf, Inf`.

### **NormLims** — Limits on normalized uncertainty

2-element row vector

Limits on the normalized uncertainty range of `ublk` used for analysis purposes, returned as a 2-element vector of the form `[min,max]`. When the uncertainty range specified in `ublk` is centered on the nominal value, `NormLims = -Inf, Inf`.

## Algorithms

Analysis functions such as `robstab` and `robgain` model uncertain real parameters as:

$$u = u_{nom} + \frac{a\Delta}{1 - b\Delta}, \quad a > 0,$$

where  $u$  is the actual value,  $u_{nom}$  is the nominal value, and  $\Delta$  is the normalized value.

When the uncertainty range is centered at the nominal value, there are no restrictions on the values  $u$  or  $\Delta$  can take. However, when the uncertainty range is skewed, there are limitations on these values. To ensure continuity, the analysis functions restrict the values  $\Delta$  and  $u$  to the ranges:

$$\Delta < \frac{1}{|b|}, \quad u > \left( u_{nom} - \left| \frac{a}{b} \right| \right), \quad \text{for } DL < DR$$
$$\Delta > -\frac{1}{|b|}, \quad u < \left( u_{nom} + \left| \frac{a}{b} \right| \right), \quad \text{for } DL < DR,$$

where  $DL$  and  $DR$  define the uncertainty range of  $u$ ,  $[u_{nom}-DL, u_{nom}+DR]$ . Note that  $b$  and  $DR-DL$  always have the same sign.

## See Also

`actual2normalized` | `normalized2actual` | `ureal`

**Introduced in R2018a**

## **getlmi**

Internal description of LMI system

### **Syntax**

```
lmiobj = getlmi
```

### **Description**

After completing the description of a given LMI system with `lmivar` and `lmiterm`, its internal representation `lmiobj` is obtained with the command

```
lmiobj = getlmi
```

This MATLAB representation of the LMI system can be forwarded to the LMI solvers or any other LMI-Lab function for subsequent processing.

### **See Also**

```
lmiterm | lmivar | newlmi | setlmi
```

**Introduced before R2006a**

# getNominal

Nominal value of uncertain model

## Syntax

```
Mnom = getNominal(M)
```

## Description

`Mnom = getNominal(M)` replaces all uncertain elements in `M` with their nominal values. All other control design blocks in `M` are unchanged.

## Examples

### Nominal Value of Uncertain Models

Create a model of a mass-spring-damper system in which the mass, spring constant, and damping constant are all uncertain.

```
m = ureal('m',3,'percent',40);
k = ureal('k',2,'percent',30);
c = ureal('c',1,'percent',20);
G = tf(1,[m,c,k])
```

```
G =
```

```
Uncertain continuous-time state-space model with 1 outputs, 1 inputs, 2 states.
```

```
The model uncertainty consists of the following blocks:
```

```
  c: Uncertain real, nominal = 1, variability = [-20,20]%, 1 occurrences
  k: Uncertain real, nominal = 2, variability = [-30,30]%, 1 occurrences
  m: Uncertain real, nominal = 3, variability = [-40,40]%, 1 occurrences
```

Type "G.NominalValue" to see the nominal value, "get(G)" to see all properties, and "G

G is a `uss` model. Extract its nominal value.

```
Gnom = getNominal(G);
```

Because `G` has only uncertain control design blocks, `Gnom` is a numeric state-space (ss) model.

Combine `G` with a tunable PID controller:

```
C = tunablePID('C','pid');  
T = feedback(G*C,1)
```

```
T =
```

```
Generalized continuous-time state-space model with 1 outputs, 1 inputs, 3 states, and 1 control input.  
C: Parametric PID controller, 1 occurrences.  
c: Uncertain real, nominal = 1, variability = [-20,20]%, 1 occurrences  
k: Uncertain real, nominal = 2, variability = [-30,30]%, 1 occurrences  
m: Uncertain real, nominal = 3, variability = [-40,40]%, 1 occurrences
```

Type `"ss(T)"` to see the current value, `"get(T)"` to see all properties, and `"T.Blocks"` to see the blocks.

`T` is a generalized state-space (`genss`) model that has both tunable and uncertain blocks. Extract the nominal value of `T`.

```
Tnom = getNominal(T)
```

```
Tnom =
```

```
Generalized continuous-time state-space model with 1 outputs, 1 inputs, 3 states, and 1 control input.  
C: Parametric PID controller, 1 occurrences.
```

Type `"ss(Tnom)"` to see the current value, `"get(Tnom)"` to see all properties, and `"Tnom.Blocks"` to see the blocks.

Extracting the nominal value of `T` preserves the tunable control design block, resulting in another `genss` model.

## Input Arguments

### **M** — Uncertain model or matrix

dynamic system model | static model



Uncertain model or matrix, specified as a dynamic system model or static model. Typically,  $M$  is a model that contains uncertainty, such as a `uss`, uncertain `genss`, or `umat` model.

## Output Arguments

### **Mnom — Nominal model or matrix**

dynamic system model | static model

Nominal value of  $M$ , returned as a dynamic system model or static model.  $Mnom$  has no uncertain blocks.

The model type of  $Mnom$  depends on the type of  $M$ . For example, if  $M$  is a `genss` model with uncertain blocks and tunable blocks, then  $Mnom$  is a `genss` model with tunable blocks.

If  $M$  contains no control design blocks other than uncertain blocks, then  $Mnom$  is a state-space (`ss`) model, an `frd` model, or a numeric array, depending on the type of  $M$ . For example, if  $M$  is a `uss` model, then  $Mnom$  is a `ss` model. If  $M$  is a `umat`, then  $Mnom$  is a numeric array.

If  $M$  has no uncertain blocks, then  $Mnom = M$ .

## See Also

`genss` | `umat` | `uss`

**Introduced in R2015b**

## gevp

Generalized eigenvalue minimization under LMI constraints

### Syntax

```
[lopt,xopt] = gevp(lmisys,nlfc,options,linit,xinit,target)
```

### Description

gevp solves the generalized eigenvalue minimization problem of minimizing  $\lambda$ , subject to:

$$C(x) < D(x)$$

$$0 < B(x)$$

$$A(x) < \lambda B(x)$$

where  $C(x) < D(x)$  and  $A(x) < \lambda B(x)$  denote systems of LMIs. Provided that “Equation 1-4” on page 1-124 and “Equation 1-5” on page 1-124 are jointly feasible, **gevp** returns the global minimum **lopt** and the minimizing value **xopt** of the vector of decision variables  $x$ . The corresponding optimal values of the matrix variables are obtained with **dec2mat**.

The argument **lmisys** describes the system of LMIs “Equation 1-4” on page 1-124 to “Equation 1-6” on page 1-124 for  $\lambda = 1$ . The LMIs involving  $\lambda$  are called the *linear-fractional constraints* while “Equation 1-4” on page 1-124 and “Equation 1-5” on page 1-124 are regular LMI constraints. The number of linear-fractional constraints “Equation 1-6” on page 1-124 is specified by **nlfc**. All other input arguments are optional. If an initial feasible pair  $(\lambda_0, x_0)$  is available, it can be passed to **gevp** by setting **linit** to  $\lambda_0$  and **xinit** to  $x_0$ . Note that **xinit** should be of length **decnbr(lmisys)** (the number of decision variables). The initial point is ignored when infeasible. Finally, the last argument **target** sets some target value for  $\lambda$ . The code terminates as soon as it has found a feasible pair  $(\lambda, x)$  with  $\lambda \leq \text{target}$ .

## Caution

When setting up your `gevp` problem, be cautious to

- Always specify the linear-fractional constraints “Equation 1-6” on page 1-124 *last* in the LMI system. `gevp` systematically assumes that the last `nlf` LMI constraints are linear fractional.
- Add the constraint  $B(x) > 0$  or any other LMI constraint that enforces it (see Remark below). This positivity constraint is required for regularity and good formulation of the optimization problem.

## Control Parameters

The optional argument `options` lets you access control parameters of the optimization code. In `gevp`, this is a five-entry vector organized as follows:

- `options(1)` sets the desired relative accuracy on the optimal value `lopt` (default =  $10^{-2}$ ).
- `options(2)` sets the maximum number of iterations allowed to be performed by the optimization procedure (100 by default).
- `options(3)` sets the feasibility radius. Its purpose and usage are the same as for `feasp`.
- `options(4)` helps speed up termination. If set to an integer value  $J > 0$ , the code terminates when the progress in  $\lambda$  over the last  $J$  iterations falls below the desired relative accuracy. Progress means the amount by which  $\lambda$  decreases. The default value is 5 iterations.
- `options(5) = 1` turns off the trace of execution of the optimization procedure. Resetting `options(5)` to zero (default value) turns it back on.

Setting `option(i)` to zero is equivalent to setting the corresponding control parameter to its default value.

## Examples

Given

$$A_1 = \begin{pmatrix} -1 & 2 \\ 1 & -3 \end{pmatrix}, A_2 = \begin{pmatrix} -0.8 & 1.5 \\ 1.3 & -2.7 \end{pmatrix}, A_3 = \begin{pmatrix} -1.4 & 0.9 \\ 0.7 & -2.0 \end{pmatrix},$$

consider the problem of finding a single Lyapunov function  $V(x) = x^T P x$  that proves stability of

$$\dot{x} = A_i x \quad (i = 1, 2, 3)$$

and maximizes the decay rate  $\frac{dV(x)}{dt}$ . This is equivalent to minimizing

$\alpha$  subject to

$$I < P$$

$$A_1^T P + P A_1 < \alpha P$$

$$A_2^T P + P A_2 < \alpha P$$

$$A_3^T P + P A_3 < \alpha P$$

To set up this problem for `gevp`, first specify the LMIs “Equation 1-8” on page 1-126 to “Equation 1-10” on page 1-126 with  $\alpha = 1$ :

```
setlmis([]);
p = lmivar(1,[2 1])

lmiterm([1 1 1 0],1)      % P > I : I
lmiterm([-1 1 1 p],1,1)   % P > I : P
lmiterm([2 1 1 p],1,a1,'s') % LFC # 1 (lhs)
lmiterm([-2 1 1 p],1,1)   % LFC # 1 (rhs)
lmiterm([3 1 1 p],1,a2,'s') % LFC # 2 (lhs)
lmiterm([-3 1 1 p],1,1)   % LFC # 2 (rhs)
lmiterm([4 1 1 p],1,a3,'s') % LFC # 3 (lhs)
lmiterm([-4 1 1 p],1,1)   % LFC # 3 (rhs)
lmis = getlmis
```

Note that the linear fractional constraints are defined last as required. To minimize  $\alpha$  subject to “Equation 1-8” on page 1-126 to “Equation 1-10” on page 1-126, call `gevp` by

```
[alpha, popt]=gevp(lmis,3)
```

This returns `alpha = -0.122` as the optimal value (the largest decay rate is therefore 0.122). This value is achieved for:

$$P = \begin{pmatrix} 5.58 & -8.35 \\ -8.35 & 18.64 \end{pmatrix}$$

## Tips

Generalized eigenvalue minimization problems involve standard LMI constraints “Equation 1-4” on page 1-124 and linear fractional constraints “Equation 1-6” on page 1-124. For well-posedness, the positive definiteness of  $B(x)$  must be enforced by adding the constraint  $B(x) > 0$  to the problem. Although this could be done automatically from inside the code, this is not desirable for efficiency reasons. For instance, the set of constraints “Equation 1-5” on page 1-124 may reduce to a single constraint as in the example above. In this case, the single extra LMI “ $P > I$ ” is enough to enforce positivity of *all* linear-fractional right sides. It is therefore left to the user to devise the least costly way of enforcing this positivity requirement.

## References

The solver `gevp` is based on Nesterov and Nemirovski's Projective Method described in

Nesterov, Y., and A. Nemirovski, *Interior Point Polynomial Methods in Convex Programming: Theory and Applications*, SIAM, Philadelphia, 1994.

The optimization is performed by the C MEX-file `fpds.mex`.

## See Also

`dec2mat` | `decnbr` | `feasp` | `mincx`

**Introduced before R2006a**

## gridureal

Grid ureal parameters uniformly over their range

### Syntax

```
B = gridureal(A,N)
[B,SampleValues] = gridureal(A,N)
[B,SampleValues] = gridureal(A,NAMES,N)
[B,SampleValues] = gridureal(A,NAMES1,N1,NAMES2,N2,...)
```

### Description

`B = gridureal(A,N)` substitutes `N` uniformly-spaced samples of the uncertain real parameters in `A`. The samples are chosen to cut “diagonally” across the cube of real parameter uncertainty space. The array `B` has size equal to `[size(A) N]`. For example, suppose `A` has 3 uncertain real parameters, say `X`, `Y` and `Z`. Let `(x1, x2, ..., xN)` denote `N` uniform samples of `X` across its range. Similar for `Y` and `Z`. Then sample `A` at the points `(x1, y1, z1)`, `(x2, y2, z2)`, and `(xN, yN, zN)` to obtain the result `B`.

If `A` depends on additional uncertain objects, then `B` will be an uncertain object.

`[B,SampleValues] = gridureal(A,N)` additionally returns the specific sampled values (as a structure whose fieldnames are the names of `A`'s uncertain elements) of the uncertain reals. Hence, `B` is the same as `usubs(A,SampleValues)`.

`[B,SampleValues] = gridureal(A,NAMES,N)` samples only the uncertain reals listed in the `NAMES` variable (cell, or char array). Any entries of `NAMES` that are not elements of `A` are simply ignored. Note that `gridureal(A,fieldnames(A.Uncertainty),N)` is the same as `gridureal(A,N)`.

`[B,SampleValues] = gridureal(A,NAMES1,N1,NAMES2,N2,...)` takes `N1` samples of the uncertain real parameters listed in `NAMES1`, and `N2` samples of the uncertain real parameters listed in `NAMES2` and so on. `size(B)` will equal `[size(A) N1 N2 ...]`.

## Examples

### Grid Open-Loop and Closed-Loop Responses of Uncertain System

Create two uncertain real parameters `gamma` and `tau`. The nominal value of `gamma` is 4 and its range is 3 to 5. The nominal value of `tau` is 0.5 and its value can vary by +/- 30 percent.

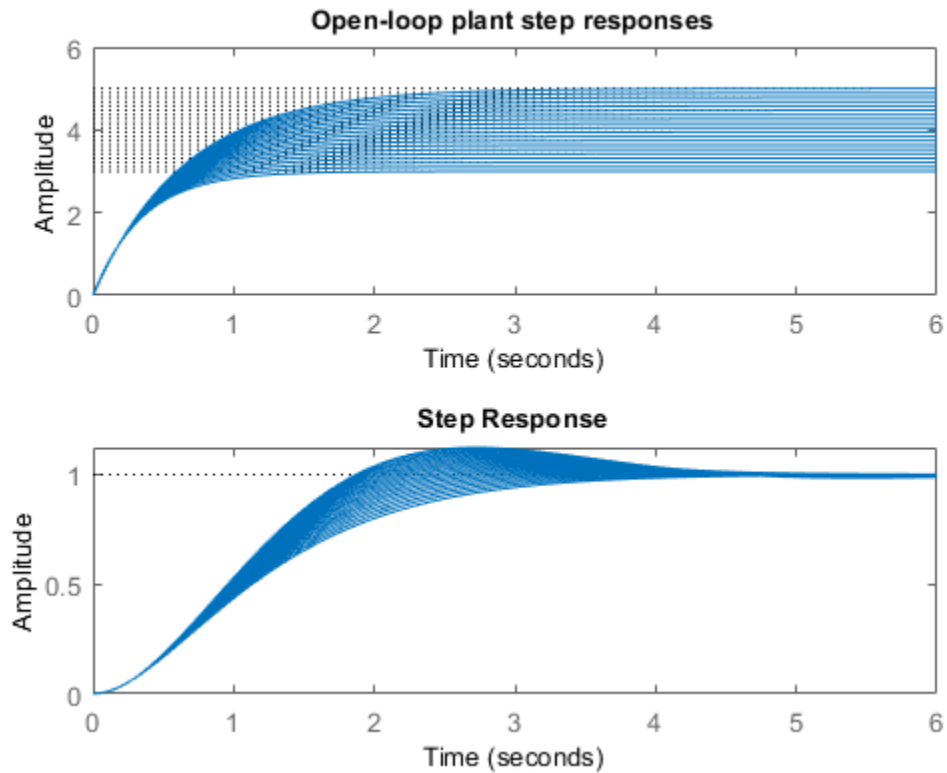
```
gamma = ureal('gamma',4);  
tau = ureal('tau',.5,'Percentage',30);
```

These uncertain parameters are used to construct an uncertain transfer function `p`. An integral controller, `c`, is synthesized for the plant `p` based on the nominal values of `gamma` and `tau`. The uncertain closed-loop system `clp` is formed.

```
p = tf(gamma,[tau 1]);  
KI = 1/(2*tau.Nominal*gamma.Nominal);  
c = tf(KI,[1 0]);  
clp = feedback(p*c,1);
```

The figure below shows the open-loop unit step response (top plot) and closed-loop response (bottom plot) for a grid of 20 values of `gamma` and `tau`.

```
subplot(2,1,1); step(gridureal(p,20),6)  
title('Open-loop plant step responses')  
subplot(2,1,2); step(gridureal(clp,20),6)
```



The plot illustrates the low-frequency closed-loop insensitivity achieved by the PI control system.

### Grid Over Multi-Dimensional Parameter Spaces

This example illustrates the different options in gridding high-dimensional (e.g.,  $n$  greater than 2) parameter spaces.

Construct an uncertain matrix,  $m$ , from four uncertain real parameters,  $a$ ,  $b$ ,  $c$ , and  $d$ , each making up the individual entries in  $m$ .



```
a = ureal('a',1);
b = ureal('b',2);
c = ureal('c',3);
d = ureal('d',4);
m = [a b;c d];
```

First, grid the (a, b) space at five places, and the (c, d) space at three places.

```
m1 = gridureal(m,{'a';'b'},5,{'c';'d'},3);
```

gridureal evaluates the uncertain matrix m at these 15 grid points, resulting in the numerical matrix m1.

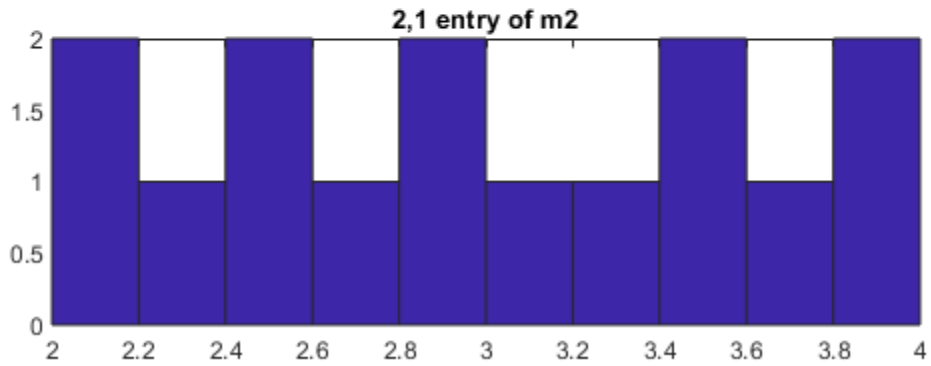
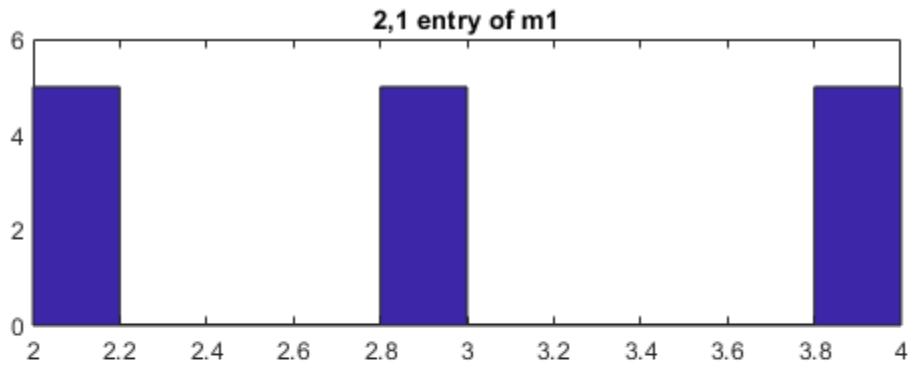
Next, grid the (a, b, c, d) space at 15 places.

```
m2 = gridureal(m,{'a';'b';'c';'d'},15);
```

gridureal samples the uncertain matrix m at these 15 points, resulting in the numerical matrix m2.

The (2,1) entry of m is just the uncertain real parameter c. Plot the histograms of the (2,1) entry of both m1 and m2. The (2,1) entry of m1 only takes on three distinct values, while the (2,1) entry of m2 takes on 15 distinct values uniformly through its range.

```
subplot(2,1,1)
hist(squeeze(m1(2,1,:)))
title('2,1 entry of m1')
subplot(2,1,2)
hist(squeeze(m2(2,1,:)))
title('2,1 entry of m2')
```



## See Also

`usample` | `usubs`

Introduced before R2006a

# h2hinfosyn

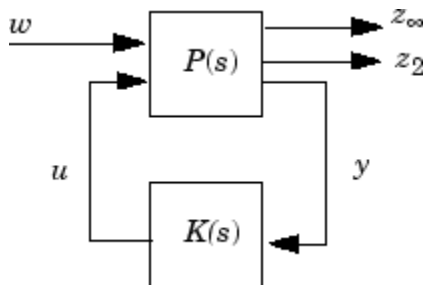
Mixed  $H_2/H_\infty$  synthesis with regional pole placement constraints

## Syntax

`[K,CL,normz,info] = h2hinfosyn(P,Nmeas,Ncon,Nz2,Wz,Name,Value)`

## Description

`[K,CL,normz,info] = h2hinfosyn(P,Nmeas,Ncon,Nz2,Wz,Name,Value)` employs LMI techniques to compute an output-feedback control law  $u = K(s)y$  for the control problem of the following illustration.



The LTI plant  $P$  has partitioned state-space form given by

$$\begin{aligned}\dot{x} &= Ax + B_1w + B_2u, \\ z_\infty &= C_1x + D_{11}w + D_{12}u, \\ z_2 &= C_2x + D_{21}w + D_{22}u, \\ y &= C_yx + D_{y1}w + D_{y2}u.\end{aligned}$$

The resulting controller  $K$ :

- Keeps the  $H_\infty$  norm  $G$  of the transfer function from  $w$  to  $z_\infty$  below the value you specify using the `Name, Value` argument 'HINFMAX'.

- Keeps the  $H_2$  norm  $H$  of the transfer function from  $w$  to  $z_2$  below the value you specify using the `Name, Value` argument 'H2MAX'.
- Minimizes a trade-off criterion of the form

$$W_1 G^2 + W_2 H^2,$$

where  $W_1$  and  $W_2$  are the first and second entries in the vector `Wz`.

- Places the closed-loop poles in the LMI region that you specify using the `Name, Value` argument 'REGION'.

Use the input arguments `Nmeas`, `Ncon`, and `Nz2` to specify the number of signals in  $y$ ,  $u$ , and  $z_2$ , respectively. You can use additional `Name, Value` pairs to specify additional options for the computation.

## Input Arguments

### **P — Plant**

LTI model

Plant, specified as an LTI model such as a `tf` or `ss` model. `P` must be a continuous-time model.

### **Nmeas — Number of measurement signals**

positive integer

Number of measurement signals, specified as a positive integer. This value is the number of signals in  $y$ .

### **Ncon — Number of control signals**

positive integer

Number of control signals, specified as a positive integer. This value is the number of signals in  $u$ .

### **Nz2 — Number of signals subject to $H_2$ constraint**

positive integers

Number of signals subject to the constraint on the  $H_2$  norm, specified as a positive integer. This value is the number of signals in  $z_2$ . If the total number of outputs of `P` is

Nout, then the first Nout - Nz2 - Nmeas outputs of P are subject to the constraint on the  $H_\infty$  norm.

### Wz — Weights for $H_\infty$ and $H_2$ performance

1-by-2 vector

Weights for  $H_\infty$  and  $H_2$  performance, specified as a 1-by-2 vector of the form [Winf, W2].

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

Example: 'REGION', reg, 'H2MAX', 1, 'HINFMAX', 1, 'DISPLAY', 'on'

### REGION — Pole placement region

[] (default) | matrix

Pole placement region, specified as a comma-separated pair consisting of 'REGION' and a matrix of the form [L, M]. This matrix specifies the pole placement region as:

$$\{z : L + zM + \bar{z}M^T < 0\}.$$

Generate the matrix using lmi reg. The default LMI region for pole placement, specified by the empty matrix [], is the open left-half plane, enforcing closed-loop stability only.

### H2MAX — Upper bound on $H_2$ norm

Inf (default) | positive scalar

Upper bound on the  $H_2$  norm of the transfer function from  $w$  to  $z_2$ , specified as a comma-separated pair consisting of 'H2MAX' and a positive scalar value or Inf. The default value Inf is equivalent to setting the limit to zero, and causes h2hinfsyn to minimize the  $H_2$  norm subject to the trade-off criterion.

Example: 'H2MAX', 1

### HINFMAX — Upper bound on $H_\infty$ norm

Inf (default) | positive scalar

Upper bound on the  $H_\infty$  norm of the transfer function from  $w$  to  $z_\infty$ , specified as a comma-separated pair consisting of 'HINFMAX' and a positive scalar value or Inf. The default value Inf is equivalent to setting the limit to zero, and causes h2hinfsyn to minimize the  $H_\infty$  norm subject to the trade-off criterion.

Example: 'HINFMAX', 1

### **DKMAX — Bound on controller feedthrough**

100 (default) | nonnegative scalar

Bound on the norm on the feedthrough matrix  $D_K$  of the controller, specified as a comma-separated pair consisting of 'DKMAX' and a nonnegative scalar value. To make the controller K strictly proper, set 'DKMAX' to 0.

Example: 'DKMAX', 0

### **TOL — Relative accuracy of trade-off criterion**

0.01 (default) | positive scalar

Desired relative accuracy on the optimal value of the trade-off criterion, specified as a comma-separated pair consisting of 'TOL' and a positive scalar value.

### **DISPLAY — Toggle for screen display**

'off' (default) | 'on'

Toggle for screen display of synthesis information, specified as a comma-separated pair consisting of 'DISPLAY' and either 'on' or 'off'.

## **Output Arguments**

### **K — Optimal output-feedback controller**

state-space model

Optimal output-feedback controller, returned as a state-space (ss) model with Nmeas inputs and Ncon outputs.

### **CL — Closed-loop system**

state-space model

Closed-loop system with synthesized controller, returned as a state-space (ss) model. The closed-loop system is  $CL = \text{lft}(P, K)$ .

**normz — Closed-loop norms**

1-by-2 vector

Closed-loop norms, returned as a 1-by-2 vector. The entries in this vector, respectively, are:

- The  $H_\infty$  norm of the closed-loop transfer function from  $w$  to  $z_\infty$ .
- The  $H_2$  norm of the closed-loop transfer function from  $w$  to  $z_2$ .

**info — Solutions of LMI solvability conditions**

structure

Solutions of LMI solvability conditions, returned as a structure containing the following fields:

- $R$  — Solution  $R$  of LMI solvability condition
- $S$  — Solution  $S$  of LMI solvability condition

**References**

- [1] Chilali, M., and P. Gahinet, "H $_\infty$  Design with Pole Placement Constraints: An LMI Approach," *IEEE Trans. Aut. Contr.*, 41 (1995), pp. 358–367.
- [2] Scherer, C., "Mixed H $_2$ /H-infinity Control," *Trends in Control: A European Perspective*, Springer-Verlag (1995), pp.173–216.

**See Also**

lmireg | msfsyn

**Introduced before R2006a**

## h2syn

$H_2$  control synthesis for LTI plant

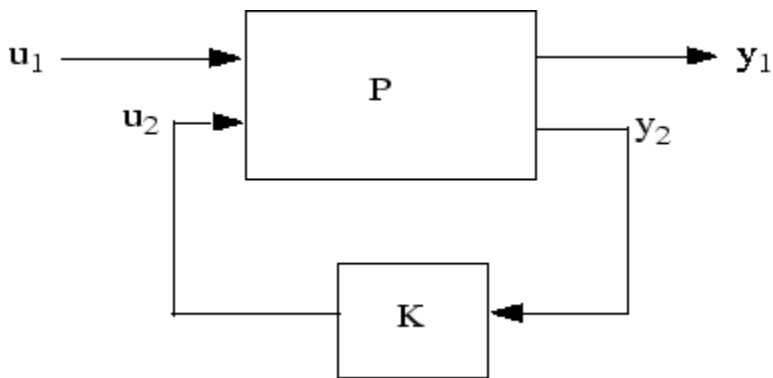
### Syntax

`[K, CL, GAM, INFO] = H2SYN(P, NMEAS, NCON)`

### Description

`[K, CL, GAM, INFO] = H2SYN(P, NMEAS, NCON)` computes a stabilizing  $H_2$  optimal controller  $K$  for a partitioned LTI plant  $P$ :

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} = \left[ \begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array} \right].$$



**$H_2$  control system  $CL = \text{lft}(P, K)$ :**

The LTI system  $P$  is partitioned such that inputs to  $B_1$  are the disturbances, inputs to  $B_2$  are the control inputs, output of  $C_1$  are the errors to be kept small, and outputs of  $C_2$  are



the output measurements provided to the controller.  $B_2$  has column size (NCON) and  $C_2$  has row size (NMEAS). The controller,  $K$ , is a state-space (ss) model and has the same number of states as  $P$ .

If  $P$  is constructed with `mktito`, you can omit NMEAS and NCON from the arguments.

The closed-loop system is returned in `CL` and the achieved  $H_2$  cost  $\gamma$  in `GAM`. `INFO` is a struct array that returns additional information about the design.

| Output Arguments            | Description                                     |
|-----------------------------|---|
| <code>K</code>              | LTI controller                                  |
| <code>CL = lft(P,K)</code>  | LTI closed-loop system $T_{y_1 u_1}$            |
| <code>GAM = norm(CL)</code> | $H_2$ optimal cost $\gamma = \ T_{y_1 u_1}\ ^2$ |
| <code>INFO</code>           | Additional output information                   |

Additional output — structure array `INFO` containing possible additional information depending on `METHOD`)

|                         |   |
|-------------------------|---|
| <code>INFO.NORMS</code> | Norms of four different quantities, full information control cost (FI), output estimation cost (OEF), direct feedback cost (DFL) and full control cost (FC). <code>NORMS = [FI OEF DFL FC]</code> ; |
| <code>INFO.KFI</code>   | Full-information gain matrix (constant feedback)<br>$u_2(t) = K_{FI}x(t)$   |
| <code>INFO.GFI</code>   | Full-information closed-loop system <code>GFI=ss(A-B2*KFI,B1,C1-D12*KFI,D11)</code>   |
| <code>INFO.HAMX</code>  | X Hamiltonian matrix (state-feedback)   |
| <code>INFO.HAMY</code>  | Y Hamiltonian matrix (Kalman filter)  |

## Examples

## Stabilizing Controller for MIMO Plant

Stabilize a 4-by-5 unstable plant with three states, two measurement signals, and one control signal.

In practice, P is an augmented plant that you have constructed by combining a model of the system to control with appropriate H2 weighting functions. For this example, use a randomly-generated model.

```
rng(0, 'twister');  
P = rss(3,4,5)';
```

This command creates a 4-output, 5-input stable model and then takes its Hermitian conjugate. This operation yields a 5-output, 4-input unstable model. For this example, assume that one of the inputs is a control signal and two of the outputs are measurements.

Confirm that P is unstable. All the poles are in the right half-plane.

```
pole(P)
```

```
ans = 3×1  
  
    0.2593  
   15.9497  
   20.7994
```

Design the stabilizing controller, assuming NMEAS = 2 and NCON = 1.

```
[K,CL,GAM] = h2syn(P,2,1);
```

Examine the closed-loop system to confirm that the plant is stabilized.

```
pole(CL)
```

```
ans = 6×1  
  
  -26.8951  
  -22.4817  
  -20.6965  
  -17.6041  
   -0.8694  
   -2.6697
```

### Mixed-Sensitivity H2 Loop Shaping

Shape the singular value plots of the sensitivity  $S = (I + GK)^{-1}$  and complementary sensitivity  $T = GK(I + GK)^{-1}$ .

To do so, find a stabilizing controller  $K$  that minimizes the  $H_2$  norm of:

$$T_{y_1 u_1} \triangleq \begin{bmatrix} W_1 S \\ (W_2/G)T \\ W_3 T \end{bmatrix}.$$

Assume the following plant and weights:

$$G(s) = \frac{s-1}{s-2}, W_1 = \frac{0.1(s+1000)}{100s+1}, W_2 = 0.1, W_3 = 0.$$

Using those values, construct the augmented plant  $P$ , as illustrated in the `mixsyn` reference page.

```
s = zpk('s');
G = 10*(s-1)/(s+1)^2;
G.u = 'u2';
G.y = 'y';

W1 = 0.1*(s+1000)/(100*s+1);
W1.u = 'y2';
W1.y = 'y11';

W2 = tf(0.1);
W2.u = 'u2';
W2.y = 'y12';

S = sumblk('y2 = u1 - y');

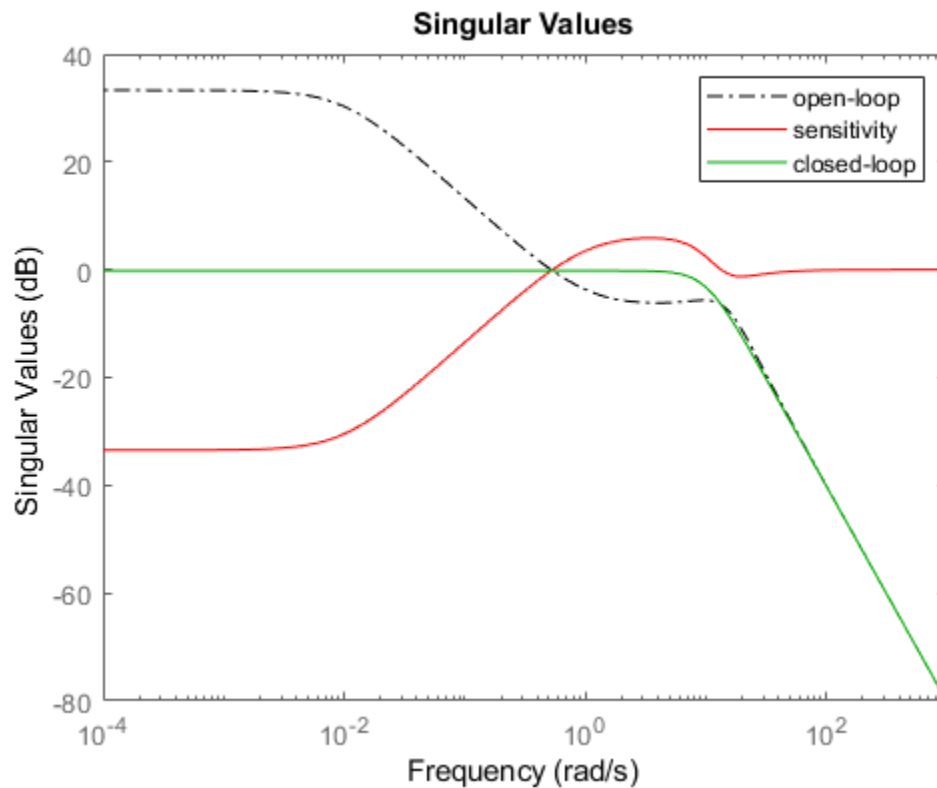
P = connect(G,S,W1,W2,{'u1','u2'},{'y11','y12','y2'});
```

Use `h2syn` to generate the controller. Note that this system has  $N_{MEAS} = 1$  and  $N_{CON} = 1$ .

```
[K,CL,GAM] = h2syn(P,1,1);
```

Examine the resulting loop shape.

```
L = G*K;  
S = inv(1+L);  
T = 1-S;  
sigmaplot(L,'k-.',S,'r',T,'g')  
legend('open-loop','sensitivity','closed-loop')
```



## Algorithms

h2syn computes the  $H_2$  optimal controller. The standard implicit assumptions for the existence of an optimal solution are:

- $(A, B_2, C_2)$  are stabilizable and detectable.
- $P_{12}$  and  $P_{21}$  have no zeros on the imaginary axis (continuous time) or unit circle (discrete time).
- $D_{12}$  has full column rank and  $D_{21}$  has full row rank. In other words,  $P_{12}$  and  $P_{21}$  have no zeros at infinity.

If the second or third of these conditions are violated, then h2syn makes slight adjustments to the plant data to enforce these conditions. This process is called regularization. It allows h2syn to compute the optimal controller for a nearby problem even when the implicit assumptions of  $H_2$  synthesis are not all satisfied.

## References

- [1] Safonov, M.G., A.J. Laub, and G. Hartmann, "Feedback Properties of Multivariable Systems: The Role and Use of Return Difference Matrix," *IEEE Trans. of Automat. Contr.*, AC-26, pp. 47-65, 1981.
- [2] Doyle, J.C., K. Glover, P. Khargonekar, and B. Francis, "State-space solutions to standard  $H_2$  and  $H_\infty$  control problems," *IEEE Transactions on Automatic Control*, vol. 34, no. 8, pp. 831-847, August 1989.
- [3] Glover, K., and J.C. Doyle, "State-space formulae for all stabilizing controllers that satisfy an  $H_\infty$  norm bound and relations to risk sensitivity," *Systems and Control Letters*, 1988. vol. 11, pp. 167-172, August 1989.

## See Also

augw | hinfsyn

**Introduced before R2006a**

## hankelmr

Hankel minimum degree approximation (MDA) without balancing

### Syntax

```
GRED = hankelmr(G)
```

```
GRED = hankelmr(G,order)
```

```
[GRED,redinfo] = hankelmr(G,key1,value1,...)
```

```
[GRED,redinfo] = hankelmr(G,order,key1,value1,...)
```

### Description

`hankelmr` returns a reduced order model `GRED` of `G` and a struct array `redinfo` containing the error bound of the reduced model and Hankel singular values of the original system.

The error bound is computed based on Hankel singular values of `G`. For a stable system Hankel singular values indicate the respective state energy of the system. Hence, reduced order can be directly determined by examining the system Hankel SV's,  $\sigma_i$ .

With only one input argument `G`, the function will show a Hankel singular value plot of the original model and prompt for model order number to reduce.

This method guarantees an error bound on the infinity norm of the *additive error*  $\|G - GRED\|_\infty$  for well-conditioned model reduced problems [1]:

$$\|G - Gred\|_\infty \leq 2 \sum_{k=1}^n \sigma_k$$

---

**Note** It seems this method is similar to the additive model reduction routines `balancmr` and `schurmr`, but actually it can produce more reliable reduced order model when the desired reduced model has nearly controllable and/or observable states (has Hankel

singular values close to machine accuracy). `hankelmr` will then select an optimal reduced system to satisfy the error bound criterion regardless the order one might naively select at the beginning.

This table describes input arguments for `hankelmr`.

| Argument | Description  |
|----------|--|
| G        | LTI model to be reduced (without any other inputs will plot its Hankel singular values and prompt for reduced order)               |
| ORDER    | (Optional) an integer for the desired order of the reduced model, or optionally a vector packed with desired orders for batch runs |

A batch run of a serial of different reduced order models can be generated by specifying `order = x:y`, or a vector of integers. By default, all the anti-stable part of a system is kept, because from control stability point of view, getting rid of unstable state(s) is dangerous to model a system.

'*MaxError*' can be specified in the same fashion as an alternative for 'ORDER'. In this case, reduced order will be determined when the sum of the tails of the Hankel sv's reaches the '*MaxError*'.

| Argument            | Value                                     | Description  |
|---------------------|---|--|
| ' <i>MaxError</i> ' | Real number or vector of different errors | Reduce to achieve $H_\infty$ error.<br>When present, ' <i>MaxError</i> ' overrides ORDER input.                                |
| ' <i>Weights</i> '  | {Wout,Win} cell array                     | Optimal 1x2 cell array of LTI weights Wout (output) and Win (input). Default for both is identity. Weights must be invertible. |
| ' <i>Display</i> '  | 'on' or 'off'                             | Display Hankel singular plots (default 'off').   |
| ' <i>Order</i> '    | Integer, vector or cell array             | Order of reduced model. Use only if not specified as 2nd argument.   |

Weights on the original model input and/or output can make the model reduction algorithm focus on some frequency range of interests. But weights have to be stable, minimum phase and invertible.

This table describes output arguments.

| Argument | Description   |
|----------|---|
| GRED     | LTI reduced order model. Become multi-dimensional array when input is a serial of different model order array.  |
| REDINFO  | A STRUCT array with 4 fields: <ul style="list-style-type: none"><li>• REDINFO.ErrorBound (bound on <math>\ G-GRED\ _\infty</math>)</li><li>• REDINFO.StabSV (Hankel SV of stable part of G)</li><li>• REDINFO.UnstabSV (Hankel SV of unstable part of G)</li><li>• REDINFO.Ganticausal (Anti-causal part of Hankel MDA)</li></ul> |

G can be stable or unstable, continuous or discrete.

---

**Note** If `size(GRED)` is not equal to the order you specified. The optimal Hankel MDA algorithm has selected the best Minimum Degree Approximate it can find within the allowable machine accuracy.

---

## Examples

Given a continuous or discrete, stable or unstable system, G, the following commands can get a set of reduced order models based on your selections:

```
rng(1234, 'twister');
G = rss(30,5,4);
[g1, redinfo1] = hankelmr(G); % display Hankel SV plot
                        % and prompt for order (try 15:20)
[g2, redinfo2] = hankelmr(G,20);
[g3, redinfo3] = hankelmr(G,[10:2:18]);
[g4, redinfo4] = hankelmr(G,'MaxError',[0.01, 0.05]);
for i = 1:4
    figure(i); eval(['sigma(G,g' num2str(i) ');']);
end
```

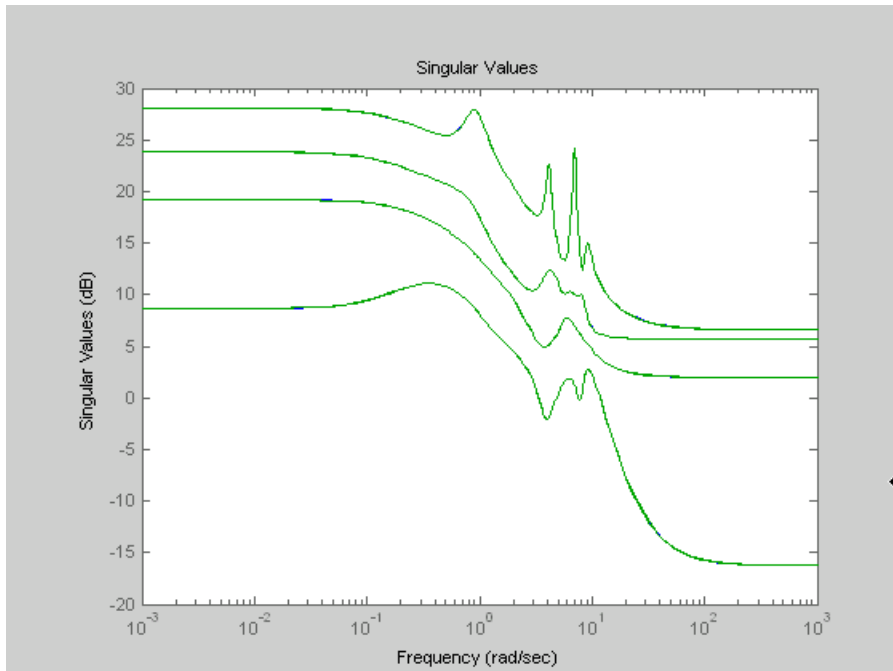
“Singular Value Bode Plot of G (30-state, 5 outputs, 4 inputs)” on page 1-147 shows a singular value Bode plot of a random system G with 20 states, 5 output and 4 inputs. The error system between G and its *Zeroth order Hankel MDA* has its infinity norm equals to an all pass function, as shown in “All-Pass Error System Between G and Zeroth Order G Anticausal” on page 1-148.



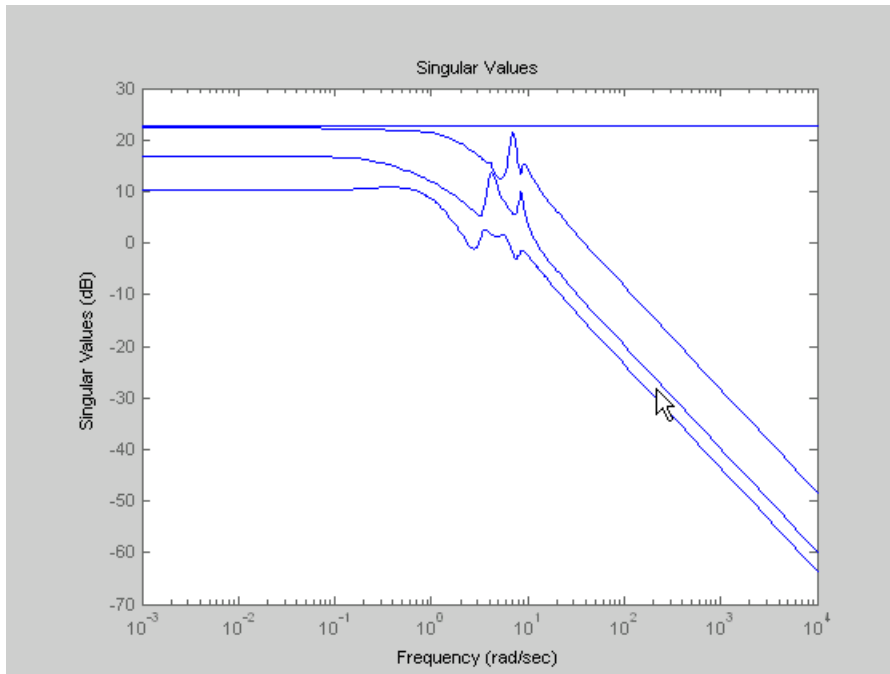
The *Zeroth order Hankel MDA* and its error system sigma plot are obtained via commands

```
[g0,redinfo0] = hankelmr(G,0);  
sigma(G-redinfo0.Ganticausal)
```

This interesting all-pass property is unique in Hankel MDA model reduction.



**Singular Value Bode Plot of G (30-state, 5 outputs, 4 inputs)**



### All-Pass Error System Between G and Zeroth Order G Anticausal

## Algorithms

Given a state-space  $(A,B,C,D)$  of a system and  $k$ , the desired reduced order, the following steps will produce a similarity transformation to truncate the original state-space system to the  $k^{th}$  order reduced model.

- 1 Find the controllability and observability grammians  $P$  and  $Q$ .
- 2 Form the descriptor

$$E = QP - \rho^2 I$$

where  $\sigma_k > \rho \geq \sigma_{k+1}$ , and descriptor state-space

Take SVD of descriptor  $E$  and partition the result into  $k^{th}$  order truncation form

$$\left[ \begin{array}{c|c} Es - \bar{A} & \bar{B} \\ \hline C & D \end{array} \right] = \left[ \begin{array}{c|c} \rho^2 A^T + QAP & QB \\ \hline CP & D \end{array} \right]$$

$$E = [U_{E1}, U_{E2}] \left[ \begin{array}{c|c} \Sigma_{E0} & 0 \\ \hline 0 & 0 \end{array} \right] \begin{bmatrix} V_{E1}^T \\ V_{E2}^T \end{bmatrix}$$

- 3 Apply the transformation to the descriptor state-space system above we have

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} U_{E1}^T \\ U_{E2}^T \end{bmatrix} (\rho^2 A^T + QAP) [V_{E1} \quad V_{E2}]$$

$$\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} U_{E1}^T \\ U_{E2}^T \end{bmatrix} [QB \quad -C^T]$$

$$[C_1 \quad C_2] = \begin{bmatrix} CP \\ -\rho B^T \end{bmatrix} [V_{E1} \quad V_{E2}]$$

$$D_1 = D$$

- 4 Form the equivalent state-space model.

$$\begin{bmatrix} \tilde{A} & \tilde{B} \\ \tilde{C} & \tilde{D} \end{bmatrix} = \begin{bmatrix} \Sigma_E^{-1} (A_{11} - A_{12} A_{22} A_{21}) & \Sigma_E^{-1} (B_1 - A_{12} A_{22} B_2) \\ C_1 - C_2 A_{22}^\dagger A_{21} & D_1 - C_2 A_{22}^\dagger B_2 \end{bmatrix}$$

The final  $k^{th}$  order Hankel MDA is the stable part of the above state-space realization. Its anticausal part is stored in `redinfo.Ganticausal`.

The proof of the Hankel MDA algorithm can be found in [2]. The error system between the original system  $G$  and the *Zeroth Order Hankel MDA*  $G_0$  is an all-pass function [1].

## References

- [1] Glover, K., "All Optimal Hankel Norm Approximation of Linear Multivariable Systems, and Their  $L_\infty$ -error Bounds," *Int. J. Control*, vol. 39, no. 6, pp. 1145-1193, 1984.

- [2] Safonov, M.G., R.Y. Chiang, and D.J.N. Limebeer, "Optimal Hankel Model Reduction for Nonminimal Systems," *IEEE Trans. on Automat. Contr.*, vol. 35, no. 4, April 1990, pp. 496-502.

## **See Also**

balancmr | bstmr | hankelsv | ncfmr | reduce | schurmr

**Introduced before R2006a**

# hankelsv

Compute Hankel singular values for stable/unstable or continuous/discrete system

## Syntax

```
hankelsv(G)
```

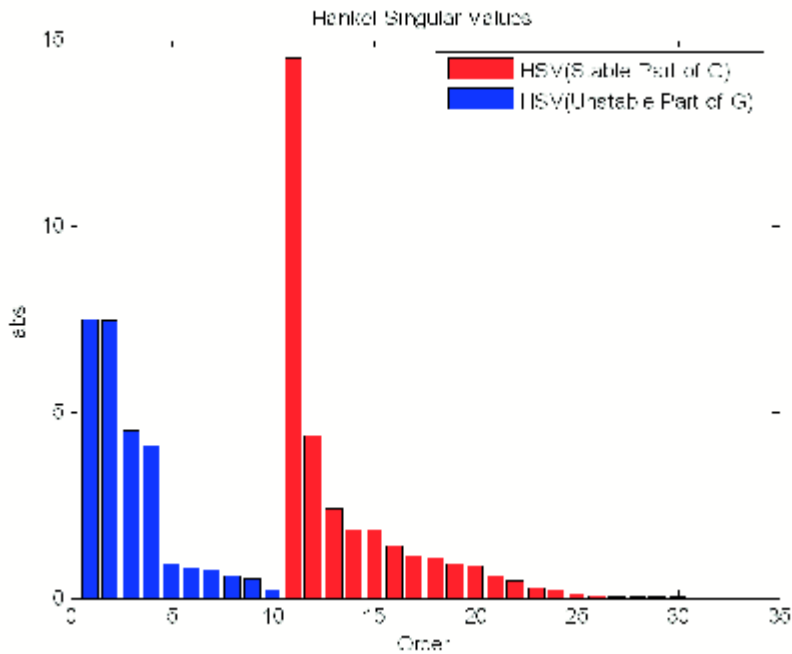
```
hankelsv(G,ErrorType,style)
```

```
[sv_stab,sv_unstab]=hankelsv(G,ErrorType,style)
```

## Description

`[sv_stab,sv_unstab]=hankelsv(G,ErrorType,style)` returns a column vector `SV_STAB` containing the Hankel singular values of the stable part of `G` and `SV_UNSTAB` of anti-stable part (if it exists). The Hankel SV's of anti-stable part `ss(a,b,c,d)` is computed internally via `ss(-a,-b,c,d)`. Discrete model is converted to continuous one via the bilinear transform.

`hankelsv(G)` with no output arguments draws a bar graph of the Hankel singular values such as the following:



This table describes optional input arguments for `hankelsvd`.

| Argument  | Value  | Description                    |
|-----------|--------|--------------------------------|
| ERRORTYPE | 'add'  | Regular Hankel SV's of G       |
|           | 'mult' | Hankel SV's of phase matrix    |
|           | 'ncf'  | Hankel SV's of coprime factors |
| STYLE     | 'abs'  | Absolute value                 |
|           | 'log'  | logarithm scale                |

## Algorithms

If `ErrorType = 'add'`, then `hankelsv` implements the numerically robust square root method to compute the Hankel singular values [1]. Its algorithm goes as follows:

Given a stable model  $G$ , with controllability and observability grammians  $P$  and  $Q$ , compute the SVD of  $P$  and  $Q$ :

```
[Up,Sp,Vp] = svd(P);
[Uq,Sq,Vq] = svd(Q);
```

Then form the square roots of the grammians:

```
Lr = Up*diag(sqrt(diag(Sp)));
Lo = Uq*diag(sqrt(diag(Sq)));
```

The Hankel singular values are simply:

```
 $\sigma_H = \text{svd}(Lo' * Lr);$ 
```

This method not only takes the advantages of robust SVD algorithm, but also ensure the computations stay well within the “square root” of the machine accuracy.

If `ErrorType = 'mult'`, then `hankelsv` computes the Hankel singular value of the phase matrix of  $G$  [2].

If `ErrorType = 'ncf'`, then `hankelsv` computes the Hankel singular value of the normalized coprime factor pair of the model [3].

## References

- [1] Safonov, M.G., and R.Y. Chiang, “A Schur Method for Balanced Model Reduction,” *IEEE Trans. on Automat. Contr.*, vol. AC-2, no. 7, July 1989, pp. 729-733.
- [2] Safonov, M.G., and R.Y. Chiang, “Model Reduction for Robust Control: A Schur Relative Error Method,” *International J. of Adaptive Control and Signal Processing*, Vol. 2, pp. 259-272, 1988.
- [3] Vidyasagar, M., *Control System Synthesis - A Factorization Approach*. London: The MIT Press, 1985.

## See Also

balancmr | bstmr | hankelmr | ncfmr | reduce | schurmr

**Introduced before R2006a**



# hifgs

Synthesis of gain-scheduled  $H_\infty$  controllers

## Syntax

```
[gopt,pdK,R,S] = hifgs(pdP,r,gmin,tol,tolred)
```

## Description

Given an affine parameter-dependent plant

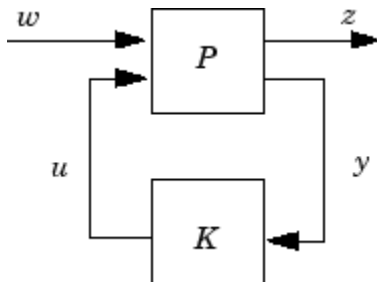
$$P \begin{cases} \dot{x} = A(p)x + B_1(p)w + B_2u \\ z = C_1(p)x + D_{11}(p)w + D_{12}u \\ y = C_2x + D_{21}w + D_{22}u \end{cases}$$

where the time-varying parameter vector  $p(t)$  ranges in a box and is measured in real time, `hifgs` seeks an affine parameter-dependent controller

$$K \begin{cases} \dot{\zeta} = A_K(p)\zeta + B_K(p)y \\ u = C_K(p)\zeta + D_K(P)y \end{cases}$$

scheduled by the measurements of  $p(t)$  and such that

- $K$  stabilizes the closed-loop system



for all admissible parameter trajectories  $p(t)$

- $K$  minimizes the closed-loop quadratic  $H_\infty$  performance from  $w$  to  $z$ .

The description `pdP` of the parameter-dependent plant  $P$  is specified with `psys` and the vector `r` gives the number of controller inputs and outputs (set `r=[p2, m2]` if  $y \in R^{p2}$  and  $u \in R^{m2}$ ). Note that `hinfgs` also accepts the polytopic model of  $P$  returned, e.g., by `aff2pol`.

`hinfgs` returns the optimal closed-loop quadratic performance `gopt` and a polytopic description of the gain-scheduled controller `pdK`. To test if a closed-loop quadratic performance  $\gamma$  is achievable, set the third input `gmin` to  $\gamma$ . The arguments `tol` and `tolred` control the required relative accuracy on `gopt` and the threshold for order reduction. Finally, `hinfgs` also returns solutions  $R, S$  of the characteristic LMI system.

## Controller Implementation

The gain-scheduled controller `pdK` is parametrized by  $p(t)$  and characterized by the values

$K_{\Pi_j}$  of  $\begin{pmatrix} A_K(p) & B_K(p) \\ C_K(p) & D_K(p) \end{pmatrix}$  at the corners  $^3_j$  of the parameter box. The command

```
Kj = psinfo(pdK, 'sys', j)
```

returns the  $j$ -th vertex controller  $K_{\Pi_j}$  while

```
pv = psinfo(pdP, 'par')
vertx = polydec(pv)
Pj = vertx(:, j)
```

gives the corresponding corner  $^3_j$  of the parameter box (`pv` is the parameter vector description).

The controller scheduling should be performed as follows. Given the measurements  $p(t)$  of the parameters at time  $t$ ,

- 1 Express  $p(t)$  as a convex combination of the  $^3_j$ :

$$p(t) = \alpha_1^3_1 + \dots + \alpha_N^3_N, \alpha_j \geq 0, \sum_{i=1}^N \alpha_j = 1$$

This convex decomposition is computed by `polydec`.

- 2 Compute the controller state-space matrices at time  $t$  as the convex combination of the vertex controllers  $K_{\Pi_j}$ :

$$\begin{pmatrix} A_K(t) & B_K(t) \\ C_K(t) & D_K(t) \end{pmatrix} = \sum_{i=1}^N \alpha_j K_{\Pi_i}.$$

- 3 Use  $A_K(t)$ ,  $B_K(t)$ ,  $C_K(t)$ ,  $D_K(t)$  to update the controller state-space equations.

## References

Apkarian, P., P. Gahinet, and G. Becker, "Self-Scheduled  $H_\infty$  Control of Linear Parameter-Varying Systems," *Automatica*, 31 (1995), pp. 1251-1261.

Becker, G., Packard, P., "Robust Performance of Linear-Parametrically Varying Systems Using Parametrically-Dependent Linear Feedback," *Systems and Control Letters*, 23 (1994), pp. 205-215.

Packard, A., "Gain Scheduling via Linear Fractional Transformations," *Syst. Contr. Letters*, 22 (1994), pp. 79-92.

## See Also

`pdsimul` | `polydec` | `psys` | `pvec`

**Introduced before R2006a**

## hinfnorm

$H_\infty$  norm of dynamic system

### Syntax

```
ninf = hinfnorm(sys)
ninf = hinfnorm(sys,tol)
[ninf,fpeak] = hinfnorm(____)
```

### Description

`ninf = hinfnorm(sys)` returns the  $H_\infty$  in absolute units of the dynamic system model, `sys`.

- If `sys` is a stable SISO system, then the  $H_\infty$  norm is the peak gain, the largest value of the frequency response magnitude.
- If `sys` is a stable MIMO system, then the  $H_\infty$  norm is the largest singular value across frequencies.
- If `sys` is an unstable system, then the  $H_\infty$  norm is defined as `Inf`.
- If `sys` is a model that has tunable or uncertain parameters, then `hinfnorm` evaluates the  $H_\infty$  norm at the current or nominal value of `sys`.
- If is a model array, then `hinfnorm` returns an array of the same size as `sys`, where `ninf(k) = hinfnorm(sys(:, :, k))`.

For stable systems, `hinfnorm(sys)` is the same as `getPeakGain(sys)`.

`ninf = hinfnorm(sys,tol)` returns the  $H_\infty$  norm of `sys` with relative accuracy `tol`.

`[ninf,fpeak] = hinfnorm(____)` also returns the frequency, `fpeak`, at which the peak gain or largest singular value occurs. You can use this syntax with any of the input arguments in previous syntaxes. If `sys` is unstable, then `fpeak = Inf`.

## Examples

### Norm of MIMO System

Compute the  $H_\infty$  norm of the following 2-input, 2-output dynamic system and the frequency at which the peak singular value occurs.

$$G(s) = \begin{bmatrix} 0 & \frac{3s}{s^2 + s + 10} \\ \frac{s+1}{s+5} & \frac{2}{s+6} \end{bmatrix}.$$

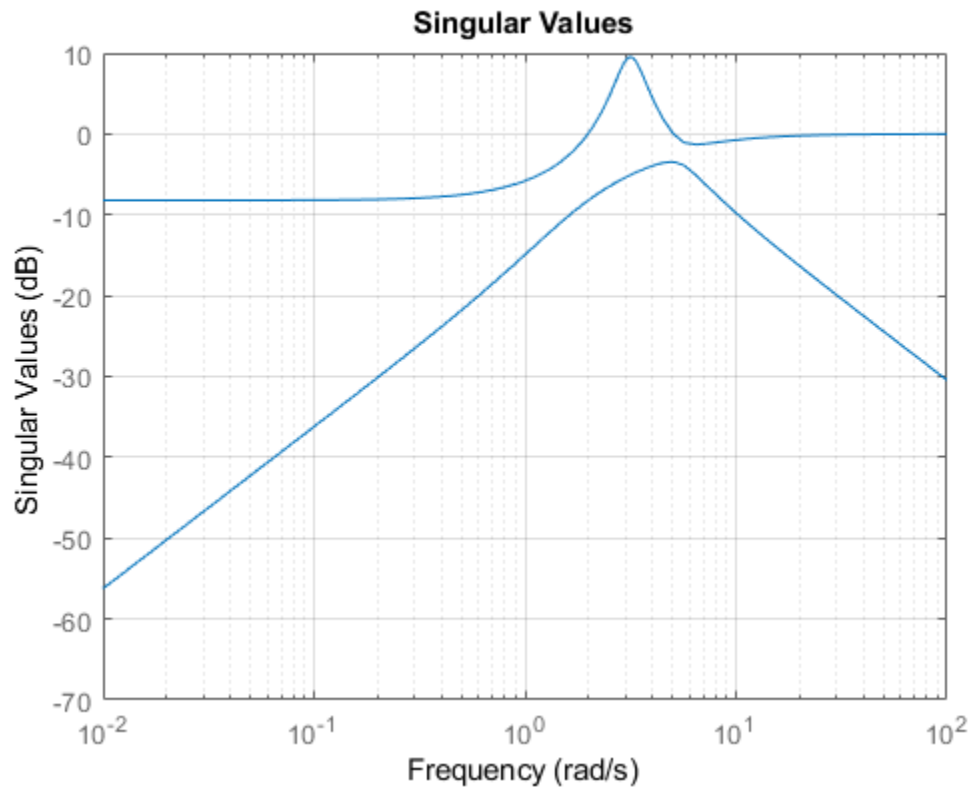
```
G = [0 tf([3 0],[1 1 10]);tf([1 1],[1 5]),tf(2,[1 6])];
[ninf,fpeak] = hinfnorm(G)
```

```
ninf = 3.0150
```

```
fpeak = 3.1623
```

The  $H_\infty$  norm of a MIMO system is its maximum singular value. Plot the singular values of  $G$  and compare the result from `hinfnorm`.

```
sigma(G),grid
```



The values `ninf` and `fpeak` are consistent with the singular value plot, which displays the values in dB.

## Input Arguments

### **sys** — Input dynamic system

dynamic system model | model array

Input dynamic system, specified as any dynamic system model or model array. `sys` can be SISO or MIMO.

**tol — Relative accuracy**

0.01 (default) | positive real scalar

Relative accuracy of the peak gain, specified as a positive real scalar value. `hinfnorm` calculates `ninf` such that the fractional difference between `ninf` and the true  $H_\infty$  norm of `sys` is no greater than `tol`.

## Output Arguments

**ninf —  $H_\infty$  norm of dynamic system**

Inf | scalar | array

$H_\infty$  norm of `sys`, returned as `Inf`, a scalar value, or an array.

- If `sys` is a single stable model, then `ninf` is a scalar value.
- If `sys` is a single unstable model, then `ninf` is `Inf`.
- If `sys` is a model array, then `ninf` is an array of the same size as `sys`, where `ninf(k) = hinfnorm(sys(:, :, k))`.

**fpeak — Frequency of peak gain or largest singular value**

Inf | nonnegative real scalar | array

Frequency at which the peak gain or largest singular value occurs, returned as `Inf`, a nonnegative real scalar value, or an array. The frequency is expressed in units of rad/TimeUnit, relative to the TimeUnit property of `sys`.

- If `sys` is a single stable model, then `fpeak` is a scalar.
- If `sys` is a single unstable model, then `fpeak` is `Inf`.
- If `sys` is a model array, then `fpeak` is an array of the same size as `sys`. In this case, `fpeak(k)` is the peak gain or largest singular value frequency of the  $k$ th model in the array.

## See Also

freqresp | getPeakGain | norm | sigma

**Introduced in R2013b**

## hinfstruct

$H_\infty$  tuning of fixed-structure controllers

### Syntax

```
CL = hinfstruct(CL0)
[CL,gamma,info] = hinfstruct(CL0)
[CL,gamma,info] = hinfstruct(CL0,options)
[C,gamma,info] = hinfstruct(P,C0,options)
```

### Description

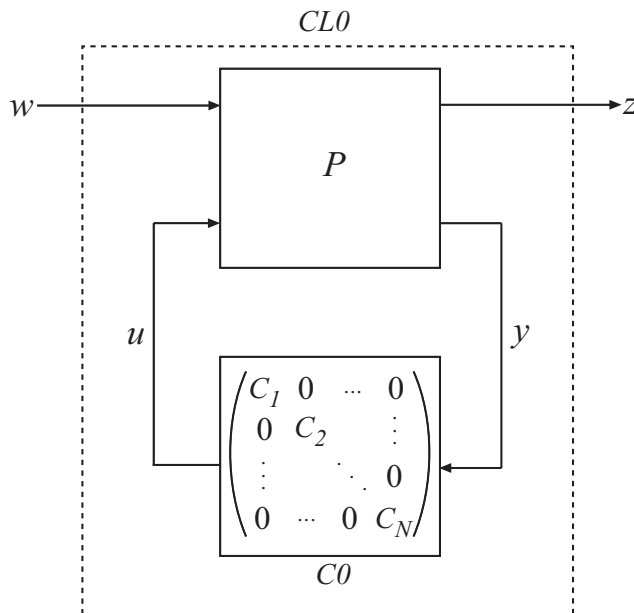
`CL = hinfstruct(CL0)` tunes the free parameters of the tunable `genss` model `CL0`. This tuning minimizes the  $H_\infty$  norm of the closed-loop transfer function modeled by `CL0`. The model `CL0` represents a closed-loop control system that includes tunable components such as controllers or filters. `CL0` can also include weighting functions that capture design requirements.

`[CL,gamma,info] = hinfstruct(CL0)` returns `gamma` (the minimum  $H_\infty$  norm) and a data structure `info` with additional information about each optimization run.

`[CL,gamma,info] = hinfstruct(CL0,options)` allows you to specify additional options for the optimizer using `hinfstructOptions`.

`[C,gamma,info] = hinfstruct(P,C0,options)` tunes the parametric controller blocks `C0`. This tuning minimizes the  $H_\infty$  norm of the closed-loop system `CL0 = lft(P,C0)`. To use this syntax, express your control system and design requirements as a Standard Form model, as in the following illustration:





$P$  is a numeric LTI model that includes the fixed elements of the control architecture.  $P$  can also include weighting functions that capture design requirements.  $C0$  can be a single tunable component (for example, a Control Design Block (Control System Toolbox) or a `genss` model) or a cell array of multiple tunable components.  $C$  is a parametric model or array of parametric models of the same types as  $C0$ .

## Input Arguments

### $CL0$

Generalized state-space (`genss`) model describing the weighted closed-loop transfer function of a control system.  $CL0$  includes both the fixed and tunable components of the control system. The tunable components of the control system are represented as control design blocks (Control System Toolbox), and are stored in the  $CL0.Blocks$  property of the `genss` model. `hinfstruct` adjusts the tunable parameters of the control design blocks in  $CL0$  to minimize the  $H_\infty$  norm of the weighted transfer function. For more information about constructing this generalized model, see “Build Tunable Closed-Loop Model for Tuning with `hinfstruct`”.

**CL0** can be a continuous-time or discrete-time model. In discrete time, the sample time must be specified ( $T_s \neq -1$ ).

**CL0** can be an array of models, allowing you to tune a set of controller parameters for multiple models simultaneously. For example, the multiple models might represent different system configurations, different operating points, or different failure modes of a system.

**CL0** can include uncertain control design blocks as well as tunable blocks. If **CL0** has uncertain blocks, `hinfstruct` performs robust tuning.

### **P**

Numeric LTI model representing the fixed elements of the control architecture to be tuned. **P** can also include weighting functions that capture design requirements. You can obtain **P** in two ways:

- In MATLAB, model the fixed elements of your control system as numeric LTI models. Then, use block-diagram building functions (such as `connect` and `feedback`) to build **P** from the modeled components. Also include any weighting functions that represent your design requirements.
- If you have a Simulink® model of your control system and have Simulink Control Design™, use `linlft` to obtain a linear model of the fixed elements of your control system. The `linlft` command linearizes your Simulink model, excluding specified Simulink blocks (the blocks that represent the controller elements you want to tune). If you are using weighting functions to represent your design requirements, connect them in series with the linear model of your plant to obtain **P**.

**P** can be a continuous-time or discrete-time model. In discrete time, the sample time must be specified ( $T_s \neq -1$ ), and must match the sample time of **C0**.

### **C0**

Single tunable component or cell array of tunable components of the control structure.

Each entry in **C0** represents one tunable element of your control architecture, such as a PID controller, a gain block, or a fixed-order transfer function. The entries of **C0** can be Control Design Blocks (Control System Toolbox) or `genss` models.

For more information and examples of creating tunable models, see “Models with Tunable Coefficients” (Control System Toolbox) in the *Control System Toolbox™ User's Guide*.

$C\theta$  can be a continuous-time or discrete-time model, as long as the sample time matches that of  $P$ .

### **options**

Set of options for `hinfstruct`. Use `hinfstructOptions` to define options. For information about the available options, see the `hinfstructOptions` reference page.

## **Output Arguments**

### **CL**

Tuned version of the generalized state-space (`genss`) model  $CL\theta$ .

The `hinfstruct` command tunes the free parameters of  $CL\theta$  to achieve a minimum  $H_\infty$  norm. `CL.Blocks` contains the same types of Control Design Blocks as `CL\theta.Blocks`, except that in `CL`, the parameters have tuned values.

To access the tuned parameter values, use `getValue`. You can also access them directly in `CL.Blocks`.

### **C**

Tuned versions of the parametric models  $C\theta$ .

When  $C\theta$  is a single parametric model, `C` is a parametric model of the same type, with tuned parameter values.

When  $C\theta$  is a cell array of parametric models, `C` is also a cell array. The entries in `C` are parametric models of the same type as the corresponding entries in  $C\theta$ .

### **gamma**

Best achieved value for the closed-loop  $H_\infty$  norm.

In some cases, `hinfstruct` performs more than one minimization run (when the `hinfstructOptions` option `RandomStarts` > 0). In such cases, `gamma` is the smallest  $H_\infty$  norm of all runs.

## info

Data structure array containing results from each optimization run. The fields of `info` are:

- `Objective` — Minimum  $H_\infty$  norm value for each run.

When `RandomStarts = 0`, `Objective = gamma`.

- `Iterations` — Number of iterations before convergence for each run.
- `TunedBlocks` — Tuned control design blocks for each run.

`TunedBlocks` differs from `C` in that `C` contains only the result from the best run. When `RandomStarts = 0`, `TunedBlocks = C`.

## Tips

- `hinfstruct` is related to `hinfosyn`, which also uses  $H_\infty$  techniques to design a controller for a MIMO plant. However, unlike `hinfstruct`, `hinfosyn` imposes no restriction on the structure and order of the controller. For that reason, `hinfosyn` always returns a smaller `gamma` than `hinfstruct`. You can therefore use `hinfosyn` to obtain a lower bound on the best achievable performance.
- Using `hinfstruct` requires some familiarity with  $H_\infty$  techniques. It requires expressing your design requirements as frequency-weighting functions on plant inputs and outputs, as described in “Formulating Design Requirements as H-Infinity Constraints”. For a simpler approach to fixed-structure tuning, use  `systune`  or  `looptune` .

## Algorithms

`hinfstruct` uses specialized nonsmooth optimization techniques to enforce closed-loop stability and minimize the  $H_\infty$  norm as a function of the tunable parameters. These techniques are based on the work in [1].

`hinfstruct` computes the  $H_\infty$  norm using the algorithm of [2] and structure-preserving eigensolvers from the SLICOT library. For more information about the SLICOT library, see <http://slicot.org>.

## References

- [1] P. Apkarian and D. Noll, "Nonsmooth H-infinity Synthesis," *IEEE Transactions on Automatic Control*, Vol. 51, Number 1, 2006, pp. 71-86.
- [2] Bruisma, N.A. and M. Steinbuch, "A Fast Algorithm to Compute the  $H_\infty$ -Norm of a Transfer Function Matrix," *System Control Letters*, 14 (1990), pp. 287-293.

## See Also

[genss](#) | [getValue](#) | [hinfstructOptions](#) | [hinfsyn](#) | [looptune](#) | [systune](#)

## Topics

["Build Tunable Closed-Loop Model for Tuning with hinfstruct"](#)  
["Fixed-Structure H-infinity Synthesis with HINFSTRUCT"](#)  
["What Is hinfstruct?"](#)  
["Formulating Design Requirements as H-Infinity Constraints"](#)  
["Structured H-Infinity Synthesis Workflow"](#)  
["Models with Tunable Coefficients" \(Control System Toolbox\)](#)

## Introduced in R2010b

## hinfstructOptions

Set options for hinfstruct

### Syntax

```
options = hinfstructOptions  
options = hinfstructOptions(Name,Value)
```

### Description

`options = hinfstructOptions` returns the default option set for the `hinfstruct` command.

`options = hinfstructOptions(Name,Value)` creates an option set with the options specified by one or more `Name,Value` pair arguments.

### Input Arguments

#### Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes ( ' '). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

`hinfstructOptions` takes the following `Name` arguments:

#### Display

Determines the amount of information to display during `hinfstruct` optimization runs.

`Display` takes the following values:

- 'off' — `hinfstruct` runs in silent mode, displaying no information during or after the run.

- `'iter'` — Display optimization progress after each iteration. The display includes the value of the closed-loop  $H_\infty$  norm after each iteration. The display also includes a `Progress` value indicating the percent change in the  $H_\infty$  norm from the previous iteration.
- `'final'` — Display a one-line summary at the end of each optimization run. The display includes the minimized value of the closed-loop  $H_\infty$  norm and the number of iterations for each run.

**Default:** `'final'`

### **MaxIter**

Maximum number of iterations in each optimization run.

**Default:** 300

### **RandomStart**

Number of additional optimizations starting from random values of the free parameters in the controller.

If `RandomStart = 0`, `hinfstruct` performs a single optimization run starting from the initial values of the tunable parameters. Setting `RandomStart = N > 0` runs  $N$  additional optimizations starting from  $N$  randomly generated parameter values.

`hinfstruct` finds a local minimum of the gain minimization problem. To increase the likelihood of finding parameter values that meet your design requirements, set `RandomStart > 0`. You can then use the best design that results from the multiple optimization runs.

Use with `UseParallel = true` to distribute independent optimization runs among MATLAB workers (requires Parallel Computing Toolbox™ software).

**Default:** 0

### **UseParallel**

Parallel processing flag.

Set to `true` to enable parallel processing by distributing randomized starts among workers in a parallel pool. If there is an available parallel pool, then the software performs independent optimization runs concurrently among workers in that pool. If no parallel pool is available, one of the following occurs:

- If **Automatically create a parallel pool** is selected in your Parallel Computing Toolbox preferences (Parallel Computing Toolbox), then the software starts a parallel pool using the settings in those preferences.
- If **Automatically create a parallel pool** is not selected in your preferences, then the software performs the optimization runs successively, without parallel processing.

If **Automatically create a parallel pool** is not selected in your preferences, you can manually start a parallel pool using `parpool` before running the tuning command.

Using parallel processing requires Parallel Computing Toolbox software.

**Default:** false

### **TargetGain**

Target  $H_\infty$  norm.

The `hinfstruct` optimization stops when the  $H_\infty$  norm (peak closed-loop gain) falls below the specified `TargetGain` value.

Set `TargetGain = 0` to optimize controller performance by minimizing the peak closed-loop gain. Set `TargetGain = Inf` to just stabilize the closed-loop system.

**Default:** 0

### **TolGain**

Relative tolerance for termination. The optimization terminates when the  $H_\infty$  norm decreases by less than `TolGain` over 10 consecutive iterations. Increasing `TolGain` speeds up termination, and decreasing `TolGain` yields tighter final values.

**Default:** 0.001

### **MaxFrequency**

Maximum closed-loop natural frequency.

Setting `MaxFrequency` constrains the closed-loop poles to satisfy  $|p| < \text{MaxFrequency}$ .

To let `hinfstruct` choose the closed-loop poles automatically based upon the system's open-loop dynamics, set `MaxFrequency = Inf`. To prevent unwanted fast dynamics or high-gain control, set `MaxFrequency` to a finite value.



Specify `MaxFrequency` in units of `1/TimeUnit`, relative to the `TimeUnit` property of the system you are tuning.

**Default:** `Inf`

### **MinDecay**

Minimum decay rate for closed-loop poles

Constrains the closed-loop poles to satisfy  $\text{Re}(p) < -\text{MinDecay}$ . Increase this value to improve the stability of closed-loop poles that do not affect the closed-loop gain due to pole/zero cancellations.

Specify `MinDecay` in units of `1/TimeUnit`, relative to the `TimeUnit` property of the system you are tuning.

**Default:** `1e-7`

## **Output Arguments**

### **options**

Option set containing the specified options for the `hinfstruct` command.

## **Examples**

### **Create Options Set for hinfstruct**

Create an options set for a `hinfstruct` run using three random restarts and a stability offset of 0.001. Also, configure the `hinfstruct` run to stop as soon as the closed-loop gain is smaller than 1.

```
options = hinfstructOptions('TargetGain',1,...  
                           'RandomStart',3, 'StableOffset',1e-3);
```

Alternatively, use dot notation to set the values of options.

```
options = hinfstructOptions;  
options.TargetGain = 1;
```

```
options.RandomStart = 3;  
options.StableOffset = 1e-3;
```

## Configure Option Set for Parallel Optimization Runs

Configure an option set for a `hinfstruct` run using 20 random restarts. Execute these independent optimization runs concurrently on multiple workers in a parallel pool.

If you have the Parallel Computing Toolbox software installed, you can use parallel computing to speed up `hinfstruct` tuning of fixed-structure control systems. When you run multiple randomized `hinfstruct` optimization starts, parallel computing speeds up tuning by distributing the optimization runs among workers.

If **Automatically create a parallel pool** is not selected in your Parallel Computing Toolbox preferences (Parallel Computing Toolbox), manually start a parallel pool using `parpool`. For example:

```
parpool;
```

If **Automatically create a parallel pool** is selected in your preferences, you do not need to manually start a pool.

Create an `hinfstructOptions` set that specifies 20 random restarts to run in parallel.

```
options = hinfstructOptions('RandomStart',20,'UseParallel',true);
```

Setting `UseParallel` to `true` enables parallel processing by distributing the randomized starts among available workers in the parallel pool.

Use the `hinfstructOptions` set when you call `hinfstruct`. For example, suppose you have already created a tunable closed loop model `CL0`. In this case, the following command uses parallel computing to tune `CL0`.

```
[CL,gamma,info] = hinfstruct(CL0,options);
```

## See Also

`hinfstruct`

**Introduced in R2010b**

## hinfosyn

Compute  $H_\infty$  optimal controller for LTI plant

### Syntax

```
[K,CL,GAM,INFO] = hinfosyn(P)
```

```
[K,CL,GAM,INFO] = hinfosyn(P,NMEAS,NCON)
```

```
[K,CL,GAM,INFO] = hinfosyn(P,NMEAS,NCON,KEY1,VALUE1,KEY2,VALUE2,...)
```

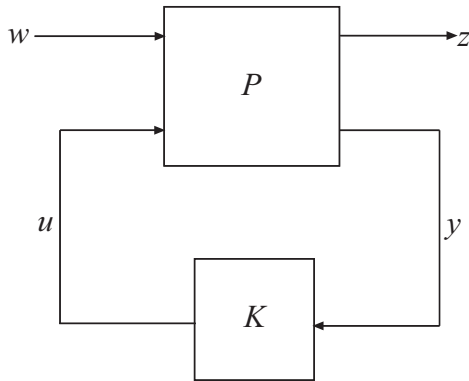
### Description

hinfosyn computes a stabilizing  $H_\infty$  optimal lti/ss controller K for a partitioned lti plant P.

$$P = \left[ \begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array} \right]$$

The controller, K, stabilizes the P and has the same number of states as P. The system P is partitioned where inputs to  $B_1$  are the disturbances, inputs to  $B_2$  are the control inputs, output of  $C_1$  are the errors to be kept small, and outputs of  $C_2$  are the output measurements provided to the controller.  $B_2$  has column size (NCON) and  $C_2$  has row size (NMEAS). The optional KEY and VALUE inputs determine tolerance, solution method and so forth.

The closed-loop system is returned in CL. This closed-loop system is given by  $CL = \text{lft}(P,K)$  as in the following diagram.



The achieved  $H_\infty$  cost  $\gamma$  is returned as GAM. The struct array INFO contains additional information about the design.

### Optional Input Arguments

| Property  | Value  | Description  |
|-----------|--------|--|
| 'GMAX'    | real   | Initial upper bound on GAM (default=Inf)   |
| 'GMIN'    | real   | Initial lower bound on GAM (default=0)   |
| 'TOLGAM'  | real   | Relative error tolerance for GAM (default=.01)   |
| 'S0'      | real   | Frequency $S0$ at which entropy is evaluated, only applies to METHOD 'maxe' (default=Inf)      |
| 'METHOD'  | 'ric'  | Standard 2-Riccati solution (default)  |
|           | 'lmi'  | LMI solution   |
|           | 'maxe' | Maximum entropy solution   |
| 'DISPLAY' | 'off'  | No command window display, or command window displays synthesis progress information (default) |
|           | 'on'   |  |

When DISPLAY='on', the hinfsyn program displays several variables indicating the progress of the algorithm. For each  $\gamma$  value being tested, the minimum magnitude, real part of the eigenvalues of the X and Y Hamiltonian matrices are displayed along with the minimum eigenvalue of  $X_\infty$  and  $Y_\infty$ , which are the solutions to the X and Y Riccati equations, respectively. The maximum eigenvalue of  $X_\infty Y_\infty$ , scaled by  $\gamma^{-2}$ , is also displayed. A # sign is placed to the right of the condition that failed in the printout.

| Output Arguments    | Description                                       |
|---------------------|---|
| K                   | l ti controller                                   |
| CL= lft(P,K)        | l ti closed-loop system $T_{y_1 u_1}$             |
| GAM = norm(CL, Inf) | $H_\infty$ cost $\gamma = \ T_{y_1 u_1}\ _\infty$ |
| INFO                | Additional output information                     |

Additional output — structure array INFO containing possible additional information depending on METHOD)

|            |   |
|------------|---|
| INFO.AS    | All solutions controller, l ti two-port LFT   |
| INFO.KFI   | Full information gain matrix (constant feedback<br>$u_2(t) = K_{FI} \begin{bmatrix} x(t) \\ u_1(t) \end{bmatrix}$ |
| INFO.KFC   | Full control gain matrix (constant output-injection; $K_{FC}$ is the dual of $K_{FI}$ )                           |
| INFO.GAMFI | $H_\infty$ cost for full information $K_{FI}$   |
| INFO.GAMFC | $H_\infty$ cost for full control $K_{FC}$   |

## Examples

### H-infinity Controller Synthesis

Synthesize a controller for a random 4-output, 5-input plant with three states, where there are two output measurements provided to the controller.

```
rng(0, 'twister'); % for reproducibility
P = rss(3,4,5);
[K,CL,GAM] = hinfsyn(P,2,2);
```

Examine the optimal  $H_\infty$  cost.

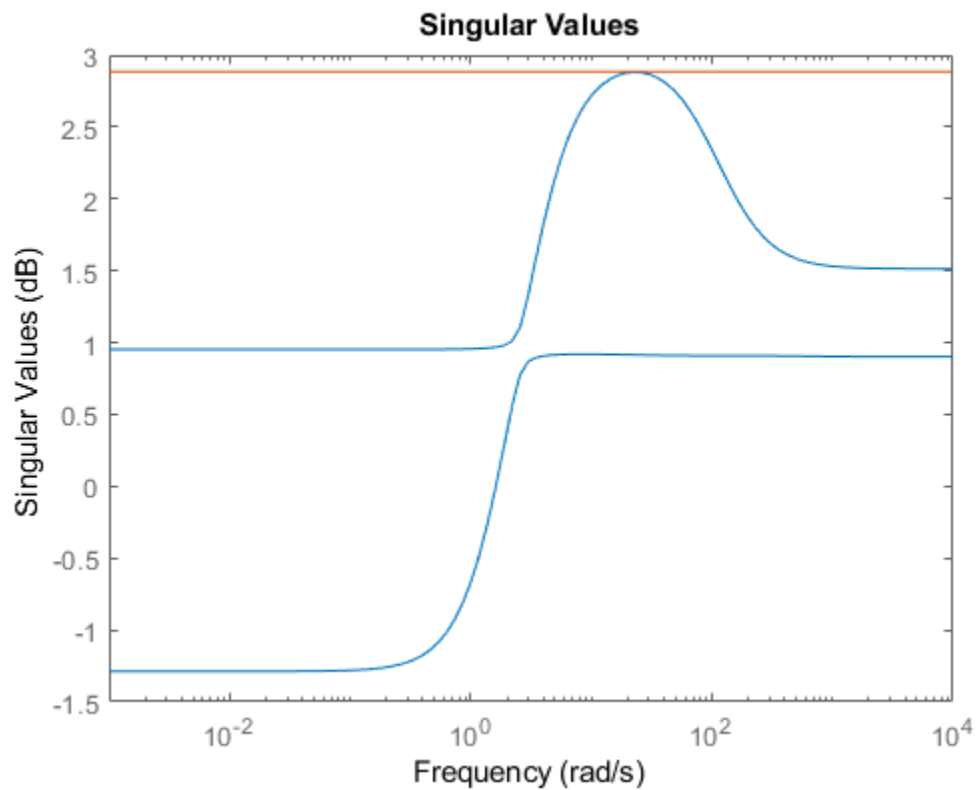
```
GAM
```

GAM = 1.3940

Use a singular-value plot to verify that

$$\|T_{y_1 u_1}\|_{\infty} = \sup_{\omega} \sigma_{\max}(T_{y_1 u_1}(j\omega)) < \gamma.$$

sigma(CL, ss(GAM))



**Mixed-Sensitivity Synthesis**

Design a mixed-sensitivity controller for the following plant, augmented by the following loop-shaping filters (see `mixsyn`).

$$G(s) = \frac{s-1}{s+1}, W_1 = \frac{0.1(s+100)}{100s+1}, W_2 = 0.1, \text{ no } W_3.$$

Define the plant, weighting filters, and augmented plant.

```
s = zpk('s');  
G = (s-1)/(s+1);  
W1 = 0.1*(s+100)/(100*s+1);  
W2 = 0.1;  
W3 = [];  
P = augw(G,W1,W2,W3);
```

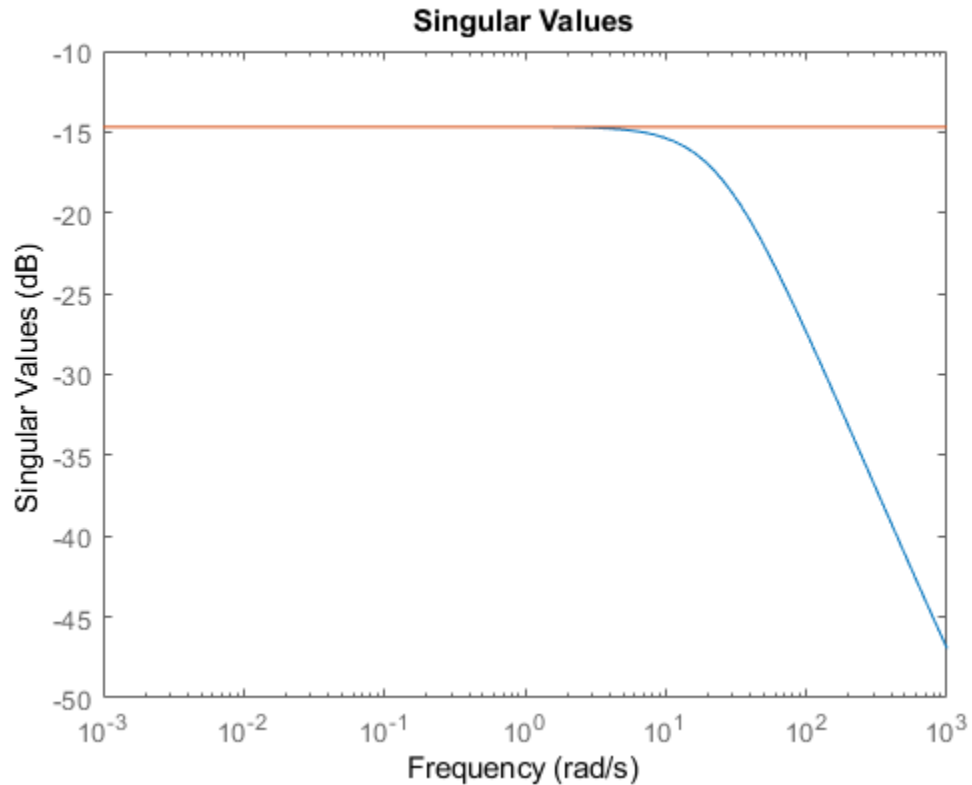
Synthesize the controller.

```
[K,CL,GAM] = hinfsyn(P);
```

Examine the singular values of the closed-loop result.

```
sigma(CL,ss(GAM))
```





For this system,  $GAM = 0.1844 = -14.6386$  dB.

Compute a new controller for the same system with no  $W_1$ .

```
W1 = [];
P = augw(G,W1,W2,W3);
[K,CL,GAM] = hinfosyn(P);
```

In this case, the resulting controller  $K$  is zero, and the closed-loop transfer function  $CL = K*(1+G*K)$  is also zero.

## Limitation

The plant must be stabilizable from the control inputs  $u$  and detectable from the measurement output  $y$ :

- $(A, B_2)$  must be stabilizable and  $(C_2, A)$  must be detectable.

Otherwise, `hinfsvn` returns an error.

## Algorithms

The default 'ric' method uses the two-Riccati formulae ([1],[2]) with loopshifting [3]. In the case of the 'lmi' method, `hinfsvn` employs the LMI technique ([4],[5],[6]). With 'METHOD' 'maxe', `K` returns the max entropy  $H_\infty$  controller that minimize an entropy integral relating to the point  $s_0$ ; i.e.,

$$\text{Entropy} = \frac{\gamma^2}{2\pi} \int_{-\infty}^{\infty} \ln \left| \det I - \gamma^{-2} T_{y_1 u_1}(j\omega)' T_{y_1 u_1}(j\omega) \right| \left[ \frac{s_0^2}{s_0^2 + \omega^2} \right] d\omega$$

where  $T_{y_1 u_1}$  is the closed-loop transfer function CL. With all methods, `hinfsvn` uses a standard  $\gamma$ -iteration technique to determine the optimal value of  $\gamma$ . Starting with high and low estimates of  $\gamma$ . The  $\gamma$ -iteration is a *bisection algorithm* that iterates on the value of  $\gamma$  in an effort to approach the optimal  $H_\infty$  control design. The stopping criterion for the bisection algorithm requires the relative difference between the last  $\gamma$  value that failed and the last  $\gamma$  value that passed be less than `TOLGAM` (default = .01)

At each value of  $\gamma$ , the algorithm employed requires tests to determine whether a solution exists for a given  $\gamma$  value. In the case of the 'ric' method, the conditions checked for the existence of a solution are:

- $H$  and  $J$  Hamiltonian matrices (which are formed from the state-space data of  $P$  and the  $\gamma$  level) must have no imaginary-axis eigenvalues.
- the stabilizing Riccati solutions  $X_\infty$  and  $Y_\infty$  associated with the Hamiltonian matrices must exist and be positive, semi-definite.
- spectral radius of  $(X_\infty, Y_\infty)$  must be less than or equal to  $\gamma^2$ .

When, `DISPLAY` is 'on', the `hinfsvn` program displays several variables, which indicate which of the above conditions are satisfied for each  $\gamma$  value being tested. In the case of

the default 'ric' method, the display includes the current value of  $\gamma$  being tested, real part of the eigenvalues of the  $X$  and  $Y$  Hamiltonian matrices along with the minimum eigenvalue of  $X_\infty$  and  $Y_\infty$ , which are the solutions to the  $X$  and  $Y$  Riccati equations, respectively. The maximum eigenvalue of  $X_\infty Y_\infty$ , scaled by  $\gamma^2$ , is also displayed. A # sign is placed to the right of the condition that failed in the printout. A similar display is produced with method 'lmi'

The algorithm works best when the following conditions are satisfied by the plant:

$D_{12}$  and  $D_{21}$  have full rank.

$$\begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix} \text{ has full column rank for all } \omega \in R.$$

$$\begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix} \text{ has full row rank for all } \omega \in R.$$

When the above rank conditions do not hold, the controller may have undesirable properties: If  $D_{12}$  and  $D_{21}$  are not full rank, the  $H_\infty$  controller  $K$  may have large high-frequency gain. If either of the latter two rank conditions does not hold at some frequency  $\omega$ , the controller may have very lightly damped poles near that frequency  $\omega$ .

In general, the solution to the infinity-norm optimal control problem is non-unique. The controller returned by hinfsyn is only one particular solution,  $K$ . When the 'ric' method is selected, the INFO.AS field of INFO contains the all-solution controller parameterization  $K_{AS}$ . All solutions to the infinity-norm control problem are parameterized

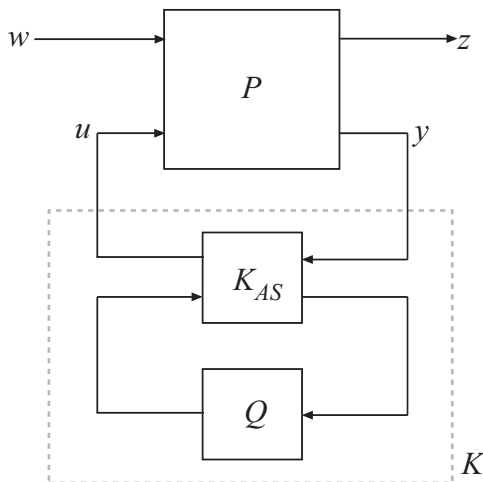
by a free stable contraction map  $Q$ , which is constrained by  $\|Q\|_\infty < 1$ . In other words, the solutions include every stabilizing controller  $K(s)$  that makes

$$\|T_{y_1 u_1}\|_\infty \triangleq \sup_{\omega} \sigma_{\max}(T_{y_1 u_1}(j\omega)) < \gamma.$$

These controllers are given by:

$$K = \text{lft}(\text{INFO.AS}, Q)$$

where  $Q$  is a stable LTI system satisfying  $\text{norm}(Q, \text{Inf}) < 1$ .



An important use of the infinity-norm control theory is for direct shaping of closed-loop singular value Bode plots of control systems. In such cases, the system  $P$  is typically the plant augmented with suitable loop-shaping filters — see `mixsyn`.

## References

- [1] Glover, K., and J.C. Doyle, "State-space formulae for all stabilizing controllers that satisfy an  $H_\infty$  norm bound and relations to risk sensitivity," *Systems & Control Letters*, vol. 11, no. 8, pp. 167-172, 1988.
- [2] Doyle, J.C., K. Glover, P. Khargonekar, and B. Francis, "State-space solutions to standard  $H_2$  and  $H_\infty$  control problems," *IEEE Transactions on Automatic Control*, vol. 34, no. 8, pp. 831-847, August 1989
- [3] Safonov, M.G., D.J.N. Limebeer, and R.Y. Chiang, "Simplifying the  $H_\infty$  Theory via Loop Shifting, Matrix Pencil and Descriptor Concepts", *Int. J. Contr.*, vol. 50, no. 6, pp. 2467-2488, 1989.
- [4] Packard, A., K. Zhou, P. Pandey, J. Leonhardson, and G. Balas, "Optimal, constant I/O similarity scaling for full-information and state-feedback problems," *Systems & Control Letters*, vol. 19, no. 4, pp. 271-280, 1992.
- [5] Gahinet, P., and P. Apkarian, "A linear matrix inequality approach to  $H_\infty$ -control," *Int J. Robust and Nonlinear Control*, vol. 4, no. 4, pp. 421-448, 1994.

- [6] Iwasaki, T., and R.E. Skelton, "All controllers for the general  $H_\infty$ -control problem: LMI existence conditions and state space formulas," *Automatica*, vol. 30, no. 8, pp. 1307-1317, 1994.

## **See Also**

augw | h2syn | hinfstruct | loopsyn | mktito | ncfsyn

**Introduced before R2006a**

## **icomplexify**

Helper function for complexify

### **Syntax**

```
DeltaR = icomplexify(DeltaCR)
```

### **Description**

`icomplexify` works on structures to extract a real value from a pair of related fields.

`DeltaR = icomplexify(DeltaCR)` affects field pairs of `DeltaCR` named `'foo'` and `'foo_cmpxfy'` where `'foo'` can be any field name. `DeltaR` is the same as `DeltaCR` except that the fields `'foo_cmpxfy'` are removed. `complexify`, by default, complexifies the real uncertainty with `ucomplex` atoms, though optionally `ultidyn` atoms can be used. If a `ucomplex` uncertainty was used to complexify the uncertain system, the real parts of `'foo_cmpxfy'` are added to the real parts of `'foo'`. If a `ultidyn` uncertainty was used to complexify the uncertain system, only the real parts of `'foo'` are returned.

### **See Also**

`complexify` | `robstab`

**Introduced in R2007a**

# iconnect

Create empty iconnect (interconnection) objects

## Syntax

`H = iconnect`

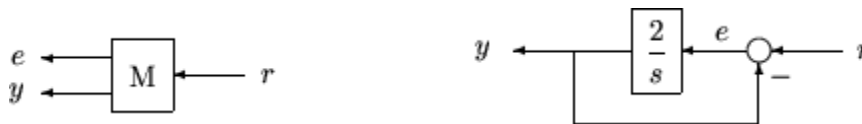
## Description

Interconnection objects (class `iconnect`) are an alternative to `sysic`, and are used to build complex interconnections of uncertain matrices and systems.

An `iconnect` object has 3 fields to be set by the user, `Input`, `Output` and `Equation`. `Input` and `Output` are `icsignal` objects, while `Equation` is a cell-array of equality constraints (using `equate`) on `icsignal` objects. Once these are specified, then the `System` property is the input/output model, implied by the constraints in `Equation`, relating the variables defined in `Input` and `Output`.

## Examples

`iconnect` can be used to create the transfer matrix `M` as described in the following figure.



Create three scalar `icsignal`: `r`, `e` and `y`. Create an empty `iconnect` object, `M`. Define the output of the interconnection to be `[e; y]`, and the input to be `r`. Define two constraints among the variables: `e = r - y`, and `y = (2/s) e`. Get the transfer function representation of the relationship between the input (`r`) and the output `[e; y]`.

```
r = icsignal(1);
e = icsignal(1);
```

```

y = icssignal(1);
M = iconnect;
M.Input = r;
M.Output = [e;y];
M.Equation{1} = equate(e,r-y);
M.Equation{2} = equate(y,tf(2,[1 0])*e);
tf(M.System)

```

The transfer functions from input to outputs are

$$\begin{aligned} \#1: & \frac{s}{s + 2} \\ \#2: & \frac{2}{s + 2} \end{aligned}$$

By not explicitly introducing  $e$ , this can be done more concisely with only one equality constraint.

```

r = icssignal(1);
y = icssignal(1);
N = iconnect;
N.Input = r;
N.Output = [r-y;y];
N.Equation{1} = equate(y,tf(2,[1 0])*(r-y));
tf(N.System)

```

You have created the same transfer functions from input to outputs.

$$\begin{aligned} \#1: & \frac{s}{s + 2} \\ \#2: & \frac{2}{s + 2} \end{aligned}$$

You can also specify uncertain, multivariable interconnections using `iconnect`. Consider two uncertain motor/generator constraints among 4 variables  $[V; I; T; W]$ ,  $V - R \cdot I - K \cdot W = 0$ , and  $T = K \cdot I$ . Find the uncertain  $2 \times 2$  matrix  $B$  so that  $[V; T] = B \cdot [W; I]$ .

```

R = ureal('R',1,'Percentage',[-10 40]);
K = ureal('K',2e-3,'Percentage',[-30 30]);
V = icssignal(1);

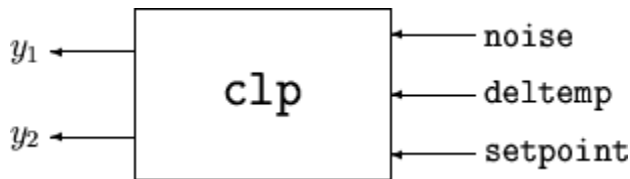
```



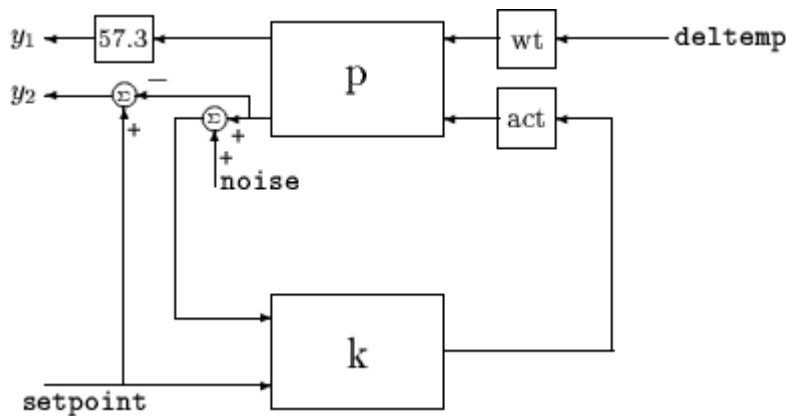
```

I = iccsignal(1);
T = iccsignal(1);
W = iccsignal(1);
M = iconnect;
M.Input = [W;I];
M.Output = [V;T];
M.Equation{1} = equate(V-R*I-K*W,iczero(1));
M.Equation{2} = equate(T,K*I);
B = M.System
UMAT: 2 Rows, 2 Columns
  K: real, nominal = 0.002, variability = [-30 30]%, 2 occurrences
  R: real, nominal = 1, variability = [-10 40]%, 1 occurrence
B.NominalValue
ans =
    0.0020    1.0000
         0     0.0020
    
```

A simple system interconnection, identical to the system illustrated in the sysic reference pages. Consider a three-input, two-output state-space matrix  $T$ ,



which has internal structure



```

P = rss(3,2,2);
K = rss(1,1,2);
A = rss(1,1,1);
    
```

```
W = rss(1,1,1);
M = iconnect;
noise = icsignal(1);
deltemp = icsignal(1);
setpoint = icsignal(1);
yp = icsignal(2);
rad2deg = 57.3
rad2deg =
    57.3000
M.Equation{1} = equate(yp,P*[W*deltemp;A*K*[noise+yp(2);setpoint]]);
M.Input = [noise;deltemp;setpoint];
M.Output = [rad2deg*yp(1);setpoint-yp(2)];
T = M.System;
size(T)
State-space model with 2 outputs, 3 inputs, and 6 states.
```

## Limitations

The syntax for `iconnect` objects and `icsignals` is very flexible. Without care, you can build inefficient (i.e., nonminimal) representations where the state dimension of the interconnection is greater than the sum of the state dimensions of the components. This is in contrast to `sysic`. In `sysic`, the syntax used to specify inputs to systems (the `input_to_ListedSubSystemName` variable) forces you to include each subsystem of the interconnection only once in the equations. Hence, interconnections formed with `sysic` are componentwise minimal. That is, the state dimension of the interconnection equals the sum of the state dimensions of the components.

## Algorithms

Each equation represents an equality constraint among the variables. You choose the input and output variables, and the `imp2exp` function makes the implicit relationship between them explicit.

## See Also

`icsignal` | `sysic`

**Introduced before R2006a**

# icsignal

Create `icsignal` object of specified dimension

## Syntax

```
v = icsignal(n);
```

```
v = icsignal(n,name)
```

## Description

`icsignal` creates an `icsignal` object, which is a symbolic column vector. The `icsignal` object is used with `iconnect` objects to specify signal constraints described by the interconnection of components.

`v = icsignal(n)` creates an `icsignal` object of vector length `n`. The value of `n` must be a nonnegative integer. `icsignal` objects are symbolic column vectors, used in conjunction with `iconnect` (interconnection) objects to specify the signal constraints described by an interconnection of components.

`v = icsignal(n,name)` creates an `icsignal` object of dimension `n`, with internal name identifier given by the character vector `name`.

## See Also

`iconnect` | `sysic`

**Introduced before R2006a**

## imp2ss

System realization via Hankel singular value decomposition

### Syntax

```
[a,b,c,d,totbnd,hsv] = imp2ss(y)
```

```
[a,b,c,d,totbnd,hsv] = imp2ss(y,ts,nu,ny,tol)
```

```
[ss,totbnd,hsv] = imp2ss(imp)
```

```
[ss,totbnd,hsv] = imp2ss(imp,tol)
```

### Description

The function `imp2ss` produces an approximate state-space realization of a given impulse response

```
imp=mksys(y,t,nu,ny,'imp');
```

using the Hankel SVD method proposed by S. Kung [2]. A continuous-time realization is computed via the inverse Tustin transform (using `bilin`) if  $t$  is positive; otherwise a discrete-time realization is returned. In the SISO case the variable  $y$  is the impulse response vector; in the MIMO case  $y$  is an  $N+1$ -column matrix containing  $N + 1$  time samples of the matrix-valued impulse response  $H_0, \dots, H_N$  of an  $nu$ -input,  $ny$ -output system stored row-wise:

$$y = [H_0(:); H_2(:); H_3(:); \dots ; H_N(:)]$$

The variable  $tol$  bounds the  $H_\infty$  norm of the error between the approximate realization ( $a, b, c, d$ ) and an exact realization of  $y$ ; the order, say  $n$ , of the realization ( $a, b, c, d$ ) is determined by the infinity norm error bound specified by the input variable  $tol$ . The inputs  $ts, nu, ny$ , and  $tol$  are optional. If omitted, they default to the values  $ts = 0, nu = 1, ny = (\text{number of rows of } y)/nu, tol = 0.01\bar{\sigma}_1$ . The output  $hsv = [\bar{\sigma}_1, \bar{\sigma}_2, \dots]$  returns the singular values (arranged in descending order of magnitude) of the Hankel matrix:

$$\Gamma = \begin{bmatrix} H_1 & H_2 & H_3 & \dots & H_N \\ H_2 & H_3 & H_4 & \dots & 0 \\ H_3 & H_4 & H_5 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ H_N & 0 & \dots & \dots & 0s \end{bmatrix}$$

Denoting by  $G_N$  a high-order exact realization of  $y$ , the low-order approximate model  $G$  enjoys the  $H_\infty$  norm bound

$$\|G - G_N\|_\infty \leq \text{totbnd}$$

where

$$\text{totbnd} = 2 \sum_{i=n+1}^N \bar{\sigma}_i.$$

## Algorithms

The realization  $(a, b, c, d)$  is computed using the Hankel SVD procedure proposed by Kung [2] as a method for approximately implementing the classical Hankel factorization realization algorithm. Kung's SVD realization procedure was subsequently shown to be equivalent to doing balanced truncation (balmr) on an exact state-space realization of the finite impulse response  $\{y(1), \dots, y(N)\}$  [3]. The infinity norm error bound for discrete balanced truncation was later derived by Al-Saggaf and Franklin [1]. The algorithm is as follows:

- 1 Form the Hankel matrix  $\Gamma$  from the data  $y$ .
- 2 Perform SVD on the Hankel matrix

$$\Gamma = U \Sigma V^* = [U_1 U_2] \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{bmatrix} V^*_{*1} \\ V^*_{*2} \end{bmatrix} = U_1 \Sigma_1 V^*_{*1}$$

where  $\Sigma_1$  has dimension  $n \times n$  and the entries of  $\Sigma_2$  are nearly zero.  $U_1$  and  $V_1$  have  $ny$  and  $nu$  columns, respectively.

- 3** Partition the matrices  $U_1$  and  $V_1$  into three matrix blocks:

$$U_1 = \begin{bmatrix} U_{11} \\ U_{12} \\ U_{13} \end{bmatrix} \begin{bmatrix} V_{11} \\ V_{12} \\ V_{13} \end{bmatrix}$$

where  $U_{11}, U_{13} \in C^{ny \times n}$  and  $V_{11}, V_{13} \in C^{nu \times n}$ .

- 4** A discrete state-space realization is computed as

$$A = \Sigma_1^{-\frac{1}{2}} \bar{U} \Sigma_1^{-\frac{1}{2}}$$

$$B = \Sigma_1^{-\frac{1}{2}} V_{11}^*$$

$$C = U_{11} \Sigma_1^{-\frac{1}{2}}$$

$$D = H_0$$

where

$$\bar{U} = \begin{bmatrix} U_{11} \\ U_{12} \end{bmatrix}' \begin{bmatrix} U_{12} \\ U_{13} \end{bmatrix}$$

- 5** If the sample time  $t$  is greater than zero, then the realization is converted to continuous time via the inverse of the Tustin transform

$$s = \frac{2}{t} \frac{z-1}{z+1}.$$

Otherwise, this step is omitted and the discrete-time realization calculated in Step 4 is returned.

## References

- [1] Al-Saggaf, U.M., and G.F. Franklin, "An Error Bound for a Discrete Reduced Order Model of a Linear Multivariable System," *IEEE Trans. on Autom. Contr.*, AC-32, 1987, p. 815-819.

- [2] Kung, S.Y., "A New Identification and Model Reduction Algorithm via Singular Value Decompositions," *Proc. Twelfth Asilomar Conf. on Circuits, Systems and Computers*, November 6-8, 1978, p. 705-714.
- [3] Silverman, L.M., and M. Bettayeb, "Optimal Approximation of Linear Systems," *Proc. American Control Conf.*, San Francisco, CA, 1980.

**Introduced before R2006a**

## **ispsys**

True for parameter-dependent systems

### **Syntax**

```
bool = ispsys(sys)
```

### **Description**

`bool = ispsys(sys)` returns 1 if `sys` is a polytopic or parameter-dependent system.

### **See Also**

`psinfo` | `psys`

**Introduced before R2006a**



# isuncertain

Check whether argument is uncertain class type

## Syntax

```
B = isuncertain(A)
```

## Description

Returns `true` if input argument is uncertain, `false` otherwise. Uncertain classes are `umat`, `ufrd`, `uss`, `ureal`, `ultidyn`, `ucomplex`, `ucomplexm`, and `udyn`.

## Examples

In this example, you verify the correct operation of `isuncertain` on `double`, `ureal`, `ss`, and `uss` objects.

```
isuncertain(rand(3,4))
ans =
    0
isuncertain(ureal('p',4))
ans =
    1
isuncertain(rss(4,3,2))
ans =
    0
isuncertain(rss(4,3,2)*[ureal('p1',4) 6;0 1])
ans =
    1
```

## Limitations

`isuncertain` only checks the class of the input argument, and does not actually verify that the input argument is truly uncertain. Create a `umat` by *lifting* a constant (i.e., not-uncertain) matrix to the `umat` class.

```
A = umat([2 3;4 5;6 7]);
```

Note that although `A` is in class `umat`, it is not actually uncertain. Nevertheless, based on class, the result of `isuncertain(A)` is `true`.

```
isuncertain(A)
ans =
     1
```

The result of `simplify(A)` is a double, and hence not uncertain.

```
isuncertain(simplify(A))
ans =
     0
```

**Introduced before R2006a**

## lftdata

Decompose uncertain objects into fixed certain and normalized uncertain parts

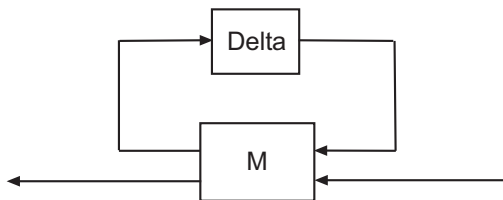
### Syntax

```
[M,Delta] = lftdata(A);
[M,Delta] = lftdata(A,List);
[M,Delta,Blkstruct] = lftdata(A);
[M,Delta,Blkstruct,Normunc] = lftdata(A);
```

### Description

`lftdata` decomposes an uncertain object into a fixed certain part and a normalized uncertain part. `lftdata` can also partially decompose an uncertain object into an uncertain part and a normalized uncertain part. Uncertain objects (`umat`, `ufrd`, `uss`) are represented as certain (i.e., not-uncertain) objects in feedback with block-diagonal concatenations of uncertain elements.

$[M, \Delta] = \text{lftdata}(A)$  separates the uncertain object  $A$  into a certain object  $M$  and a normalized uncertain matrix  $\Delta$  such that  $A$  is equal to  $\text{lft}(\Delta, M)$ , as shown below.



If  $A$  is a `umat`, then  $M$  will be `double`; if  $A$  is a `uss`, then  $M$  will be `ss`; if  $A$  is a `ufrd`, then  $M$  will be `frd`. In all cases,  $\Delta$  is a `umat`.

$[M, \Delta] = \text{lftdata}(A, \text{List})$  separates the uncertain object  $A$  into an uncertain object  $M$ , in feedback with a normalized uncertain matrix  $\Delta$ . `List` is a cell (or char)

array of names of uncertain elements of *A* that make up *Delta*. All other uncertainty in *A* remains in *M*.

`lftdata(A, fieldnames(A.Uncertainty))` is the same as `lftdata(A)`.

`[M, DELTA, BLKSTRUCT] = lftdata(A)` returns an *N*-by-1 structure array *BLKSTRUCT*, where *BLKSTRUCT(i)* describes the *i*-th normalized uncertain element. This uncertainty description can be passed directly to the low-level structured singular value analysis function `mussv`.

`[M, DELTA, BLKSTRUCT, NORMUNC] = lftdata(A)` returns the cell array *NORMUNC* of normalized uncertain elements. Each normalized element has 'Normalized' appended to its original name to avoid confusion. Note that `lft(blkdiag(NORMUNC{:}), M)` is equivalent to *A*.

## Examples

Create an uncertain matrix *A* with 3 uncertain parameters *p1*, *p2* and *p3*. You can decompose *A* into its certain, *M*, and normalized uncertain parts, *Delta*.

```
p1 = ureal('p1', -3, 'perc', 40);
p2 = ucomplex('p2', 2);
A = [p1 p1+p2; 1 p2];
[M, Delta] = lftdata(A);
```

You can inspect the difference between the original uncertain matrix, *A*, and the result formed by combining the two results from the decomposition.

```
simplify(A-lft(Delta,M))
ans =
     0     0
     0     0
M
M =
     0         0    1.0954    1.0954
     0         0         0    1.0000
  1.0954    1.0000   -3.0000   -1.0000
     0    1.0000    1.0000    2.0000
```

You can check the worst-case norm of the uncertain part using `wcnorm`. Compare samples of the uncertain part *A* with the uncertain matrix *A*.

```

wcn = wcnorm(Delta)
wcn =
    lbound: 1.0000
    ubound: 1.0001
usample(Delta,5)
ans(:,:,1) =
    0.8012          0
         0      0.2499 + 0.6946i
ans(:,:,2) =
    0.4919          0
         0      0.2863 + 0.6033i
ans(:,:,3) =
   -0.1040          0
         0      0.7322 - 0.3752i
ans(:,:,4) =
    0.8296          0
         0      0.6831 + 0.1124i
ans(:,:,5) =
    0.6886          0
         0      0.0838 + 0.3562i

```

## Uncertain Systems

Create an uncertain matrix A with 2 uncertain real parameters v1 and v2 and create an uncertain system G using A as the dynamic matrix and simple matrices for the input and output.

```

A = [ureal('p1',-3,'perc',40) 1;1 ureal('p2',-2)];
sys = ss(A,[1;0],[0 1],0);
sys.InputGroup.ActualIn = 1;
sys.OutputGroup.ActualOut = 1;

```

You can decompose G into a certain system, Msys, and a normalized uncertain matrix, Delta. You can see from Msys that it is certain and that the input and output groups have been adjusted.

```

[Msys,Delta] = lftdata(sys);
Msys

```

```

a =
      x1  x2
x1   -3   1
x2    1  -2

```

```
b =
      u1    u2    u3
x1  1.095    0    1
x2    0      1    0
```

```
c =
      x1    x2
y1  1.095    0
y2    0      1
y3    0      1
```

```
d =
      u1  u2  u3
y1    0   0   0
y2    0   0   0
y3    0   0   0
```

```
Input groups:
  Name      Channels
ActualIn      3
p1_NC         1
p2_NC         2
```

```
Output groups:
  Name      Channels
ActualOut      3
p1_NC         1
p2_NC         2
```

Continuous-time model.

You can compute the norm on samples of the difference between the original uncertain matrix and the result formed by combining Msys and Delta.

```
norm(usample(sys-lft(Delta,Msys),'p1',4,'p2',3),'inf')
ans =
      0      0      0
      0      0      0
      0      0      0
      0      0      0
```

## Partial Decomposition

Create an uncertain matrix A and derive an uncertain matrix B using an implicit-to-explicit conversion, `imp2exp`. Note that B has 2 uncertain parameters R and K. You can decompose B into certain, M, and normalized uncertain parts, Delta.

```
R = ureal('R',1,'Percentage',[-10 40]);
K = ureal('K',2e-3,'Percentage',[-30 30]);
A = [1 -R 0 -K;0 -K 1 0];
Yidx = [1 3];
Uidx = [4 2];
B = imp2exp(A,Yidx,Uidx);
[M,Delta] = lftdata(B);
```

The same operation can be performed by defining the uncertain parameters, K and R, to be extracted.

```
[MK,DeltaR] = lftdata(B,'R');
MK
UMAT: 3 Rows, 3 Columns
  K: real, nominal = 0.002, variability = [-30 30]%, 2 occurrences
[MR,DeltaK] = lftdata(B,'K');
MR
UMAT: 4 Rows, 4 Columns
  R: real, nominal = 1, variability = [-10 40]%, 1 occurrence

simplify(B-lft(Delta,M))
ans =
    0     0
    0     0
simplify(B-lft(DeltaR,MK))
ans =
    0     0
    0     0
simplify(B-lft(DeltaK,MR))
ans =
    0     0
    0     0
```

Sample and inspect the uncertain part as well as the difference between the original uncertain matrix and the sampled matrix. You can see the result formed by combining the two results from the decomposition.

```
[Mall,Deltaall] = lftdata(B,{'K';'R'});
simplify(Mall)-M
```

```
ans =  
  0      0      0      0      0  
  0      0      0      0      0  
  0      0      0      0      0  
  0      0      0      0      0  
  0      0      0      0      0
```

## See Also

`lft` | `ssdata`

**Introduced before R2006a**



# lmiedit

Specify or display systems of LMIs as MATLAB expressions

## Syntax

`lmiedit`

## Description

`lmiedit` is a graphical user interface for the symbolic specification of LMI problems. Typing `lmiedit` calls up a window with two editable text areas and various buttons. To specify an LMI system,

- 1 Give it a name (top of the window).
- 2 Declare each matrix variable (name and structure) in the upper half of the window. The structure is characterized by its type (S for symmetric block diagonal, R for unstructured, and G for other structures) and by an additional structure matrix similar to the second input argument of `lmivar`. Please use one line per matrix variable in the text editing areas.
- 3 Specify the LMIs as MATLAB expressions in the lower half of the window. An LMI can stretch over several lines. However, do not specify more than one LMI per line.

Once the LMI system is fully specified, you can perform the following operations by pressing the corresponding button:

- Visualize the sequence of `lmivar/lmiterm` commands needed to describe this LMI system (`view commands` buttons)
- Conversely, display the symbolic expression of the LMI system produced by a particular sequence of `lmivar/lmiterm` commands (click the `describe...` buttons)
- Save the symbolic description of the LMI system (`save` button). This description can be reloaded later on by pressing the `load` button
- Read a sequence of `lmivar/lmiterm` commands from a file (`read` button). The matrix expression of the LMI system specified by these commands is then displayed by clicking on `describe the LMIs...`

- Write in a file the sequence of `lmivar`/`lmiterm` commands needed to specify a particular LMI system (`write` button)
- Generate the internal representation of the LMI system by pressing `create`. The result is written in a MATLAB variable with the same name as the LMI system

### Tips

Editable text areas have built-in scrolling capabilities. To activate the scroll mode, click in the text area, maintain the mouse button down, and move the mouse up or down. The scroll mode is only active when all visible lines have been used.

### See Also

`lmiinfo` | `lmiterm` | `lmivar` | `newlmi`

**Introduced before R2006a**

# lmiinfo

Information about variables and term content of LMIs

## Syntax

`lmiinfo`

## Description

`lmiinfo` provides qualitative information about the system of LMIs `lmisys`. This includes the type and structure of the matrix variables, the number of diagonal blocks in the inner factors, and the term content of each block.

`lmiinfo` is an interactive facility where the user seeks specific pieces of information. General LMIs are displayed as

$$N' * L(x) * N < M' * R(x) * M$$

where  $N, M$  denote the outer factors and  $L, R$  the left and right inner factors. If the outer factors are missing, the LMI is simply written as

$$L(x) < R(x)$$

If its right side is zero, it is displayed as

$$N' * L(x) * N < 0$$

Information on the block structure and term content of  $L(x)$  and  $R(x)$  is also available. The term content of a block is symbolically displayed as

$$C1 + A1*X2*B1 + B1'*X2*A1' + a2*X1 + x3*Q1$$

with the following conventions:

- $X1, X2, x3$  denote the problem variables. Upper-case  $X$  indicates matrix variables while lower-case  $x$  indicates scalar variables. The labels 1,2,3 refer to the first, second, and third matrix variable in the order of declaration.

- $C_j$  refers to constant terms. Special cases are  $I$  and  $-I$  ( $I$  = identity matrix).
- $A_j$ ,  $B_j$  denote the left and right coefficients of variable terms. Lower-case letters such as  $a_2$  indicate a scalar coefficient.
- $Q_j$  is used exclusively with scalar variables as in  $x_3 * Q_1$ .

The index  $j$  in  $A_j$ ,  $B_j$ ,  $C_j$ ,  $Q_j$  is a dummy label. Hence  $C_1$  may appear in several blocks or several LMIs without implying any connection between the corresponding constant terms. Exceptions to this rule are the notations  $A_1 * X_2 * A_1'$  and  $A_1 * X_2 * B_1 + B_1' * X_2' * A_1'$  which indicate symmetric terms and symmetric pairs in diagonal blocks.

## Examples

Consider the LMI

$$0 \begin{pmatrix} -2X + A^T Y B + B^T Y^T A + I & X C \\ C^T X & -z I \end{pmatrix}$$

where the matrix variables are  $X$  of Type 1,  $Y$  of Type 2, and  $z$  scalar. If this LMI is described in `lmis`, information about  $X$  and the LMI block structure can be obtained as follows:

```
lmiinfo(lmis)
```

```
          LMI ORACLE
-----
```

```
This is a system of 1 LMI with 3 variable matrices
```

```
Do you want information on
  (v) matrix variables   (l) LMIs   (q) quit
```

```
?> v
```

```
Which variable matrix (enter its index k between 1 and 3) ? 1
X1 is a 2x2 symmetric block diagonal matrix
its (1,1)-block is a full block of size 2
```

```
-----
```

This is a system of 1 LMI with 3 variable matrices

Do you want information on

(v) matrix variables      (l) LMIs      (q) quit

?> l

Which LMI (enter its number k between 1 and 1) ? 1

This LMI is of the form

$$0 < R(x)$$

where the inner factor(s) has 2 diagonal block(s)

Do you want info on the right inner factor ?

(w) whole factor      (b) only one block  
(o) other LMI      (t) back to top level

?> w

Info about the right inner factor

block (1,1) :  $I + a1*X1 + A2*X2*B2 + B2'*X2'*A2'$

block (2,1) :  $A3*X1$

block (2,2) :  $x3*A4$

(w) whole factor      (b) only one block  
(o) other LMI      (t) back to top level

-----

This is a system of 1 LMI with 3 variable matrices

Do you want information on

(v) matrix variables      (l) LMIs      (q) quit

?> q

It has been a pleasure serving you!

Note that the prompt symbol is `?>` and that answers are either indices or letters. All blocks can be displayed at once with option `(w)`, or you can prompt for specific blocks with option `(b)`.

## Tips

`lmiinfo` does not provide access to the numerical value of LMI coefficients.

## See Also

`decinfo` | `decnbr` | `lminbr` | `matnbr`

**Introduced before R2006a**

## **lminbr**

Return number of LMIs in LMI system

### **Syntax**

```
k = lminbr(lmisys)
```

### **Description**

`lminbr` returns the number `k` of linear matrix inequalities in the LMI problem described in `lmisys`.

### **See Also**

`lmiinfo` | `matnbr`

**Introduced before R2006a**

## lmireg

Specify LMI regions for pole placement

### Syntax

```
region = lmireg
```

```
region = lmireg(reg1,reg2,...)
```

### Description

`lmireg` is an interactive facility to specify the LMI regions involved in multi-objective  $H_\infty$  synthesis with pole placement constraints (see `msfsyn` and `h2hinfsyn`). An LMI region is any convex subset  $D$  of the complex plane that can be characterized by an LMI in  $z$  and  $z^*$ , i.e.,

$$D = \{z \in \mathbb{C} : L + Mz + M^T \bar{z} < 0\}$$

for some fixed real matrices  $M$  and  $L = L^T$ . This class of regions encompasses half planes, strips, conic sectors, disks, ellipses, and any intersection of the above.

Calling `lmireg` without argument starts an interactive query/answer session where you can specify the region of your choice. The matrix `region = [L, M]` is returned upon termination. This matrix description of the LMI region can be passed directly to `msfsyn` for synthesis purposes.

The function `lmireg` can also be used to intersect previously defined LMI regions `reg1`, `reg2`, . . . . The output region is then the  $[L, M]$  description of the intersection of these regions.

### Examples



## Define LMI Region for Pole Placement

For LMI controller synthesis with functions like `msfsyn` and `h2hinfyn`, you can restrict the eigenvalues of the closed-loop system to an LMI region. The region is specified as a matrix of the form  $[L \ M]$ . In this example, use `lmireg` interactively to generate a matrix you can use to restrict the poles of the closed-loop system to  $\text{Re}(z) < -1$ .

Start the interactive process.

```
region = lmireg
```

```
Select a region among the following:
```

```
h)  Half-plane
d)  Disk
c)  Conic sector
e)  Ellipsoid
p)  Parabola
s)  Horizontal strip
m)  Matrix description of the LMI region
q)  Quit
choice:
```

The software prompts you to select the geometry of the region. For this example, enter `h` to specify a half-plane region. The software now prompts you to specify left half-plane ( $\text{Re}(z)$  less than some value) or right half-plane ( $\text{Re}(z)$  greater than some value).

```
Orientation (x < x0 -> l , x > x0 -> r):
```

Enter `l` to specify a left half-plane. The software prompts you to specify a value for `x0`.

```
Specify x0:
```

Enter `-1`. You have now completely specified the restriction  $\text{Re}(z) < -1$ . If you want to specify additional regional constraints on the pole locations, you can select another geometry now and follow the prompts. For this example, enter `q` to generate the LMI region matrix corresponding to  $\text{Re}(z) < -1$ .

```
region =
```

```
  2.0000 + 1.0000i   1.0000 + 0.0000i
```

You can now use `region` with `msfsyn` or `h2hinfosyn`.

## **See Also**

`h2hinfosyn` | `msfsyn`

**Introduced before R2006a**

# lmiterm

Specify term content of LMIs

## Syntax

`lmiterm(termID,A,B,flag)`

## Description

`lmiterm` specifies the term content of an LMI one term at a time. Recall that *LMI term* refers to the elementary additive terms involved in the block-matrix expression of the LMI. Before using `lmiterm`, the LMI description must be initialized with `setlmis` and the matrix variables must be declared with `lmivar`. Each `lmiterm` command adds one extra term to the LMI system currently described.

LMI terms are one of the following entities:

- outer factors
- constant terms (fixed matrices)
- variable terms  $AXB$  or  $AX^TB$  where  $X$  is a matrix variable and  $A$  and  $B$  are given matrices called the term coefficients.

When describing an LMI with several blocks, remember to specify **only the terms in the blocks on or below the diagonal** (or equivalently, only the terms in blocks on or above the diagonal). For instance, specify the blocks (1,1), (2,1), and (2,2) in a two-block LMI.

In the calling of `lmiterm`, `termID` is a four-entry vector of integers specifying the term location and the matrix variable involved.

$$\text{termID}(1) = \begin{cases} +p \\ -p \end{cases}$$

where positive  $p$  is for terms on the *left-side* of the  $p$ -th LMI and negative  $p$  is for terms on the *right-side* of the  $p$ -th LMI.

Recall that, by convention, the left side always refers to the smaller side of the LMI. The index  $p$  is relative to the order of declaration and corresponds to the identifier returned by `newlmi`.

$$\text{termID}(2 : 3) = \begin{cases} [0,0] & \text{for outer factors} \\ [i,j] & \text{for terms in the } (i,j)\text{-th block of the left or right inner factor} \end{cases}$$

$$\text{termID}(4) = \begin{cases} 0 & \text{for outer factors} \\ x & \text{for variable terms } AXB \\ -x & \text{for variable terms } AX^T B \end{cases}$$

where  $x$  is the identifier of the matrix variable  $X$  as returned by `lmivar`.

The arguments  $A$  and  $B$  contain the numerical data and are set according to:

| Type of Term      | A                    | B                    |
|-------------------|----------------------|----------------------|
| outer factor $N$  | matrix value of $N$  | omit                 |
| constant term $C$ | matrix value of $C$  | omit                 |
| variable term     | matrix value of $A$  | matrix value of $B$  |
| $AXB$ or $AX^T B$ | (1 if $A$ is absent) | (1 if $B$ is absent) |

Note that identity outer factors and zero constant terms need not be specified.

The extra argument `flag` is optional and concerns only conjugated expressions of the form

$$(AXB) + (AXB^T) = AXB + B^T X^T A^T$$

in *diagonal blocks*. Setting `flag = 's'` allows you to specify such expressions with a single `lmiterm` command. For instance,

```
lmiterm([1 1 1 X],A,1,'s')
```

adds the symmetrized expression  $AX + X^T A^T$  to the (1,1) block of the first LMI and summarizes the two commands

```
lmiterm([1 1 1 X],A,1)
lmiterm([1 1 1 -X],1,A')
```

Aside from being convenient, this shortcut also results in a more efficient representation of the LMI.

## Examples

### Specify LMI Terms

Consider the LMI

$$\begin{pmatrix} 2AX_2A^T - x_3E + DD^T & B^T X_1 \\ X_1^T B & -I \end{pmatrix} < M^T \begin{pmatrix} CX_1C^T + CX_1^T C^T & 0 \\ 0 & -fX_2 \end{pmatrix} M$$

where  $X_1$ ,  $X_2$  are matrix variables of Types 2 and 1, respectively, and  $x_3$  is a scalar variable (Type 1).

After you initialize the LMI description using `setlmis` and declare the matrix variables using `lmivar`, specify the terms on the left side of this LMI.

```
lmiterm([1 1 1 X2],2*A,A') % 2*A*X2*A'
lmiterm([1 1 1 x3],-1,E) % -x3*E
lmiterm([1 1 1 0],D*D') % D*D'
lmiterm([1 2 1 -X1],1,B) % X1'*B
lmiterm([1 2 2 0],-1) % -I
```

Here  $X1$ ,  $X2$ , and  $x3$  are the variable identifiers returned by `lmivar` when you declare the variables.

Similarly, specify the term content of the right side.

```
lmiterm([-1 0 0 0],M) % outer factor M
lmiterm([-1 1 1 X1],C,C','s') % C*X1*C'+C*X1'*C'
lmiterm([-1 2 2 X2],-f,1) % -f*X2
```

Note that  $CX_1C^T + CX_1^T C^T$  is specified by a single `lmiterm` command with the flag 's' to ensure proper symmetrization.

## See Also

`getlmis` | `lmiedit` | `lmivar` | `newlmi` | `setlmis`

**Introduced before R2006a**

# lmivar

Specify matrix variables in LMI problem

## Syntax

```
X = lmivar(type,struct)
```

```
[X,n,sX] = lmivar(type,struct)
```

## Description

`lmivar` defines a new matrix variable  $X$  in the LMI system currently described. The optional output  $X$  is an identifier that can be used for subsequent reference to this new variable.

The first argument `type` selects among available types of variables and the second argument `struct` gives further information on the structure of  $X$  depending on its type. Available variable types include:

**type=1:** Symmetric matrices with a block-diagonal structure. Each diagonal block is either full (arbitrary symmetric matrix), scalar (a multiple of the identity matrix), or identically zero.

If  $X$  has  $R$  diagonal blocks, `struct` is an  $R$ -by-2 matrix where

- `struct(r,1)` is the size of the  $r$ -th block
- `struct(r,2)` is the type of the  $r$ -th block (1 for full, 0 for scalar, -1 for zero block).

**type=2:** Full  $m$ -by- $n$  rectangular matrix. Set `struct = [m,n]` in this case.

**type=3:** Other structures. With Type 3, each entry of  $X$  is specified as zero or  $\pm x$  where  $x_n$  is the  $n$ -th decision variable.

Accordingly, `struct` is a matrix of the same dimensions as  $X$  such that

- `struct(i,j)=0` if  $X(i,j)$  is a hard zero

- `struct(i,j)=n` if  $X(i,j) = x_n$
- `struct(i,j)=-n` if  $X(i,j) = -x_n$

Sophisticated matrix variable structures can be defined with Type 3. To specify a variable  $X$  of Type 3, first identify how many *free independent entries* are involved in  $X$ . These constitute the set of decision variables associated with  $X$ . If the problem already involves  $n$  decision variables, label the new free variables as  $x_{n+1}, \dots, x_{n+p}$ . The structure of  $X$  is then defined in terms of  $x_{n+1}, \dots, x_{n+p}$  as indicated above. To help specify matrix variables of Type 3, `lmivar` optionally returns two extra outputs: (1) the total number  $n$  of scalar decision variables used so far and (2) a matrix `sX` showing the entry-wise dependence of  $X$  on the decision variables  $x_1, \dots, x_n$ .

## Examples

### Type 1 and Type 2 Matrix Variables

Consider an LMI system with three matrix variables  $X_1$ ,  $X_2$ , and  $X_3$  such that

- $X_1$  is a 3-by-3 symmetric matrix (unstructured),
- $X_2$  is a 2-by-4 rectangular matrix (unstructured),
- $X_3 =$

$$\begin{pmatrix} \Delta & 0 & 0 \\ 0 & \delta_1 & 0 \\ 0 & 0 & \delta_2 I_2 \end{pmatrix},$$

where  $\Delta$  is an arbitrary 5-by-5 symmetric matrix,  $\delta_1$  and  $\delta_2$  are scalars, and  $I_2$  denotes the identity matrix of size 2.

Define these three variables using `lmivar`.

```
setlmis([])
X1 = lmivar(1,[3 1]);           % Type 1
```



```
X2 = lmivar(2,[2 4]);           % Type 2 of dimension 2-by-4
X3 = lmivar(1,[5 1;1 0;2 0]); % Type 1
```

The last command defines  $X_3$  as a variable of Type 1 with one full block of size 5 and two scalar blocks of sizes 1 and 2, respectively.

### Type 3 Matrix Variables

Combined with the extra outputs  $n$  and  $sX$  of `lmivar`, Type 3 allows you to specify fairly complex matrix variable structures. For instance, consider a matrix variable  $X$  with structure given by:

$$X = \begin{pmatrix} X_1 & 0 \\ 0 & X_2 \end{pmatrix}$$

where  $X_1$  and  $X_2$  are 2-by-3 and 3-by-2 rectangular matrices, respectively. Specify this structure as follows.

Define the rectangular variables  $X_1$  and  $X_2$ .

```
setlmis([])
[X1,n,sX1] = lmivar(2,[2 3]);
[X2,n,sX2] = lmivar(2,[3 2]);
```

The outputs  $sX1$  and  $sX2$  give the decision variable content of  $X_1$  and  $X_2$ .

$sX1$

$sX1 = 2 \times 3$

|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |

$sX2$

$sX2 = 3 \times 2$

```
7      8
9      10
11     12
```

For instance,  $sX2(1,1) = 7$  means that the (1,1) entry of  $X_2$  is the seventh decision variable.

Next, use Type 3 to specify the matrix variable  $X$ , and define its structure in terms of the structures of  $X_1$  and  $X_2$ .

```
[X,n,sX] = lmmivar(3,[sX1,zeros(2);zeros(3),sX2]);
```

Confirm that the resulting  $X$  has the desired structure.

sX

sX = 5×5

```
1      2      3      0      0
4      5      6      0      0
0      0      0      7      8
0      0      0      9     10
0      0      0     11     12
```

## See Also

[delmvar](#) | [getlms](#) | [lmedit](#) | [lmiterm](#) | [setlms](#) | [setmvar](#) | [skewdec](#)

**Introduced before R2006a**

# loopmargin

Stability margin analysis of LTI and Simulink feedback loops

## Syntax

```
[cm, dm, mm] = loopmargin(L)
[m1, ..., mn] = loopmargin(L, MFLAG)
[cmi, dmi, mmi, cmo, dmo, mmo, mmio] = loopmargin(P, C)
[m1, ..., mn] = loopmargin(P, C, MFLAG)
[cm, dm, mm] = loopmargin(Model, Blocks, Ports)
[cm, dm, mm, info] = loopmargin(Model, Blocks, Ports, OP)
[m1, ..., mn, info] = loopmargin(Model, Blocks, Ports, MFLAG)
[m1, ..., mn, info] = loopmargin(Model, Blocks, Ports, OP, MFLAG)
```

## Description

`[cm, dm, mm] = loopmargin(L)` analyzes the multivariable feedback loop consisting of the loop transfer matrix  $L$  (size  $N$ -by- $N$ ) in negative feedback with an  $N$ -by- $N$  identity matrix.

`cm`, or classical gain and phase margins, is an  $N$ -by-1 structure corresponding to loop-at-a-time gain and phase margins for each channel.  $L$  is an LTI model. Use `-L` to specify positive feedback.

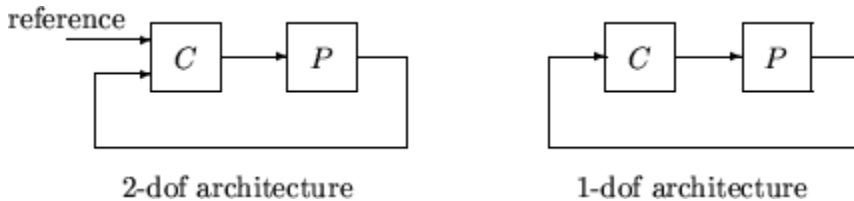
`dm` is an  $N$ -by-1 structure corresponding to loop-at-a-time disk gain and phase margins for each channel. The disk margin for the  $i$ -th feedback channel defines a circular region centered on the negative real axis at the average GainMargin (GM), e.g. ,  $(GM_{low} + GM_{high})/2$ , such that  $L(i, i)$  does not enter that region. Gain and phase disk margin bounds are derived from the radius of the circle, calculated based on the balanced sensitivity function.

`mm`, the multiloop disk margin, is a structure. `mm` describes how much independent and concurrent gain and phase variation can occur independently in each feedback channel while maintaining stability of the closed-loop system. Note that `mm` is a single structure, independent of the number of channels. This is because variations in all channels are

considered simultaneously. As in the case for disk margin, the guaranteed bounds are calculated based on a balanced sensitivity function.

`[m1,...,mn] = loopmargin(L,MFLAG)` returns a subset of the margins, specified by the character vector `MFLAG`. This optional argument may be any combination, in any order, of the 3 characters 'c', 'd' and 'm'. For example, `[m1,m2] = loopmargin(L,'m,c')` returns the multi-loop disk margin ('m') in `m1`, and the classical margins ('c') in `m2`. Use 'd' to specify the disk margin.

`[cmi,dmi,mmi,cmo,dmo,mmo,mmio] = loopmargin(P,C)` analyzes the multivariable feedback loop consisting of the controller `C` in negative feedback with the plant, `P`. `C` should only be the compensator in the feedback path, without reference channels, if it is a 2-dof architecture. That is, if the closed-loop system has a 2-dof architecture the reference channel of the controller should be eliminated, resulting in a 1-dof architecture, as shown.



`cmi`, `dmi` and `mmi` structures correspond to the classical loop-at-a-time gain and phase margins, disk margins and multiloop channel margins at the plant input respectively. The structures `cmo`, `dmo` and `mmo` have the same fields as described for `cmi`, `dmi` and `mmi` though they correspond to the plant outputs. `mmio`, or multi-input/multi-output margins, is a structure corresponding to simultaneous, independent, variations in all the individual input and output channels of the feedback loops. `mmio` has the same fields as `mmi` and `mmo`.

`[m1,...,mn] = loopmargin(P,C,MFLAG)` returns a subset of the margins, specified by `MFLAG`. This optional argument may be any combination, in any order, of the 7 character pairs 'ci', 'di', 'mi', 'co', 'do', 'mo', and 'mm'. For example, `[m1,m2,m3] = loopmargin(P,C,'mo,ci,mm')` returns the multi-loop disk margin at the plant output ('mo') in `m1`, the classical margins at the plant input ('ci') in `m2`, and the multi-loop disk margins for simultaneous, independent variations in all input and output channels ('mm') in `m3`.

## Usage with Simulink

`[cm,dm,mm] = loopmargin(Model,Blocks,Ports)` does a multi-loop stability margin analysis using Simulink Control Design software. `Model` specifies the name of the Simulink diagram for analysis. The margin analysis points are defined at the output ports (`Ports`) of blocks (`Blocks`) within the model. `Blocks` is a cell array of full block path names and `Ports` is a vector of the same dimension as `Blocks`. If all `Blocks` have a single output port, then `Ports` would be a vector of ones with the same length as `Blocks`.

Three types of stability margins are computed: loop-at-a-time classical gain and phase margins (`cm`), loop-at-a-time disk margins (`dm`) and a multi-loop disk margin (`mm`).

`[cm,dm,mm] = loopmargin(Model,Blocks,Ports,OP)` uses the operating point object `OP` to create linearized systems from the Simulink `Model`.

`[cm,dm,mm,info] = loopmargin(Model,Blocks,Ports,OP)` returns `info` in addition to the margins. `info` is a structure with fields `OperatingPoint`, `LinearizationIO` and `SignalNames` corresponding to the analysis.

`[m1,...,mn,info] = loopmargin(Model,Blocks,Ports,MFLAG)` and `[m1,...,mn,info] = loopmargin(Model,Blocks,Ports,OP,MFLAG)` return a subset of the margins, specified by the character vector `MFLAG`. This optional argument may be any combination, in any order, of the 3 characters 'c', 'd' and 'm'. For example, `[m1,m2] = loopmargin(Model,Blocks,Ports,'m,c')` returns the multi-loop disk margin ('m') in `m1`, and the classical margins ('c') in `m2`. Use 'd' to specify the disk margin.

## Basic Syntax

`[cm,dm,mm] = loopmargin(L)` `cm` is calculated using the `allmargin` command and has the same fields as `allmargin`. The output `cm` is an N-by-1 structure of classical gain and phase margins for each feedback channel with all other loops closed. `cm` has the following fields:

| Field       | Description   |
|-------------|---|
| GMFrequency | All -180 deg crossover frequencies (in radians-per-second)                  |
| GainMargin  | Corresponding gain margins ( $GM = 1/L$ where $L$ is the gain at crossover) |

| <b>Field</b> | <b>Description</b>   |
|--------------|--|
| PhaseMargin  | Corresponding phase margins (in degrees)   |
| PMFrequency  | All 0 dB crossover frequencies (in radians-per-second)   |
| DelayMargin  | Delay margins (in seconds for continuous-time systems, and multiples of the sample time for discrete-time systems)   |
| Stable       | 1 if nominal closed loop is stable, 0 otherwise. If L is a frd or ufrd object, the <b>Stable</b> flag is set to NaN. |

dm, or Disk Margin, is an N-by-1 structure of disk margins for each feedback channel with all other loops closed. dm has the following fields:

| <b>Field</b> | <b>Description</b>   |
|--------------|--|
| GainMargin   | Smallest gain variation (GM) such that a disk centered at the point - (GM(1) + GM(2))/2 just touches the Nyquist plot of the loop transfer function.   |
| PhaseMargin  | Smallest phase variation, in degrees, corresponding to the disk described in the GainMargin field.   |
| Frequency    | Frequency with the weakest disk margin, in rad/TimeUnit, where TimeUnit is the TimeUnit property of L.<br><br>For frd models, loopmargin computes margins at all the frequency points in the model, and returns the frequency with the weakest margin of these values. |

mm is a structure with the following fields.

| <b>Field</b> | <b>Description</b>   |
|--------------|--|
| GainMargin   | Guaranteed bound on simultaneous, independent, gain variations allowed in all plant channels.            |
| PhaseMargin  | Guaranteed bound on simultaneous, independent, phase variations allowed in all plant channels (degrees). |

| Field     | Description  |
|-----------|--|
| Frequency | Frequency with the weakest disk margin, in rad/TimeUnit, where TimeUnit is the TimeUnit property of L.<br><br>For frd models, loopmargin computes margins at all the frequency points in the model, and returns the frequency with the weakest margin of these values. |

## Relationship Between Disk Margin and Gain and Phase Margins

The disk margin is based on a multiplicative uncertainty model in which the loop gain  $L$  of each loop channel becomes

$$L \rightarrow L \frac{(1 + \Delta)}{(1 - \Delta)}, \quad |\Delta| < \alpha < 1,$$

where  $\Delta$  is complex. The uncertainty size  $\alpha$  is the disk margin. The uncertain quantity  $(1 + \Delta)/(1 - \Delta)$  has a gain component and a phase component. Thus, enforcing a disk margin  $\alpha$  also enforces minimum gain and phase margins given by

$$GM = \frac{1 + \alpha}{1 - \alpha},$$

$$PM = 2 \arctan(\alpha),$$

with  $GM$  in absolute units and  $PM$  in degrees. The gain and phase margins are therefore related by

$$GM = \frac{1 + \tan(PM/2)}{1 - \tan(PM/2)}.$$

When you specify independent gain and phase margins for tuning, the software chooses the smallest  $\alpha$  that enforces both values, which is

$$\alpha = \max \left[ \frac{GM - 1}{GM + 1}, \tan(PM/2) \right].$$

Note that  $GM$  and  $PM$  are not the same as the classical gain and phase margins. Rather, they provide stronger guarantees of stability, because both of the following can occur at the same time without loss of stability:

- The loop gain can increase or decrease by a factor of  $GM$ , and
- The loop phase can increase or decrease by  $PM$  degrees.

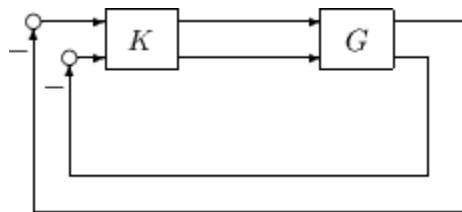
By contrast, the classical gain and phase margins consider only gain variations or phase variations at a single frequency, the crossover frequency.

## Examples

### MIMO Loop-at-a-Time Margins

This example shows how to compute loop-at-a-time margins (gain, phase, and/or distance to  $-1$ ), and also illustrates that such margins can be inaccurate measures of multivariable robustness margins. Margins of individual loops can be very sensitive to small perturbations within other loops.

Consider the nominal closed-loop system of the following illustration.



$G$  and  $K$  are 2-by-2 (MIMO) systems, given by:

$$G = \frac{1}{s^2 + \alpha^2} \begin{bmatrix} s - \alpha^2 & \alpha(s + 1) \\ -\alpha(s + 1) & s - \alpha^2 \end{bmatrix}, K = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}.$$

Set  $\alpha = 10$ , construct  $G$  in state-space form, and compute the loop margins.

$$\begin{aligned} \mathbf{a} &= [0 \ 10; -10 \ 0]; \\ \mathbf{b} &= \text{eye}(2); \end{aligned}$$



```
c = [1 8;-10 1];
d = zeros(2,2);
G = ss(a,b,c,d);
K = [1 -2;0 1];
[cmi,dmi,mmi,cmo,dmo,mmo,mmio] = loopmargin(G,K);
```

First consider the margins at the input to the plant. The first input channel has infinite gain margin and 90 degrees of phase margin based on the results from the `loopmargin` command, `cmi(1)`.

```
cmi(1)
```

```
ans = struct with fields:
  GainMargin: [1x0 double]
  GMFrequency: [1x0 double]
  PhaseMargin: 90
  PMFrequency: 21
  DelayMargin: 0.0748
  DMFrequency: 21
  Stable: 1
```

The disk margin analysis, `dmi`, of the first channel provides similar results.

```
dmi(1)
```

```
ans = struct with fields:
  GainMargin: [0 Inf]
  PhaseMargin: [-90 90]
  Frequency: 0
```

The second input channel has a gain margin of 2.105 and infinite phase margin based on the single-loop analysis, `cmi(2)`.

```
cmi(2)
```

```
ans = struct with fields:
  GainMargin: 2.1053
  GMFrequency: 0
  PhaseMargin: [1x0 double]
  PMFrequency: [1x0 double]
  DelayMargin: [1x0 double]
  DMFrequency: [1x0 double]
  Stable: 1
```

The disk margin analysis, `dmi(2)`, which allows for simultaneous gain and phase variations a loop-at-a-time results in maximum gain margin variations of 0.475 and 2.105 and phase margin variations of +/- 39.18 degs.

`dmi(2)`

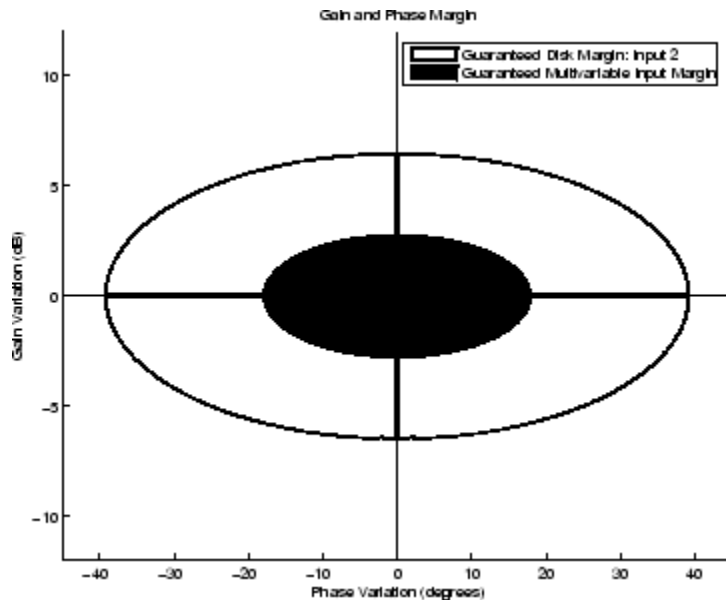
```
ans = struct with fields:
    GainMargin: [0.4750 2.1053]
    PhaseMargin: [-39.1846 39.1846]
    Frequency: 0
```

The multiple margin analysis of the plant inputs corresponds to allowing simultaneous, independent gain and phase margin variations in each channel. Allowing independent variation of the input channels further reduces the tolerance of the closed-loop system to variations at the input to the plant. The multivariable margin analysis, `mmi`, leads to a maximum allowable gain margin variation of 0.728 and 1.373 and phase margin variations of +/- 17.87 deg. Hence even though the first channel had infinite gain margin and 90 degrees of phase margin, allowing variation in both input channels leads to a factor of two reduction in the gain and phase margin.

`mmi`

```
mmi = struct with fields:
    GainMargin: [0.7288 1.3721]
    PhaseMargin: [-17.8304 17.8304]
    Frequency: 0
```

The guaranteed region of phase and gain variations for the closed-loop system can be illustrated graphically. The disk margin analysis, `dmi(2)`, indicates the closed-loop system will remain stable for simultaneous gain variations of 0.475 and 2.105 ( $\pm 6.465$  dB) and phase margin variations of  $\pm 39.18$  deg in the second input channel. This is denoted by the region associated with the large ellipse in the following figure. The multivariable margin analysis at the input to the plant, `mmi`, indicates that the closed-loop system will be stable for independent, simultaneous, gain margin variation up to 0.728 and 1.373 ( $\pm 2.753$  dB) and phase margin variations up to  $\pm 17.87$  deg (the dark ellipse region) in both input channels.



The output channels have single-loop margins of infinite gain and 90 deg phase variation. The output multivariable margin analysis, `mimo`, leads to a maximum allowable gain margin variation of 0.607 and 1.649 and phase margin variations of  $\pm 27.53$  degs. Hence even though both output channels had infinite gain margin and 90 degrees of phase margin, simultaneous variations in both channels significantly reduce the margins at the plant outputs.

`mimo`

```
mimo = struct with fields:
  GainMargin: [0.6070 1.6474]
  PhaseMargin: [-27.4826 27.4826]
  Frequency: 0.2663
```

The margins when all the input and output channels are allowed to vary independently are in the output `mimo`. For this system, this output shows that the allowable gain margin variations are 0.827 and 1.210 and allowable phase margin variations are  $\pm 10.84$  deg.

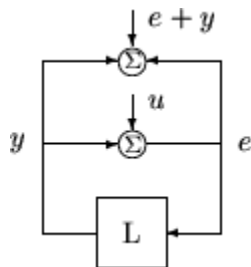
`mimo`

```
mimo = struct with fields:
  GainMargin: [0.8270 1.2092]
```

PhaseMargin: [-10.8190 10.8190]  
 Frequency: 0

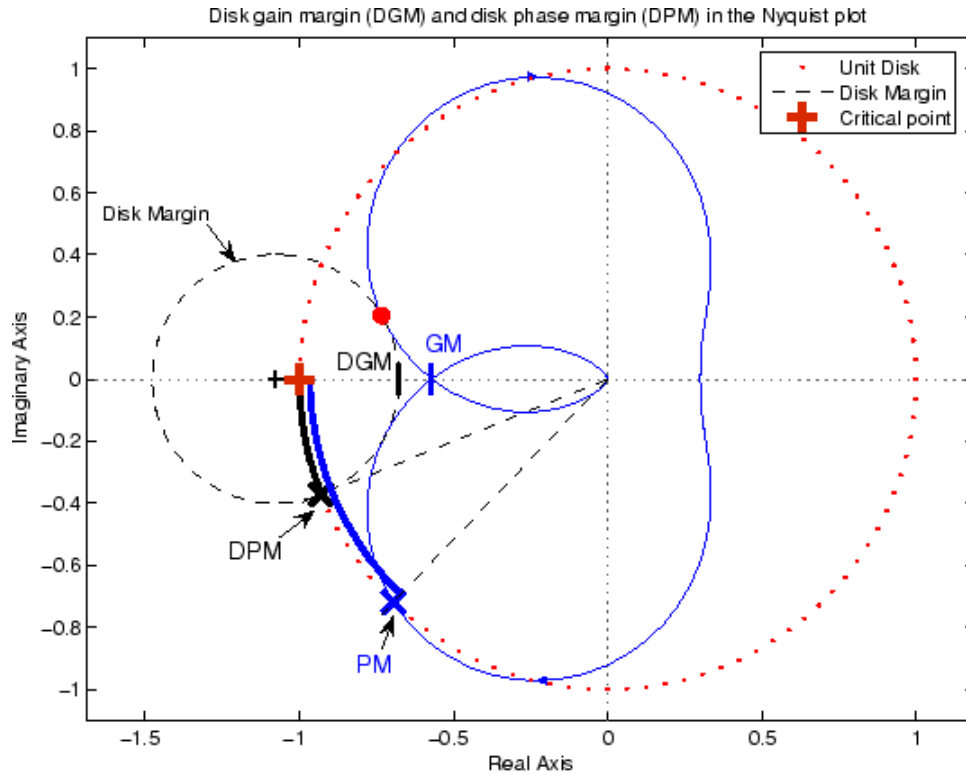
## Algorithms

Two well-known loop robustness measures are based on the sensitivity function  $S=(I-L)^{-1}$  and the complementary sensitivity function  $T=L(I-L)^{-1}$  where  $L$  is the loop gain matrix associated with the input or output loops broken simultaneously. In the following figure,  $S$  is the transfer matrix from summing junction input  $u$  to summing junction output  $e$ .  $T$  is the transfer matrix from  $u$  to  $y$ . If signals  $e$  and  $y$  are summed, the transfer matrix from  $u$  to  $e+y$  is given by  $(I+L) \cdot (I-L)^{-1}$ , the balanced sensitivity function. It can be shown (Dailey, 1991, Blight, Daily and Gangass, 1994) that each broken-loop gain can be perturbed by the complex gain  $(1+\Delta)(1-\Delta)$  where  $|\Delta| < 1/\mu(S+T)$  or  $|\Delta| < 1/\sigma_{\max}(S+T)$  at each frequency without causing instability at that frequency. The peak value of  $\mu(S+T)$  or  $\sigma_{\max}(S+T)$  gives a robustness guarantee for all frequencies, and for  $\mu(S+T)$  the guarantee is nonconservative (Blight, Daily and Gangass, 1994).



$$\begin{aligned} e &= (I - L)^{-1}u &= Su \\ y &= L(I - L)^{-1}u &= Tu \\ e + y &= (I + L) \cdot (I - L)^{-1}u &= (S + T)u \end{aligned}$$

This figure shows a comparison of a disk margin analysis with the classical notations of gain and phase margins.



The Nyquist plot is of the loop transfer function  $L(s)$

$$L(s) = \frac{\frac{s}{30} + 1}{(s+1)(s^2 + 1.6s + 16)}$$

- The Nyquist plot of  $L$  corresponds to the blue line.
- The unit disk corresponds to the dotted red line.
- GM and PM indicate the location of the classical gain and phase margins for the system  $L$ .
- DGM and DPM correspond to the disk gain and phase margins. The disk margins provide a lower bound on classical gain and phase margins.

- The disk margin circle corresponds to the dashed black line. The disk margin corresponds to the largest disk centered at  $(GMD + 1/GMD)/2$  that just touches the loop transfer function  $L$ . This location is indicated by the red dot.

The disk margin and multiple channel margins calculation involve the balanced sensitivity function  $S+T$ . For a given peak value of  $\mu(S+T)$ , any simultaneous phase and gain variations applied to each loop independently will not destabilize the system if the perturbations remain inside the corresponding circle or disk. This corresponds to the disk margin calculation to find `dmi` and `dmo`.

Similarly, the multiple channel margins calculation involves the balanced sensitivity function  $S+T$ . Instead of calculating  $\mu(S+T)$  a single loop at a time, all the channels are included in the analysis. A  $\mu$ -analysis problem is formulated with each channel perturbed by an independent, complex perturbation. The peak  $\mu(S+T)$  value guarantees that any simultaneous, independent phase and gain variations applied to each loop simultaneously will not destabilize the system if they remain inside the corresponding circle or disk of size  $\mu(S+T)$ .

For frequency-response data (`frd`) models, `loopmargin` uses the techniques of  $\mu$ -analysis to compute the disk margin at each frequency point in the model, and returns the weakest margin of these values. For all other models, the  $\mu$ -analysis computation identifies the frequency with the weakest margin.

## References

Barrett, M.F., Conservatism with robustness tests for linear feedback control systems, Ph.D. Thesis, Control Science and Dynamical Systems, University of Minnesota, 1980.

Blight, J.D., R.L. Dailey, and D. Gangsass, "Practical control law design for aircraft using multivariable techniques," *International Journal of Control*, Vol. 59, No. 1, 1994, pp. 93-137.

Bates, D., and I. Postlethwaite, "Robust Multivariable Control of Aerospace Systems," *Delft University Press*, Delft, The Netherlands, ISBN: 90-407-2317-6, 2002.

## See Also

`allmargin` | `bode` | `loopsens` | `mussv` | `robstab` | `wcgain` | `wcmargin` | `wcsens`

**Introduced before R2006a**

## loopsens

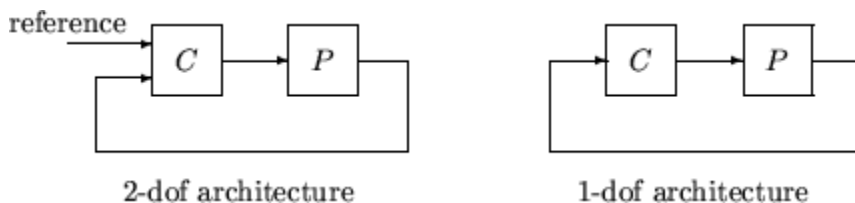
Sensitivity functions of plant-controller feedback loop

### Syntax

```
loops = loopsens(P,C)
```

### Description

`loops = loopsens(P,C)` creates a struct, `loops`, whose fields contain the multivariable sensitivity, complementary and open-loop transfer functions. The closed-loop system consists of the controller `C` in negative feedback with the plant `P`. `C` should only be the compensator in the feedback path, not any reference channels, if it is a 2-Dof controller as seen in the figure below. The plant and compensator `P` and `C` can be constant matrices, double, lti objects, frd/ss/tf/zpk, or uncertain objects umat/ufrd/uss.



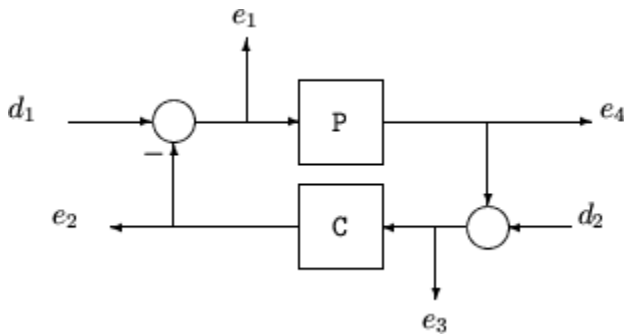
The `loops` returned variable is a structure with fields:

| Field  | Description   |
|--------|---|
| Poles  | Closed-loop poles. NaN for frd/ufrd objects                               |
| Stable | 1 if nominal closed loop is stable, 0 otherwise. NaN for frd/ufrd objects |
| Si     | Input-to-plant sensitivity function                                       |
| Ti     | Input-to-plant complementary sensitivity function                         |
| Li     | Input-to-plant loop transfer function                                     |



| Field | Description  |
|-------|--|
| So    | Output-to-plant sensitivity function                   |
| To    | Output-to-plant complementary sensitivity function     |
| Lo    | Output-to-plant loop transfer function                 |
| PSi   | Plant times input-to-plant sensitivity function        |
| CSo   | Compensator times output-to-plant sensitivity function |

The multivariable closed-loop interconnection structure, shown below, defines the input/output sensitivity, complementary sensitivity, and loop transfer functions.



| Description   | Equation          |
|---|-------------------|
| Input sensitivity ( $TF_{e1 \leftarrow d1}$ )                 | $(I + CP)^{-1}$   |
| Input complementary sensitivity ( $TF_{e2 \leftarrow d1}$ )   | $CP(I + CP)^{-1}$ |
| Output sensitivity ( $TF_{e3 \leftarrow d2}$ )                | $(I + PC)^{-1}$   |
| Output complementary sensitivity ( $-TF_{e4 \leftarrow d2}$ ) | $PC(I + PC)^{-1}$ |
| Input loop transfer function                                  | $CP$              |
| Output loop transfer function                                 | $PC$              |

## Examples

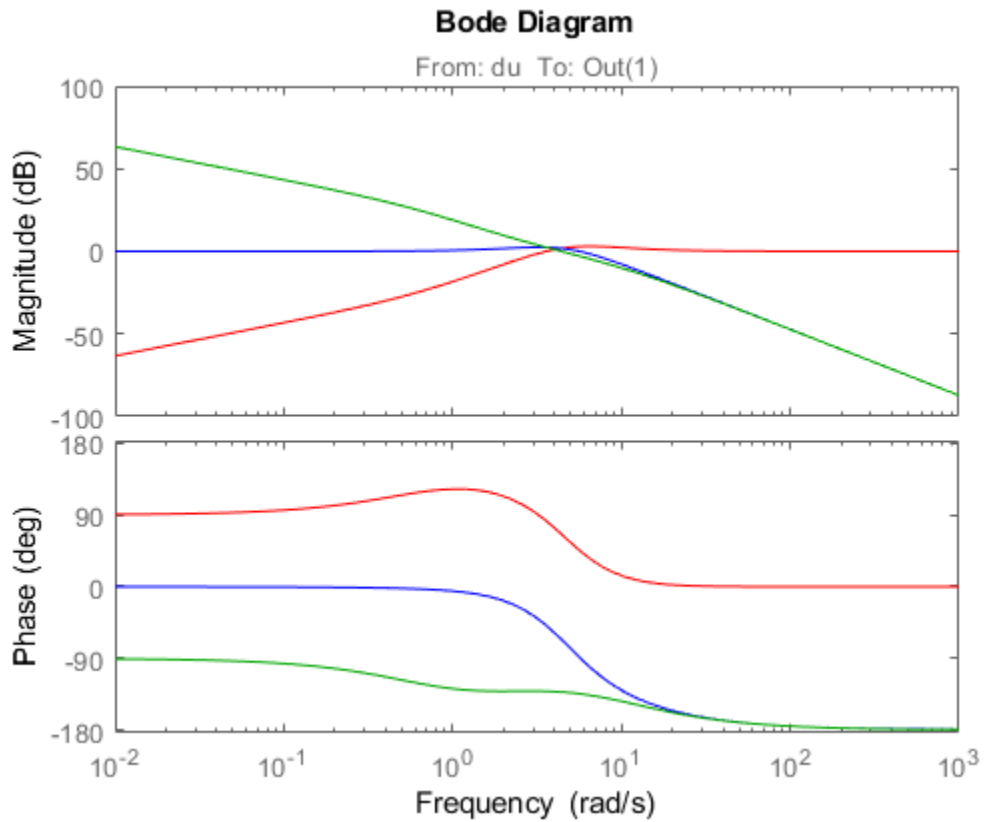
**Single Input, Single Output (SISO) Loop Sensitivities**

Consider PI controller for a dominantly 1st-order plant, with the closed-loop bandwidth of 2.5 rads/sec. Since the problem is SISO, all gains are the same at input and output.

```
gamma = 2; tau = 1.5; taufast = 0.1;
P = tf(gamma,[tau 1])*tf(1,[taufast 1]);
taucp = 0.4;
xicp = 0.8;
wnclp = 1/(taucp*xicp);
KP = (2*xicp*wnclp*tau - 1)/gamma;
KI = wnclp^2*tau/gamma;
C = tf([KP KI],[1 0]);
```

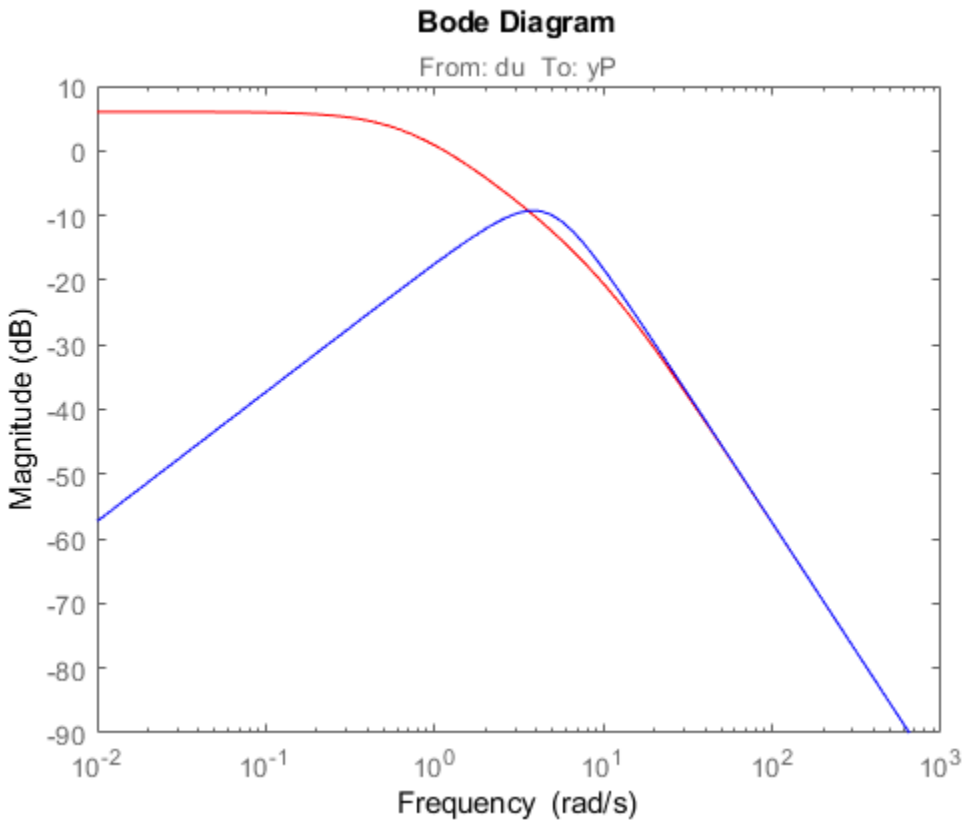
Form the closed-loop (and open-loop) systems with `loopsens`, and plot Bode plots using the gains at the plant input.

```
loops = loopsens(P,C);
bode(loops.Si, 'r', loops.Ti, 'b', loops.Li, 'g')
```



Finally, compare the open-loop plant gain to the closed-loop value of  $PSi$ .

```
bodemag(P, 'r', loops.PSi, 'b')
```

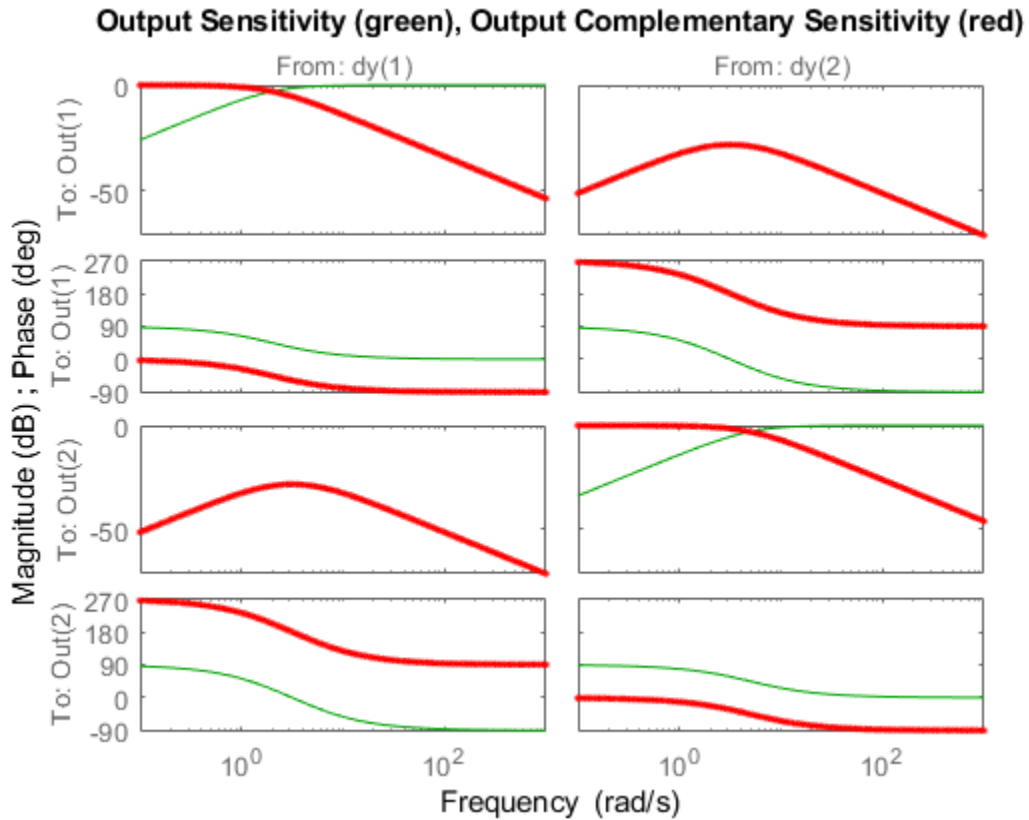


### Multi Input, Multi Output (MIMO) Loop Sensitivities

Consider an integral controller for a constant-gain, 2-input, 2-output plant. For purposes of illustration, the controller is designed via inversion, with different bandwidths in each rotated channel.

```
P = ss([2 3;-1 1]);
BW = diag([2 5]);
[U,S,V] = svd(P.d); % get SVD of Plant Gain
Csvd = V*inv(S)*BW*tf(1,[1 0])*U'; % inversion based on SVD
loops = loopsens(P,Csvd);
```

```
bode(loops.So,'g',loops.To,'r.',logspace(-1,3,120))
title('Output Sensitivity (green), Output Complementary Sensitivity (red)');
```



## See Also

[loopmargin](#) | [robstab](#) | [wcmargin](#) | [wcsens](#)

Introduced before R2006a

## loopsyn

$H_\infty$  optimal controller synthesis for LTI plant

### Syntax

`[K, CL, GAM, INFO]=loopsyn(G, Gd)`

`[K, CL, GAM, INFO]=loopsyn(G, Gd, RANGE)`

### Description

loopsyn is an  $H_\infty$  optimal method for loopshaping control synthesis. It computes a stabilizing  $H_\infty$  controller  $K$  for plant  $G$  to shape the sigma plot of the loop transfer function  $GK$  to have desired loop shape  $G_d$  with accuracy  $\gamma = \text{GAM}$  in the sense that if  $\omega_0$  is the 0 db crossover frequency of the sigma plot of  $G_d(j\omega)$ , then, roughly,

$$\underline{\sigma}(G(j\omega)K(j\omega)) \geq \frac{1}{\gamma} \underline{\sigma}(G_d(j\omega)) \text{ for all } \omega > \omega_0$$

$$\underline{\sigma}(G(j\omega)K(j\omega)) \leq \gamma \underline{\sigma}(G_d(j\omega)) \text{ for all } \omega > \omega_0$$

The STRUCT array **INFO** returns additional design information, including a MIMO stable min-phase shaping pre-filter  $W$ , the shaped plant  $G_s = GW$ , the controller for the shaped plant  $K_s = WK$ , as well as the frequency range  $\{\omega_{\min}, \omega_{\max}\}$  over which the loop shaping is achieved

| Input Argument | Description  |
|----------------|--|
| G              | LTI plant  |
| Gd             | Desired loop-shape (LTI model)   |
| RANGE          | (optional, default $\{0, \text{Inf}\}$ ) Desired frequency range for loop-shaping, a 1-by-2 cell array $\{\omega_{\min}, \omega_{\max}\}$ ; $\omega_{\max}$ should be at least ten times $\omega_{\min}$ |

| Output Argument | Description  |
|-----------------|--|
| K               | LTI controller   |
| CL= G*K/(I +GK) | LTI closed-loop system   |
| GAM             | Loop-shaping accuracy ( $GAM \geq 1$ , with $GAM=1$ being perfect fit)   |
| INFO            | Additional output information  |
| INFO.W          | LTI pre-filter $W$ satisfying $\sigma(G_d) = \sigma(GW)$ for all $\omega$ ;<br><br>$W$ is always minimum-phase.  |
| INFO.Gs         | LTI shaped plant: $G_s = GW$ .   |
| INFO.Ks         | LTI controller for the shaped plant: $K_s = WK$ .  |
| INFO.range      | $\{\omega_{\min}, \omega_{\max}\}$ cell-array containing the approximate frequency range over which loop-shaping could be accurately achieved to with accuracy $G$ . The output INFO.range is either the same as or a subset of the input range. |

## Examples

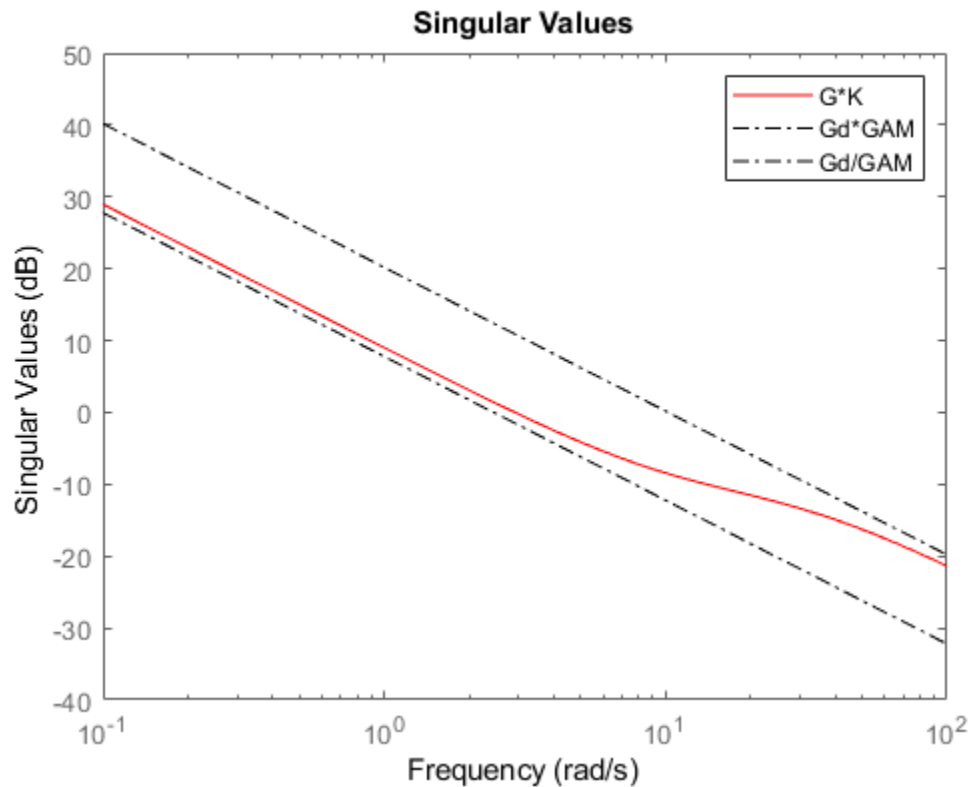
### Optimal loopsyn Loop-Shaping Control

Calculate the optimal loopsyn loop shaping control for a 5-state, 4-output, 5-input plant with a full-rank nonminimum-phase zero at  $s = 10$ .

```

rng(0, 'twister');
s = tf('s');
w0 = 5;
Gd = 5/s;                                % desired bandwidth w0=5
G = ((s-10)/(s+100))*rss(3,4,5);         % 4-by-5 non-min-phase plant
[K,CL,GAM,INFO] = loopsyn(G,Gd);
sigma(G*K, 'r', Gd*GAM, 'k-.', Gd/GAM, 'k-.', {.1,100}) % plot result
legend('G*K', 'Gd*GAM', 'Gd/GAM')

```



This plot shows that the controller K optimally fits  $\sigma(G*K)$ . The controller falls between  $\sigma(Gd) + \text{GAM}$  and  $\sigma(Gd) - \text{GAM}$  (expressed in dB). In this example,  $\text{GAM} = 2.0423 = 6.2026 \text{ dB}$ .

## Limitations

The plant G must be stabilizable and detectable, must have at least as many inputs as outputs, and must be full rank; i.e.,

- $\text{size}(G,2) \geq \text{size}(G,1)$



- $\text{rank}(\text{freqresp}(G, w)) = \text{size}(G, 1)$  for some frequency  $w$ .

The order of the controller  $K$  can be large. Generically, when  $G_d$  is given as a SISO LTI, then the order  $N_K$  of the controller  $K$  satisfies

$$\begin{aligned} N_K &= N_{G_s} + N_W \\ &= N_y N_{G_d} + N_{RHP} + N_W \\ &= N_y N_{G_d} + N_{RHP} + N_G \end{aligned}$$

where

- $N_y$  denotes the number of outputs of the plant  $G$ .
- $N_{RHP}$  denotes the total number of nonstable poles and nonminimum-phase zeros of the plant  $G$ , including those on the stability boundary and at infinity.
- $N_G, N_{G_s}, N_{G_d}$  and  $N_W$  denote the respective orders of  $G, G_s, G_d$  and  $W$ .

Model reduction can help reduce the order of  $K$  — see `reduce` and `ncfmr`.

## Algorithms

Using the GCD formula of Le and Safonov [1], `loopsyn` first computes a stable-minimum-phase loop-shaping, squaring-down prefilter  $W$  such that the shaped plant  $G_s = GW$  is square, and the desired shape  $G_d$  is achieved with good accuracy in the frequency range  $\{\omega_{\min}, \omega_{\max}\}$  by the shaped plant; i.e.,

$$\sigma(G_d) \approx \sigma(G_s) \text{ for all } \omega \in \{\omega_{\min}, \omega_{\max}\}.$$

Then, `loopsyn` uses the Glover-McFarlane [2] normalized-coprime-factor control synthesis theory to compute an optimal “loop-shaping” controller for the shaped plant via  $K_s = \text{ncfsyn}(G_s)$ , and returns  $K = W * K_s$ .

If the plant  $G$  is a continuous time LTI and

- 1  $G$  has a full-rank D-matrix, and
- 2 no finite zeros on the  $j\omega$ -axis, and
- 3  $\{\omega_{\min}, \omega_{\max}\} = [0, \infty]$ ,

then  $GW$  theoretically achieves a perfect accuracy fit  $\sigma(G_d) = \sigma(GW)$  for all frequency  $\omega$ . Otherwise, `loopsyn` uses a bilinear pole-shifting bilinear transform [3] of the form

```
Gshifted=bilin(G,-1,'S_Tust',[omega_min,omega_max]),
```

which results in a perfect fit for transformed `Gshifted` and an approximate fit over the smaller frequency range  $[\omega_{\min}, \omega_{\max}]$  for the original unshifted  $G$  provided that  $\omega_{\max} \gg \omega_{\min}$ . For best results, you should choose  $\omega_{\max}$  to be at least 100 times greater than  $\omega_{\min}$ . In some cases, the computation of the optimal  $W$  for `Gshifted` may be singular or ill-conditioned for the range  $[\omega_{\min}, \omega_{\max}]$ , as when `Gshifted` has undamped zeros or, in the continuous-time case only, `Gshifted` has a  $D$ -matrix that is rank-deficient); in such cases, `loopsyn` automatically reduces the frequency range further, and returns the reduced range  $[\omega_{\min}, \omega_{\max}]$  as a cell array in the output `INFO.range`={ $\omega_{\min}, \omega_{\max}$ }

## References

- [1] Le, V.X., and M.G. Safonov. Rational matrix GCD's and the design of squaring-down compensators—a state space theory. *IEEE Trans. Autom. Control*, AC-36(3):384-392, March 1992.
- [2] Glover, K., and D. McFarlane. Robust stabilization of normalized coprime factor plant descriptions with  $H_\infty$ -bounded uncertainty. *IEEE Trans. Autom. Control*, AC-34(8): 821-830, August 1992.
- [3] Chiang, R.Y., and M.G. Safonov.  $H_\infty$  synthesis using a bilinear pole-shifting transform. *AIAA J. Guidance, Control and Dynamics*, 15(5):1111-1115, September-October 1992.

## See Also

`mixsyn` | `ncfsyn`

## Topics

Loop Shaping of HIMAT Pitch Axis Controller

**Introduced before R2006a**

## ltiarray2uss

Compute uncertain system bounding given LTI ss array

---

**Note** ltiarray2uss will be removed in a future release. Use ucover instead.

---

### Syntax

```
usys = ltiarray2uss(P,Parray,ord)
[usys,wt] = ltiarray2uss(P,Parray,ord)
[usys,wt,diffdata] = ltiarray2uss(P,Parray,ord)
[usys,wt,diffdata] = ltiarray2uss(P,Parray,ord,'InputMult')
[usys,wt,diffdata] = ltiarray2uss(P,Parray,ord,'OutputMult')
[usys,wt,diffdata] = ltiarray2uss(P,Parray,ord,'Additive')
```

### Description

The command ltiarray2uss, calculates an uncertain system usys with nominal value P, and whose range of behavior includes the given array of systems, Parray.

usys = ltiarray2uss(P,Parray,ord), usys is formulated as an input multiplicative uncertainty model,

$usys = P*(I + wt*ultidyn('IMult',[size(P,2) \ size(P,2)]))$ , where wt is a stable scalar system, whose magnitude overbounds the relative difference,  $(P - Parray)/P$ . The state order of the weighting function used to bound the multiplicative difference between P and Parray is ord. Both P and Parray must be in the classes ss/tf/zpk/frd. If P is an frd then usys will be a ufrd object, otherwise usys will be a uss object. The ultidyn atom is named based on the variable name of Parray in the calling workspace.

`[usys,wt] = ltiarray2uss(P,Parray,ord)`, returns the weight `wt` used to bound the infinity norm of  $((P - Parray)/P)$ .

`[usys,wt] = ltiarray2uss(P,Parray,ord, 'OutputMult')`, uses multiplicative uncertainty at the plant output (as opposed to input multiplicative uncertainty). The formula for `usys` is

```
usys = (I + wt*ultidyn('Name',[size(P,1) size(P,1)])*P).
```

`[usys,wt] = ltiarray2uss(P,Parray,ord, 'Additive')`, uses additive uncertainty.

`usys = P + wt*ultidyn('Name',[size(P,1) size(P,2)])`. `wt` is a frequency domain overbound of the infinity norm of  $(Parray - P)$ .

`[usys,wt] = ltiarray2uss(P,Parray,ord, 'InputMult')`, uses multiplicative uncertainty at the plant input (this is the default). The formula for `usys` is `usys = P*(I + wt*ultidyn('Name',[size(P,2) size(P,2)]))`.

`[usys,wt,diffdata] = ltiarray2uss(P,Parray,ord,type)` returns the norm of the difference (absolute difference for additive, and relative difference for multiplicative uncertainty) between the nominal model `P` and `Parray`. `wt` satisfies `diffdata(w_i) < |wt(w_i)|` at all frequency points.

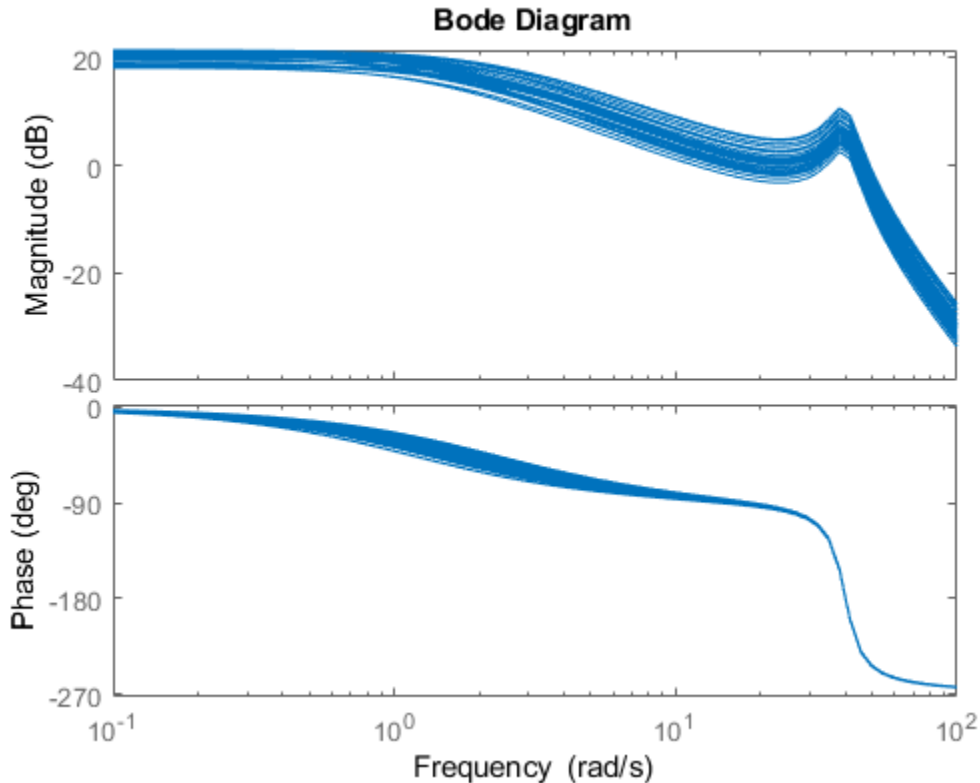
## Examples

### Uncertain System Bounding an LTI Array

Consider a third order transfer function with an uncertain gain, filter time constant and a lightly damped flexible mode. This model is used to represent a physical system from which frequency response data is acquired.

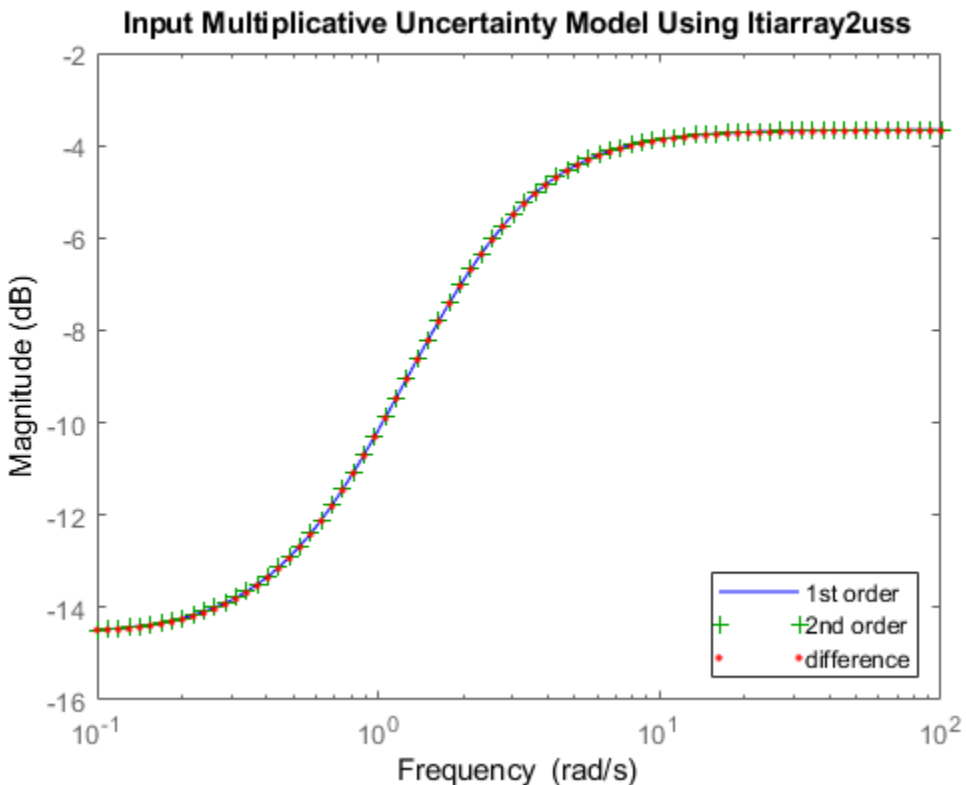
```
gain = ureal('gain',10,'Perc',20);
tau = ureal('tau',.6,'Range',[.42 .9]);
wn = 40;
zeta = 0.1;
usys = tf(gain,[tau 1])*tf(wn^2,[1 2*zeta*wn wn^2]);
sysnom = usys.NominalValue;
parray = usample(usys,30);
om = logspace(-1,2,80);
```

```
parrayg = frd(parray,om);
bode(parrayg)
```



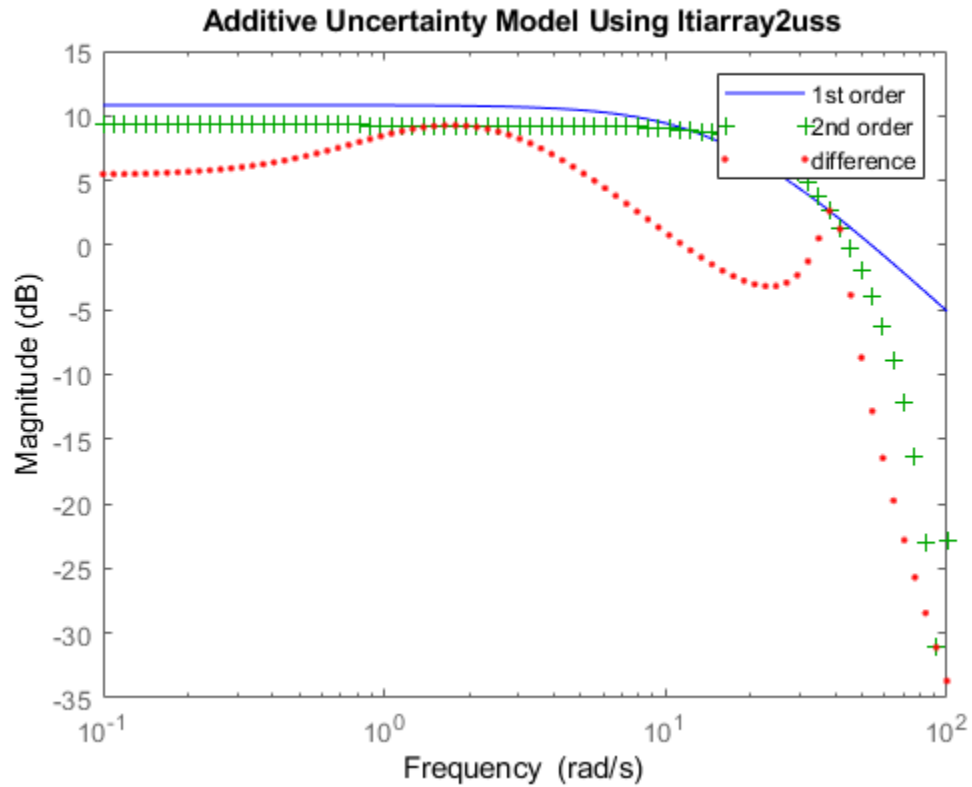
The frequency response data in `parray` represents 30 experiments performed on the system. The command `ltiarray2uss` is used to generate an uncertain model, `umod`, based on the frequency response data. Initially an input multiplicative uncertain model is used to characterize the collection of 30 frequency responses. First and second order input multiplicative uncertainty weight are calculated from the data.

```
[umodIn1,wtIn1,diffdataIn] = ltiarray2uss(sysnom,parrayg,1);
[umodIn2,wtIn2,diffdataIn] = ltiarray2uss(sysnom,parrayg,2);
bodemag(wtIn1,'b-',wtIn2,'g+',diffdataIn,'r.',om)
title('Input Multiplicative Uncertainty Model Using ltiarray2uss')
legend('1st order','2nd order','difference','Location','SouthEast')
```



Alternatively, an additive uncertain model is used to characterize the collection of 30 frequency responses.

```
[umodAdd1,wtAdd1,diffdataAdd] = ltiarray2uss(sysnom,parray,1,'Additive');
[umodAdd2,wtAdd2,diffdataAdd] = ltiarray2uss(sysnom,parray,2,'Additive');
bodemag(wtAdd1,'b-',wtAdd2,'g+',diffdataAdd,'r.',om)
title('Additive Uncertainty Model Using ltiarray2uss')
legend('1st order','2nd order','difference')
```



- “First-Cut Robust Design”

## See Also

`fitmagfrd` | `ultidyn` | `uss`

## Topics

“First-Cut Robust Design”

**Introduced in R2006a**

## ltrsyn

LQG loop transfer-function recovery (LTR) control synthesis

### Syntax

$[K, SVL, W1] = \text{ltrsyn}(G, F, XI, THETA, RH0)$

$[K, SVL, W1] = \text{ltrsyn}(G, F, XI, THETA, RH0, W)$

$[K, SVL, W1] = \text{ltrsyn}(G, F, XI, THETA, RH0, OPT)$

$[K, SVL, W1] = \text{ltrsyn}(G, F, XI, THETA, RH0, W, OPT)$

### Description

$[K, SVL, W1] = \text{ltrsyn}(G, F, XI, TH, RH0)$  computes a reconstructed-state output-feedback controller  $K$  for LTI plant  $G$  so that  $K*G$  asymptotically recovers plant-input full-state feedback loop transfer function  $L(s) = F(Is-A)^{-1}B+D$ ; that is, at any frequency  $\omega > 0$ ,  $\max(\sigma(K*G-L, \omega)) \rightarrow 0$  as  $\rho \rightarrow \infty$ , where  $L = \text{ss}(A, B, F, D)$  is the LTI full-state feedback loop transfer function.

$[K, SVL, W1] = \text{ltrsyn}(G, F1, Q, R, RH0, 'OUTPUT')$  computes the solution to the 'dual' problem of filter loop recovery for LTI plant  $G$  where  $F$  is a Kalman filter gain matrix. In this case, the recovery is at the plant output, and  $\max(\sigma(G*K-L, \omega)) \rightarrow 0$  as  $\rho \rightarrow \infty$ , where  $L1$  denotes the LTI filter loop feedback loop transfer function  $L1 = \text{ss}(A, F, C, D)$ .

Only the LTI controller  $K$  for the final value  $RH0$  (end) is returned.

| Inputs |   |
|--------|---|
| G      | LTI plant   |
| F      | LQ full-state-feedback gain matrix                                    |
| XI     | plant noise intensity,<br>or, if OPT='OUTPUT' state-cost matrix XI=Q, |



| Inputs  |   |
|---------|---|
| THETA   | sensor noise intensity<br>or, if OPT='OUTPUT' control-cost matrix THETA=R,  |
| RHO     | vector containing a set of recovery gains   |
| W       | (optional) vector of frequencies (to be used for plots); if input W is not supplied, then a reasonable default is used            |
| Outputs |   |
| K       | $K(s)$ — LTI LTR (loop-transfer-recovery) output-feedback, for the last element of RHO (i.e., RHO(end))                           |
| SVL     | sigma plot data for the recovered loop transfer function if G is MIMO or, for SISO G only, Nyquist loci SVL = [re(1:nr) im(1:nr)] |
| W1      | frequencies for SVL plots, same as W when present   |

## Examples

```
s=tf('s');G=ss(1e4/((s+1)*(s+10)*(s+100)));[A,B,C,D]=ssdata(G);
F=lqr(A,B,C'*C,eye(size(B,2)));
L=ss(A,B,F,0*B);
XI=100*C'*C; THETA=eye(size(C,1));
RHO=[1e3,1e6,1e9,1e12];W=logspace(-2,2);
nyquist(L,'k-.');hold;
[K,SVL,W1]=ltrsyn(G,F,XI,THETA,RHO,W);
```

See also `ltrdemo`

## Limitations

The `ltrsyn` procedure may fail for non-minimum phase plants. For full-state LTR (default OPT='INPUT'), the plant should not have fewer outputs than inputs. Conversely for filter LTR (when OPT='OUTPUT'), the plant should not have fewer inputs than outputs. The plant must be strictly proper, i.e., the  $D$ -matrix of the plant should be all zeros. `ltrsyn` is only for continuous time plants ( $Ts==0$ )

## Algorithms

For each value in the vector RHO,  $[K, SVL, W1] = \text{ltrsyn}(G, F, XI, THETA, RHO)$  computes the full-state-feedback (default  $OPT='INPUT'$ ) LTR controller

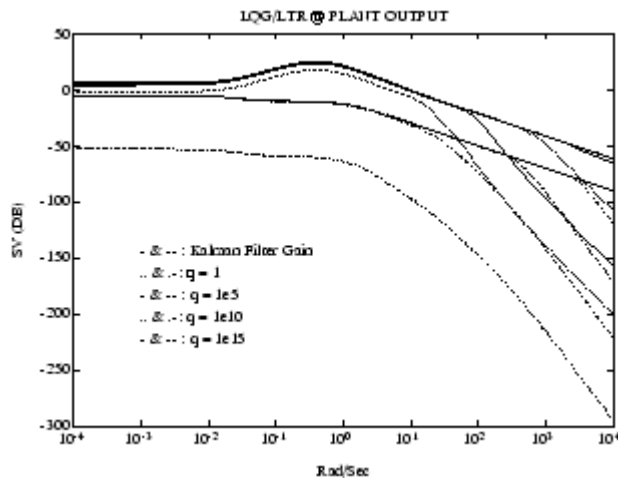
$$K(s) = \left[ K_c (Is - A + BK_c + K_f C - K_f DK_c)^{-1} K_f \right]$$

where  $K_c = F$  and  $K_f = \text{lqr}(A', C', XI + RHO(i) * B * B', THETA)$ . The “fictitious noise” term  $RHO(i) * B * B'$  results in loop-transfer recovery as  $RHO(i) \rightarrow \infty$ . The Kalman filter

gain is  $K_f = \Sigma C^T \Theta^{-1}$  where  $\Sigma$  satisfies the Kalman filter Riccati equation

$$0 = \Sigma A^T + A \Sigma - \Sigma C^T \Theta^{-1} C \Sigma + \Xi + \rho B B^T. \text{ See [1] for further details.}$$

Similarly for the 'dual' problem of filter loop recovery case,  $[K, SVL, W1] = \text{ltrsyn}(G, F, Q, R, RHO, 'OUTPUT')$  computes a filter loop recovery controller of the same form, but with  $K_f = F$  is being the input filter gain matrix and the control gain matrix  $K_c$  computed as  $K_c = \text{lqr}(A, B, Q + RHO(i) * C' * C, R)$ .



**Example of LQG/LTR at Plant Output.**

## References

- [1] Doyle, J., and G. Stein, "Multivariable Feedback Design: Concepts for a Classical/Modern Synthesis," *IEEE Trans. on Automat. Contr.*, AC-26, pp. 4-16, 1981.

## See Also

h2syn | hinfsyn | loopsyn | lqg | ncfsyn

**Introduced before R2006a**

## makeweight

First-order weighting function with specified DC gain, crossover frequency, and high-frequency gain

`makeweight` is a convenient way to specify loop shapes, target gain profiles, or weighting functions for applications such as controller synthesis and control system tuning.

### Syntax

```
W = makeweight(dcgain,wc,hfgain)
W = makeweight(dcgain,wc,hfgain,Ts)
```

### Description

`W = makeweight(dcgain,wc,hfgain)` creates a stable, first-order, continuous-time state-space model whose frequency response has the specified low-frequency gain, crossover frequency, and high-frequency gain. In other words, the response of `W` satisfies:

$$\begin{aligned}W(j \cdot 0) &= \text{dcgain} \\ |W(j \cdot \text{wc})| &= 1 \\ W(j \cdot \infty) &= \text{hfgain}.\end{aligned}$$

The low-frequency gain and the high-frequency gain must satisfy either  $|\text{dcgain}| < 1 < |\text{hfgain}|$  or  $|\text{hfgain}| < 1 < |\text{dcgain}|$ .

`W = makeweight(dcgain,wc,hfgain,Ts)` creates a stable, first-order, discrete-time state-space model with the specified sample time. The response of `W` satisfies:

$$\begin{aligned}W(e^{j \cdot 0 \cdot \text{Ts}}) &= \text{dcgain} \\ |W(e^{j \cdot \text{wc} \cdot \text{Ts}})| &= 1 \\ W(e^{j\pi}) &= \text{hfgain}.\end{aligned}$$

As in the continuous-time case, the low-frequency gain and the high-frequency gain must satisfy either  $|dcgain| < 1 < |hfgain|$  or  $|hfgain| < 1 < |dcgain|$ . In addition, the crossover frequency must satisfy  $\omega_c T_s < \pi$ .

## Examples

### Continuous-Time Weighting Functions

Create continuous-time weighting functions by specifying low-frequency gain, high-frequency gain, and crossover frequencies.

Create a weighting function with gain of 40 dB at low frequency, rolling off to -20 dB at high frequency. Specify a crossover frequency of 0.4 rad/s.

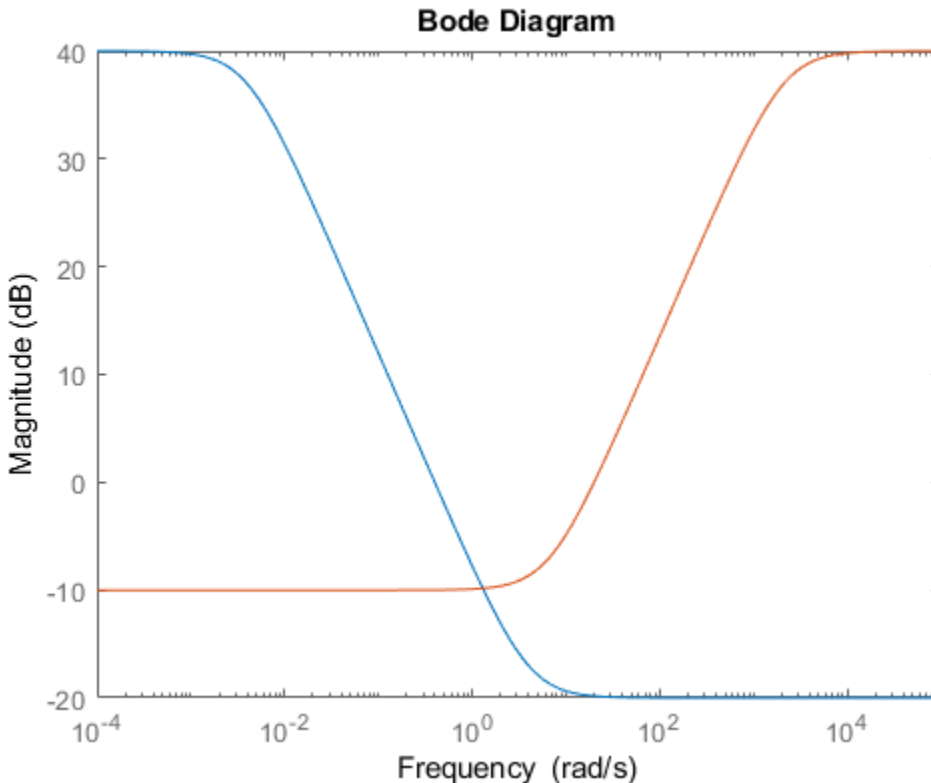
```
Wl = makeweight(100, .4, .10);
```

Create a weighting function with gain of -10 dB at low frequency, rising to 40 dB at high frequency. Specify a crossover frequency of 20 rad/s.

```
Wh = makeweight(0.316, 20, 100);
```

Plot the magnitudes of the weighting functions to confirm that they meet the response specifications.

```
bodemag(Wl, Wh)
```



### Weighting Functions With Roll-Off

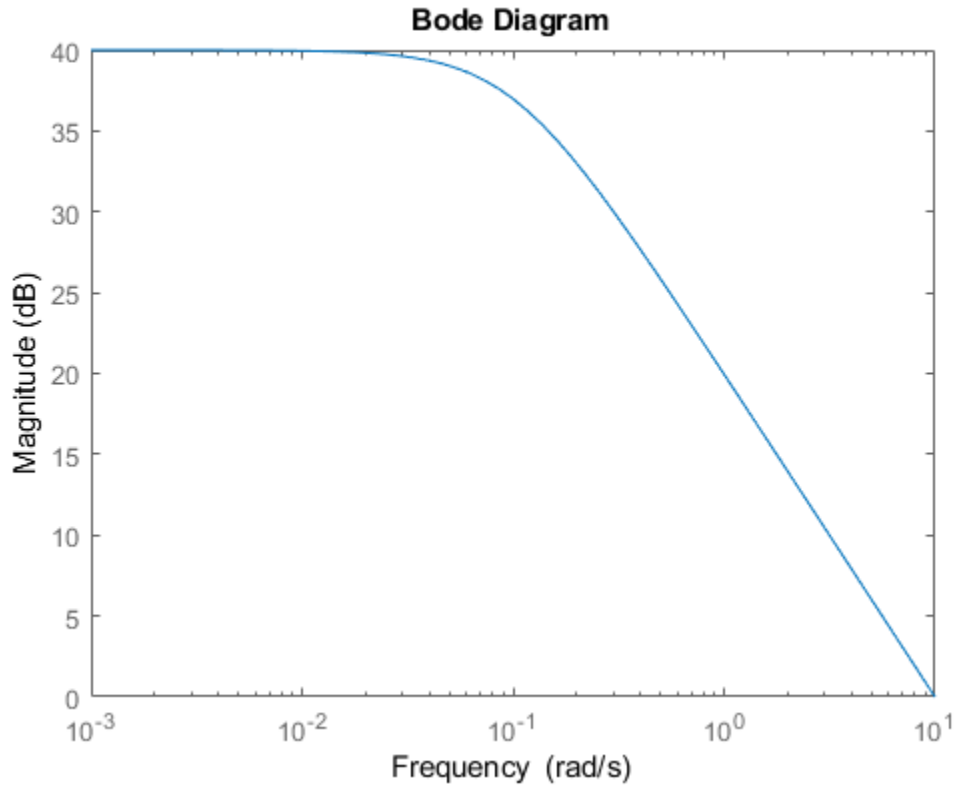
Create a gain profile that rolls off at high frequency without flattening. Specify a gain of 40 dB at low frequency and a crossover frequency of 10 rad/s.

```
W = makeweight(100,10,0);
```

Specifying a high-frequency gain of 0 ensures that the frequency response rolls off at high frequencies without leveling off.

Plot the gain profile to confirm the shape.

```
bodemag(W)
```



### Discrete-Time Weighting Functions

Create discrete-time weighting functions by specifying low-frequency gain, high-frequency gain, crossover frequencies, and sample time.

Create a weighting function with gain of 40 dB at low frequency, rolling off to -20 dB at high frequency. Specify a crossover frequency of 0.4 rad/s and a sample time of 0.1 s.

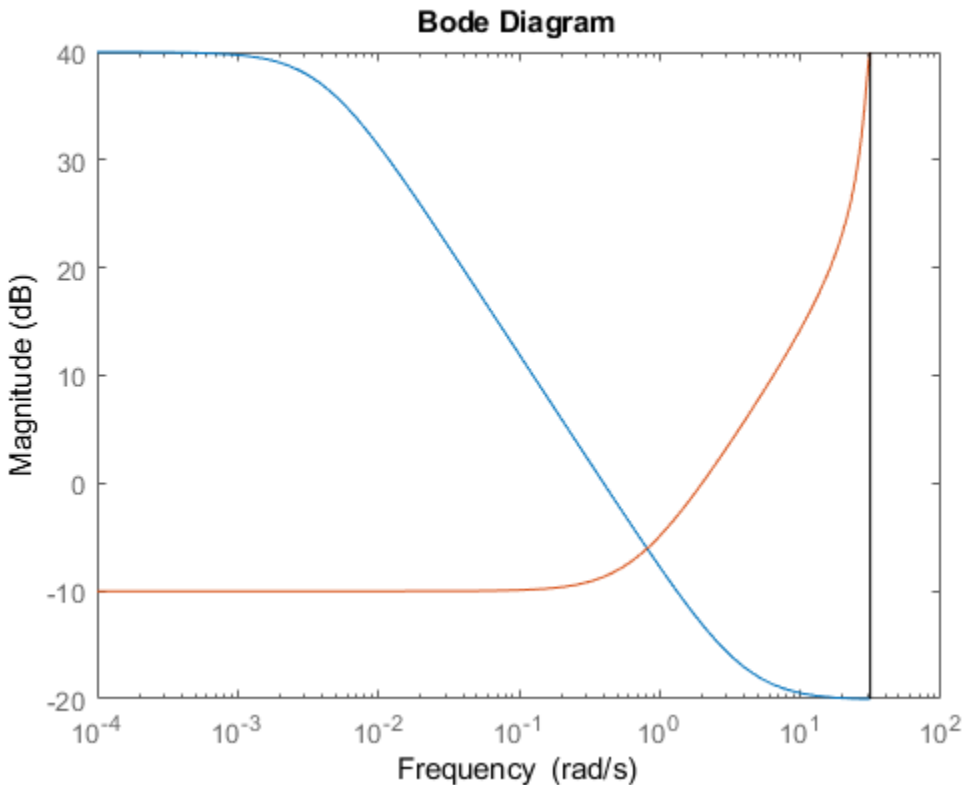
```
Wl = makeweight(100, .4, .10, 0.1);
```

Create a weighting function with gain of -10 dB at low frequency, rising to 40 dB at high frequency. Specify a crossover frequency of 2 rad/s and a sample time of 0.1 s.

```
Wh = makeweight(0.316,2,100,0.1);
```

Plot the magnitudes of the weighting functions to confirm that they meet the response specifications.

```
bodemag(WL,Wh)
```



The high-frequency leveling of Wh is somewhat distorted due to the proximity of its crossover frequency to the Nyquist frequency.



## Input Arguments

### **dcgain** — Low-frequency gain

real scalar

Low-frequency gain of the weighting function, specified as a real scalar value. The gain is expressed in absolute units. For example, to specify a low-frequency gain of 20 dB, set `dcgain = 10`.

The low-frequency gain and the high-frequency gain must satisfy either  $|dcgain| < 1 < |hfgain|$  or  $|hfgain| < 1 < |dcgain|$ .

### **wc** — Crossover frequency

positive scalar

Crossover frequency of the weighting function in radians/second, specified as a positive scalar value.

For discrete-time weighting functions, the crossover frequency must satisfy  $wc * Ts < \pi$ .

### **hfgain** — High-frequency gain

real scalar

High-frequency gain of the weighting function, specified as a real scalar value. The gain is expressed in absolute units. For example, to specify a high-frequency gain of -20 dB, set `hfgain = 0.1`.

The low-frequency gain and the high-frequency gain must satisfy either  $|dcgain| < 1 < |hfgain|$  or  $|hfgain| < 1 < |dcgain|$ .

### **Ts** — sample time

positive scalar | -1

sample time of discrete-time weighting function, specified as a positive scalar value or as -1. A positive value sets the sample time in seconds. A specified sample time must satisfy  $wc * Ts < \pi$ . The value -1 creates a discrete-time state-space model with unspecified sample time.

## Output Arguments

### **W** — Weighting function

state-space model

Weighting function, returned as a state-space (ss) model. For continuous-time weighting functions, the response of **W** satisfies:

$$W(j \cdot 0) = \text{dcgain}$$

$$|W(j \cdot \omega_c)| = 1$$

$$W(j \cdot \infty) = \text{hfgain}.$$

For discrete-time weighting functions, the response of **W** satisfies:

$$W(e^{j \cdot 0 \cdot T_s}) = \text{dcgain}$$

$$|W(e^{j \cdot \omega_c \cdot T_s})| = 1$$

$$W(e^{j\pi}) = \text{hfgain}.$$

## See Also

`TuningGoal.LoopShape` | `TuningGoal.WeightedGain` |  
`TuningGoal.WeightedVariance` | `dksyn` | `hinfstruct` | `hinfosyn` | `ss`

**Introduced before R2006a**

## matnbr

Number of matrix variables in system of LMIs

### Syntax

```
K = matnbr(lmisys)
```

### Description

matnbr returns the number K of matrix variables in the LMI problem described by `lmisys`.

### See Also

`decinfo` | `decnbr` | `lmiinfo`

**Introduced before R2006a**

## mat2dec

Extract vector of decision variables from matrix variable values

### Syntax

```
decvec = mat2dec(lmisys,X1,X2,X3,...)
```

### Description

Given an LMI system `lmisys` with matrix variables  $X_1, \dots, X_K$  and given values  $X1, \dots, Xk$  of  $X_1, \dots, X_K$ , `mat2dec` returns the corresponding value `decvec` of the vector of decision variables. Recall that the decision variables are the independent entries of the matrices  $X_1, \dots, X_K$  and constitute the free scalar variables in the LMI problem.

This function is useful, for example, to initialize the LMI solvers `mincx` or `gevp`. Given an initial guess for  $X_1, \dots, X_K$ , `mat2dec` forms the corresponding vector of decision variables `xinit`.

An error occurs if the dimensions and structure of  $X1, \dots, Xk$  are inconsistent with the description of  $X_1, \dots, X_K$  in `lmisys`.

### Examples

Consider an LMI system with two matrix variables  $X$  and  $Y$  such that

- $X$  is a symmetric block diagonal with one 2-by-2 full block and one 2-by-2 scalar block.
- $Y$  is a 2-by-3 rectangular matrix.

Particular instances of  $X$  and  $Y$  are

$$X_0 = \begin{pmatrix} 1 & 3 & 0 & 0 \\ 3 & -1 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{pmatrix}, \quad Y_0 = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

and the corresponding vector of decision variables is given by

```
decv = mat2dec(lmisys,X0,Y0)
```

```
decv'
```

```
ans =  
      1      3     -1      5      1      2      3      4      5      6
```

Note that `decv` is of length 10 since  $Y$  has 6 free entries while  $X$  has 4 independent entries due to its structure. Use `decinfo` to obtain more information about the decision variable distribution in  $X$  and  $Y$ .

## See Also

`dec2mat` | `decinfo` | `decnbr`

**Introduced before R2006a**

## **mincx**

Minimize linear objective under LMI constraints

### **Syntax**

```
[copt,xopt] = mincx(lmisys,c,options,xinit,target)
```

### **Description**

The function `mincx` solves the convex program

minimize  $c^T x$  subject to  $N^T L(x) N \leq M^T R(x) M$

where  $x$  denotes the vector of scalar decision variables.

The system of LMIs is described by `lmisys`. The vector `c` must be of the same length as  $x$ . This length corresponds to the number of decision variables returned by the function `decnbr`. For linear objectives expressed in terms of the matrix variables, the adequate `c` vector is easily derived with `defcx`.

The function `mincx` returns the global minimum `copt` for the objective  $c^T x$ , as well as the minimizing value `xopt` of the vector of decision variables. The corresponding values of the matrix variables is derived from `xopt` with `dec2mat`.

The remaining arguments are optional. The vector `xinit` is an initial guess of the minimizer `xopt`. It is ignored when infeasible, but may speed up computations otherwise. Note that `xinit` should be of the same length as `c`. As for `target`, it sets some target for the objective value. The code terminates as soon as this target is achieved, that is, as soon as some feasible  $x$  such that  $c^T x \leq \text{target}$  is found. Set `options` to `[]` to use `xinit` and `target` with the default options.

## Control Parameters

The optional argument `options` gives access to certain control parameters of the optimization code. In `mincx`, this is a five-entry vector organized as follows:

- `options(1)` sets the desired relative accuracy on the optimal value `lopt` (default =  $10^{-2}$ ).
- `options(2)` sets the maximum number of iterations allowed to be performed by the optimization procedure (100 by default).
- `options(3)` sets the feasibility radius. Its purpose and usage are as for `feasp`.
- `options(4)` helps speed up termination. If set to an integer value  $J > 0$ , the code terminates when the objective  $c^T x$  has not decreased by more than the desired relative accuracy during the last  $J$  iterations.
- `options(5) = 1` turns off the trace of execution of the optimization procedure. Resetting `options(5)` to zero (default value) turns it back on.

Setting `option(i)` to zero is equivalent to setting the corresponding control parameter to its default value. See `feasp` for more detail.

## Tip for Speed-Up

In LMI optimization, the computational overhead per iteration mostly comes from solving a least-squares problem of the form

$$\min_x \|Ax - b\|$$

where  $x$  is the vector of decision variables. Two methods are used to solve this problem: Cholesky factorization of  $A^T A$  (default), and QR factorization of  $A$  when the normal equation becomes ill conditioned (when close to the solution typically). The message

```
* switching to QR
```

is displayed when the solver has to switch to the QR mode.

Since QR factorization is incrementally more expensive in most problems, it is sometimes desirable to prevent switching to QR. This is done by setting `options(4) = 1`. While not guaranteed to produce the optimal value, this generally achieves a good trade-off between speed and accuracy.

## Memory Problems

QR-based linear algebra (see above) is not only expensive in terms of computational overhead, but also in terms of memory requirement. As a result, the amount of memory required by QR may exceed your swap space for large problems with numerous LMI constraints. In such case, MATLAB issues the error

```
??? Error using ==> pds  
Out of memory. Type HELP MEMORY for your options.
```

You should then ask your system manager to increase your swap space or, if no additional swap space is available, set `options(4) = 1`. This will prevent switching to QR and `mincx` will terminate when Cholesky fails due to numerical instabilities.

## References

The solver `mincx` implements Nesterov and Nemirovski's Projective Method as described in

Nesterov, Yu, and A. Nemirovski, *Interior Point Polynomial Methods in Convex Programming: Theory and Applications*, SIAM, Philadelphia, 1994.

Nemirovski, A., and P. Gahinet, "The Projective Method for Solving Linear Matrix Inequalities," *Proc. Amer. Contr. Conf.*, 1994, Baltimore, Maryland, pp. 840-844.

The optimization is performed by the C-MEX file `pds.mex`.

## See Also

`dec2mat` | `decnbr` | `defcx` | `feasp` | `gevp` | `mincx`

**Introduced before R2006a**



# mixsyn

$H_\infty$  mixed-sensitivity synthesis method for robust control loopshaping design

## Syntax

```
[K,CL,GAM,INFO] = mixsyn(G,W1,W2,W3)
```

```
[K,CL,GAM,INFO] = mixsyn(G,W1,W2,W3,KEY1,VALUE1,KEY2,VALUE2,...)
```

## Description

`[K,CL,GAM,INFO]=mixsyn(G,W1,W2,W3)` computes a controller  $K$  that minimizes the  $H_\infty$  norm of the closed-loop transfer function the weighted mixed sensitivity

$$T_{y1u1} \triangleq \begin{bmatrix} W_1 S \\ W_2 R \\ W_3 T \end{bmatrix}$$

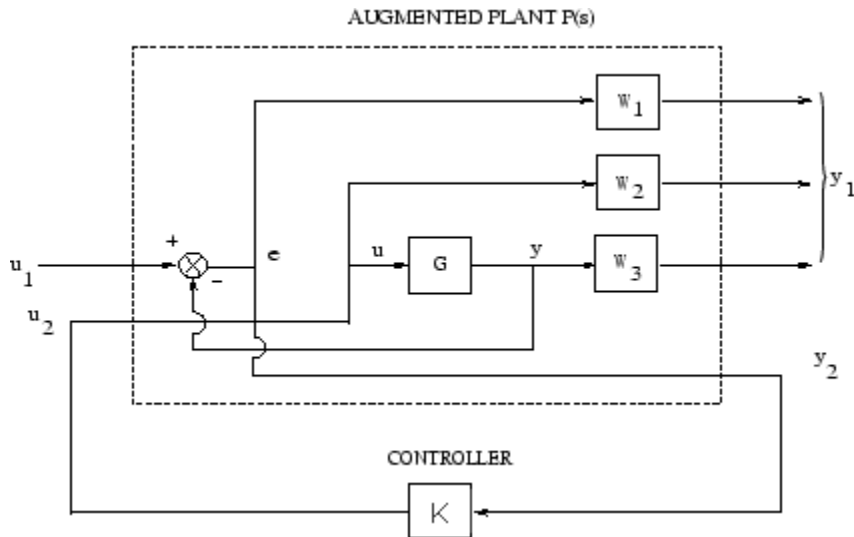
where  $S$  and  $T$  are called the sensitivity and complementary sensitivity, respectively.  $R$  measures the control effort, the transfer function from the reference input to the plant input.  $S$ ,  $R$ , and  $T$  are given by

$$S = (I + GK)^{-1}$$

$$R = K(I + GK)^{-1}$$

$$T = GK(I + GK)^{-1}$$

The following figure shows the relevant control architecture.



**Closed-loop transfer function  $T_{y_1u_1}$  for mixed sensitivity mixsyn.**

The returned controller  $K$  is such that  $S$ ,  $R$ , and  $T$  satisfy the following loop-shaping inequalities:

$$\begin{aligned} \bar{\sigma}(S(j\omega)) &\leq \gamma \underline{\sigma}(W_1^{-1}(j\omega)) \\ \bar{\sigma}(R(j\omega)) &\leq \gamma \underline{\sigma}(W_2^{-1}(j\omega)) \\ \bar{\sigma}(T(j\omega)) &\leq \gamma \underline{\sigma}(W_3^{-1}(j\omega)) \end{aligned}$$

where  $\gamma = \text{GAM}$ . Thus, the inverses of  $W_1$  and  $W_3$  determine the shapes of sensitivity  $S$  and complementary sensitivity  $T$ . Typically, you choose a  $W_1$  that is large inside the desired control bandwidth to achieve good disturbance attenuation (i.e., performance). Similarly, you typically choose a  $W_3$  that is large outside the control bandwidth, which helps to ensure good stability margin (i.e., robustness).

For dimensional compatibility, each of the three weights  $W_1$ ,  $W_2$  and  $W_3$  must be either empty, scalar (SISO) or have respective input dimensions  $N_Y$ ,  $N_U$ , and  $N_Y$  where  $G$  is  $N_Y$ -by- $N_U$ . If one of the weights is not needed, you may simply assign an empty matrix  $[\ ]$ ; e.g.,  $P = \text{AUGW}(G, W_1, [\ ], W_3)$  is SYS but without the second row (without the row containing  $W_2$ ).

## Examples

### Loop Shaping with mixsyn

This example shows the use of `mixsyn` for sensitivity and complementary sensitivity loop shaping.

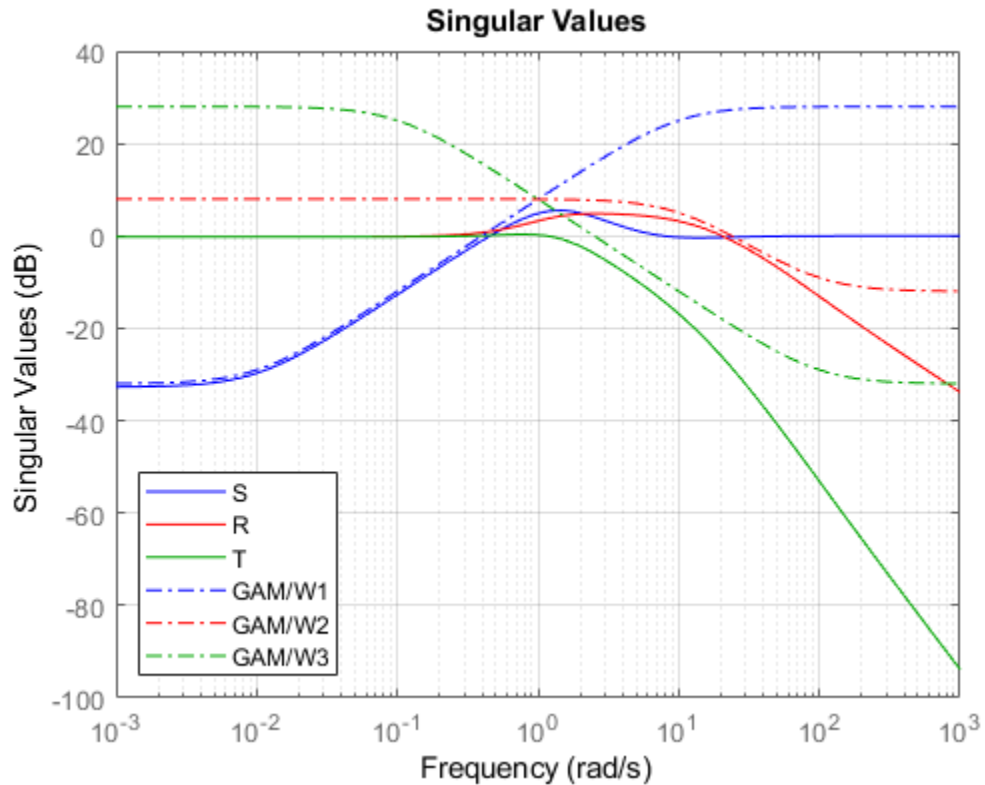
Create a plant model and weighting functions.

```
s = zpk('s');
G = (s-1)/(s+1)^2;
W1 = 10*(0.1*s+1)/(100*s+1);
W2 = 0.1*(0.1*s+1)/(0.01*s+1);
W3 = 0.01*(10*s+1)/(0.01*s+1);

[K,CL,GAM] = mixsyn(G,W1,W2,W3);
```

`mixsyn` shapes the singular values of the sensitivity function  $S$ , the complementary sensitivity function  $T$ , and the control effort  $R = K*S$ . Examine the results of the synthesis and the shapes of these transfer functions.

```
S = feedback(1,G*K);
R = K*S;
T = 1-S;
sigma(S, 'b', R, 'r', T, 'g', GAM/W1, 'b-.', ss(GAM/W2), 'r-.', GAM/W3, 'g-.', {1e-3, 1e3})
legend('S', 'R', 'T', 'GAM/W1', 'GAM/W2', 'GAM/W3', 'Location', 'SouthWest')
grid
```



## Limitations

The transfer functions  $G$ ,  $W_1$ ,  $W_2$  and  $W_3$  must be proper, i.e., bounded as  $s \rightarrow \infty$  or, in the discrete-time case, as  $z \rightarrow \infty$ . Additionally,  $W_1$ ,  $W_2$  and  $W_3$  should be stable. The plant  $G$  should be stabilizable and detectable; else,  $P$  will not be stabilizable by any  $K$ .

## Algorithms

`[K,CL,GAM,INFO] = mixsyn(G,W1,W2,W3,KEY1,VALUE1,KEY2,VALUE2,...)`

is equivalent to

```
[K,CL,GAM,INFO] = hifsyn(augw(G,W1,W2,W3),KEY1,VALUE1,KEY2,VALUE2,...)
```

mixsyn accepts all the same key value pairs as hifsyn.

## See Also

augw | hifsyn

**Introduced before R2006a**

## mkfilter

Generate Bessel, Butterworth, Chebyshev, or RC filter

### Syntax

```
sys = mkfilter(fc,ord,type)
```

```
sys = mkfilter(fc,ord,type,psbndr)
```

### Description

`sys = mkfilter(fc,ord,type)` returns a single-input, single-output analog low pass filter `sys` as an `ss` object. The cutoff frequency (Hertz) is `fc` and the filter order is `ord`, a positive integer. The argument `type` specifies the type of filter and can be one of the following:

| <b>type value</b> | <b>Description</b>                   |
|-------------------|--------------------------------------|
| 'butterw'         | Butterworth filter                   |
| 'cheby'           | Chebyshev filter                     |
| 'bessel'          | Bessel filter                        |
| 'rc'              | Series of resistor/capacitor filters |

The dc gain of each filter (except even-order Chebyshev) is set to unity.

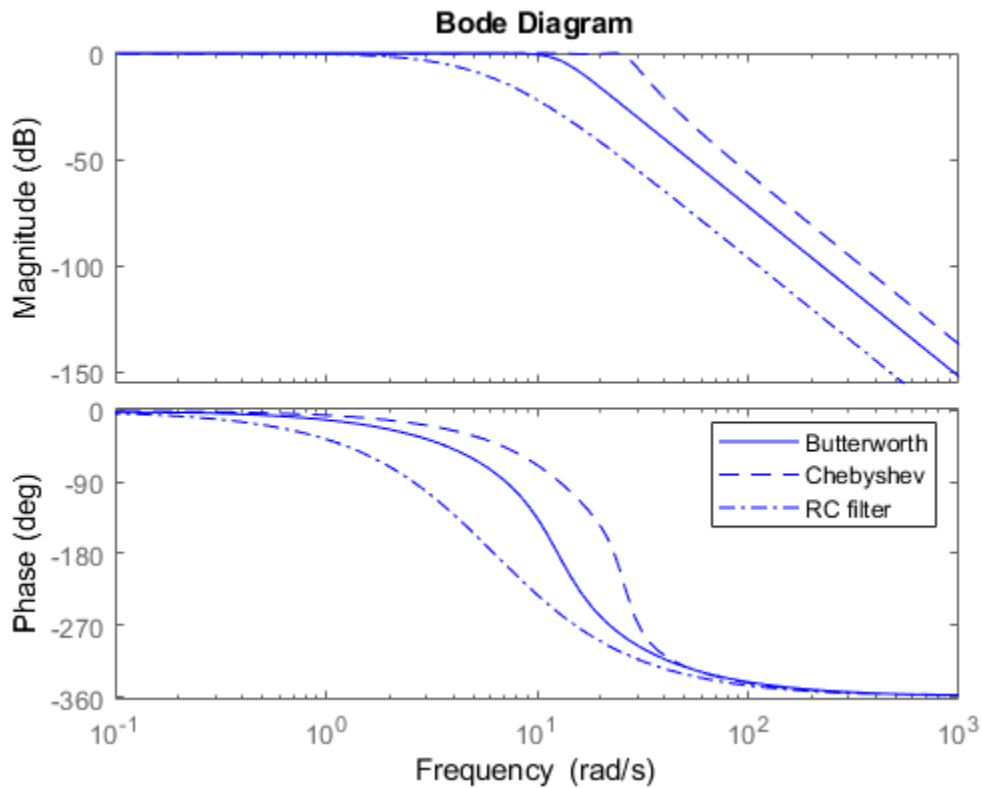
`sys = mkfilter(fc,ord,type,psbndr)` contains the input argument `psbndr` that specifies the Chebyshev passband ripple (in dB). At the cutoff frequency, the magnitude is `-psbndr` dB. For even-order Chebyshev filters the DC gain is also `-psbndr` dB.

## Examples

### Generate Filters

Generate several different types of filters and compare their frequency responses.

```
butw = mkfilter(2,4,'butterw');  
cheb = mkfilter(4,4,'cheby',0.5);  
rc = mkfilter(1,4,'rc');  
bode(butw,'-',cheb,'--',rc,'-.-')  
legend('Butterworth','Chebyshev','RC filter')
```



## Limitations

The Bessel filters are calculated using the recursive polynomial formula. This is poorly conditioned for high order filters (order > 8).

## **See Also**

augw

**Introduced before R2006a**



# mktito

Partition LTI system into two-input/two-output system

## Syntax

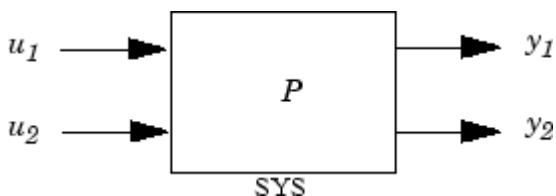
```
SYS=mktito(SYS,NMEAS,NCONT)
```

## Description

`SYS=mktito(SYS,NMEAS,NCONT)` adds TITO (two-input/two-output) partitioning to LTI system `SYS`, assigning `OutputGroup` and `InputGroup` properties such that

$$NMEAS = \dim(y_2)$$

$$NCONT = \dim(u_2)$$



Any preexisting `OutputGroup` or `InputGroup` properties of `SYS` are overwritten. TITO partitioning simplifies syntax for control synthesis functions like `hinfsyn` and `h2syn`.

## Examples

You can type

```
P=rss(2,4,5); P=mktito(P,2,2);
disp(P.OutputGroup); disp(P.InputGroup);
```

to create a 4-by-5 LTI system `P` with `OutputGroup` and `InputGroup` properties

```
U1: [1 2 3]
U2: [4 5]
Y1: [1 2]
Y2: [3 4]
```

## Algorithms

```
[r,c]=size(SYS);
set(SYS,'InputGroup', struct('U1',1:c-NCONT,'U2',c-NCONT+1:c));
set(SYS,'OutputGroup',struct('Y1',1:r-NMEAS,'Y2',r-NMEAS+1:r));
```

## See Also

augw | h2syn | hinfsyn | sdhinfsyn

**Introduced before R2006a**

# modreal

Modal form realization and projection

## Syntax

`[G1,G2] = modreal(G,cut)`

## Description

`[G1,G2] = modreal(G,cut)` returns a set of state-space LTI objects G1 and G2 in modal form given a state-space G and the model size of G1, cut.

The modal form realization has its A matrix in block diagonal form with either 1x1 or 2x2 blocks. The real eigenvalues will be put in 1x1 blocks and complex eigenvalues will be put in 2x2 blocks. These diagonal blocks are ordered in ascending order based on eigenvalue magnitudes.

The complex eigenvalue  $a+bj$  is appearing as 2x2 block

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$$

This table describes input arguments for `modreal`.

| Argument | Description   |
|----------|---|
| G        | LTI model to be reduced.  |
| cut      | (Optional) an integer to split the realization. Without it, a complete modal form realization is returned |

This table lists output arguments.

| Argument | Description              |
|----------|--------------------------|
| G1, G2   | LTI models in modal form |

$G$  can be stable or unstable.  $G_1 = (A_1, B_1, C_1, D_1)$ ,  $G_2 = (A_2, B_2, C_2, D_2)$  and  $D_1 = D + C_2(-A_2)^{-1}B_2$  is calculated such that the system DC gain is preserved.

## Examples

Given a continuous stable or unstable system,  $G$ , the following commands can get a set of modal form realizations depending on the split index -- cut:

```
rng(1234, 'twister');  
G = rss(50,2,2);  
[G1,G2] = modreal(G,2); % cut = 2 for two rigid body modes  
G1.D = zeros(2,2); % remove the DC gain of the system from G1  
sigma(G,G1,G2)
```

## Algorithms

Using a real eigen structure decomposition `reig` and ordering the eigenvectors in ascending order according to their eigenvalue magnitudes, we can form a similarity transformation out of these ordered real eigenvectors such that the resulting systems  $G_1$  and/or  $G_2$  are in block diagonal modal form.

---

**Note** This routine is extremely useful when model has  $j\omega$ -axis singularities, e.g., rigid body dynamics. It has been incorporated inside Hankel based model reduction routines - `hankelmr`, `balancmr`, `bstmr`, and `schurmr` to isolate those  $j\omega$ -axis poles from the actual model reduction process.

---

## See Also

`balancmr` | `bstmr` | `hankelmr` | `hankelsv` | `ncfmr` | `reduce` | `schurmr`

**Introduced before R2006a**

# msfsyn

Multi-model/multi-objective state-feedback synthesis

## Syntax

`[gopt,h2opt,K,Pcl,X] = msfsyn(P,r,obj,region,tol)`

## Description

Given an LTI plant  $P$  with state-space equations

$$\begin{cases} \dot{x} = Ax + B_1w + B_2u \\ z_\infty = C_1x + D_{11}w + D_{12}u \\ z_2 = C_2x + D_{22}u \end{cases}$$

`msfsyn` computes a state-feedback control  $u = Kx$  that

- Maintains the RMS gain ( $H_\infty$  norm) of the closed-loop transfer function  $T_\infty$  from  $w$  to  $z_\infty$  below some prescribed value  $\gamma_0 > 0$
- Maintains the  $H_2$  norm of the closed-loop transfer function  $T_2$  from  $w$  to  $z_2$  below some prescribed value  $\nu_0 > 0$
- Minimizes an  $H_2/H_\infty$  trade-off criterion of the form

$$\alpha \|T_\infty\|_\infty^2 + \beta \|T_2\|_2^2$$

- Places the closed-loop poles inside the LMI region specified by `region` (see `lmireg` for the specification of such regions). The default is the open left-half plane.

Set `r = size(d22)` and `obj = [ $\gamma_0$ ,  $\nu_0$ ,  $\alpha$ ,  $\beta$ ]` to specify the problem dimensions and the design parameters  $\gamma_0$ ,  $\nu_0$ ,  $\alpha$ , and  $\beta$ . You can perform pure pole placement by setting `obj = [0 0 0 0]`. Note also that  $z_\infty$  or  $z_2$  can be empty.

On output, `gopt` and `h2opt` are the guaranteed  $H_\infty$  and  $H_2$  performances, `K` is the optimal

state-feedback gain, `Pcl` the closed-loop transfer function from  $w$  to  $\begin{pmatrix} z_\infty \\ z_2 \end{pmatrix}$ , and `X` the corresponding Lyapunov matrix.

The function `msfsyn` is also applicable to multi-model problems where `P` is a polytopic model of the plant:

$$\begin{cases} \dot{x} = A(t)x + B_1(t)w + B_2(t)u \\ z_\infty = C_1(t)x + D_{11}(t)w + D_{12}(t)u \\ z_2 = C_2(t)x + D_{22}(t)u \end{cases}$$

with time-varying state-space matrices ranging in the polytope

$$\begin{pmatrix} A(t) & B_1(t) & B_2(t) \\ C_1(t) & D_{11}(t) & D_{12}(t) \\ C_2(t) & 0 & D_{22}(t) \end{pmatrix} \in \text{Co} \left\{ \begin{pmatrix} A_k & B_k & C_k \\ C_{1k} & D_{11k} & D_{12k} \\ C_{2k} & 0 & D_{22k} \end{pmatrix} : k = 1, \dots, K \right\}$$

In this context, `msfsyn` seeks a state-feedback gain that robustly enforces the specifications over the entire polytope of plants. Note that polytopic plants should be defined with `psys` and that the closed-loop system `Pcl` is itself polytopic in such problems. Affine parameter-dependent plants are also accepted and automatically converted to polytopic models.

## See Also

`lmireg` | `psys`

**Introduced before R2006a**

## mussv

Compute bounds on structured singular value ( $\mu$ )

### Syntax

```
bounds = mussv(M,BlockStructure)
[bounds,muinfo] = mussv(M,BlockStructure)
[bounds,muinfo] = mussv(M,BlockStructure,Options)
[ubound,q] = mussv(M,F,BlockStructure)
[ubound,q] = mussv(M,F,BlockStructure,'s')
```

### Description

`bounds = mussv(M,BlockStructure)` calculates upper and lower bounds on the structured singular value, or  $\mu$ , for a given block structure. `M` is a `double` array, an `frd` model, or a state-space (`ss`) model.

- If `M` is an N-D array (with  $N \geq 3$ ), then the computation is performed pointwise along the third and higher array dimensions.
- If `M` is a `frd` model, then the computations are performed pointwise in frequency (as well as any array dimensions).
- If `M` is a `ss` model, the computations are performed using state-space algorithms. Frequencies are adaptively selected, and upper bounds are guaranteed to hold over each interval between frequencies. `M` must be a single system, without array dimensions.

`BlockStructure` is a matrix specifying the perturbation block structure. `BlockStructure` has 2 columns, and as many rows as uncertainty blocks in the perturbation structure. The  $i$ -th row of `BlockStructure` defines the dimensions of the  $i$ -th perturbation block.

- If `BlockStructure(i,:) = [-r 0]`, then the  $i$ -th block is an  $r$ -by- $r$  repeated, diagonal real scalar perturbation;

- if `BlockStructure(i, :) = [r 0]`, then the  $i$ -th block is an  $r$ -by- $r$  repeated, diagonal complex scalar perturbation;
- if `BlockStructure(i, :) = [r c]`, then the  $i$ -th block is an  $r$ -by- $c$  complex full-block perturbation.
- If `BlockStructure` is omitted, its default is `ones(size(M,1),2)`, which implies a perturbation structure of all 1-by-1 complex blocks. In this case, if `size(M,1)` does not equal `size(M,2)`, an error results.

If  $M$  is a two-dimensional matrix, then `bounds` is a 1-by-2 array containing an upper (first column) and lower (second column) bound of the structured singular value of  $M$ . For all matrices  $\Delta$  with block-diagonal structure defined by `BlockStructure` and with norm less than  $1/\text{bounds}(1)$  (upper bound), the matrix  $I - M\Delta$  is not singular. Moreover, there is a matrix  $\Delta_S$  with block-diagonal structure defined by `BlockStructure` and with norm equal to  $1/\text{bounds}(2)$  (lower bound), for which the matrix  $I - M\Delta_S$  is singular.

The format used in the 3rd output argument from `lftdata` is also acceptable for describing the block structure.

If  $M$  is an `frd`, the computations are always performed pointwise in frequency. The output argument `bounds` is a 1-by-2 `frd` of upper and lower bounds at each frequency. Note that `bounds.Frequency` equals `M.Frequency`.

If  $M$  is an N-D array (either `double` or `frd`), the upper and lower bounds are computed pointwise along the 3rd and higher array dimensions (as well as pointwise in frequency, for `frd`). For example, suppose that `size(M)` is  $r \times c \times d_1 \times \dots \times d_F$ . Then `size(bounds)` is  $1 \times 2 \times d_1 \times \dots \times d_F$ . Using single index notation, `bounds(1,1,i)` is the upper bound for the structured singular value of  $M(:, :, i)$ , and `bounds(1,2,i)` is the lower bound for the structured singular value of  $M(:, :, i)$ . Here, any  $i$  between 1 and  $d_1 \cdot d_2 \dots d_F$  (the product of the  $d_k$ ) would be valid.

If  $M$  is a `ss` model, `bounds` is returned as an `frd` model.

`bounds = mussv(M,BlockStructure,Options)` specifies computation options. `Options` is a character vector, containing any combination of the following characters:

| Option | Meaning   |
|--------|---|
| 'a'    | Upper bound to greatest accuracy, using LMI solver. This is the default behavior when the number of decision variables within the D/G scalings is less than 45. |



| Option | Meaning  |
|--------|--|
| 'f'    | Force fast upper bound (typically not as tight as the default)   |
| 'G'    | Force upper bound to use gradient method. This is the default behavior when the number of decision variables within the D/G scalings is greater than or equal to 45.   |
| 'U'    | Upper-bound “only” (lower bound uses a fast/cheap algorithm).  |
| 'gN'   | Use gain-based lower bound method multiple times. The value of $N$ sets the number of times, according to $10+N*10$ . For example, 'g6' uses gain-based lower bound 70 times. Larger numbers typically give better lower bounds.<br><br>If all uncertainty blocks described by <code>blk</code> are real, then the default is 'g1'. If at least one uncertainty block is complex, then <code>mussv</code> uses power iteration lower bound by default. |
| 'i'    | Reinitialize lower bound computation at each new matrix (only relevant if <code>M</code> is <code>ND</code> array or <code>frd</code> ).   |
| 'mN'   | Randomly reinitialize lower bound iteration multiple times. $N$ is an integer between 1 and 9. For example, 'm7' randomly reinitializes the lower bound iteration 7 times. Larger numbers are typically more computationally expensive, but often give better lower bounds.  |
| 'p'    | Use power iteration method to compute lower bound. When at least one of the uncertainty blocks described by <code>BlockStructure</code> is complex, then 'p' is the default lower bound method.  |
| 's'    | Suppress progress information (silent).  |
| 'd'    | Display warnings.  |
| 'x'    | Decrease iterations in lower bound computation (faster but not as tight as default). Use 'U' for an even faster lower bound.   |
| 'an'   | Same as 'a', but without automatic prescaling.   |
| 'o'    | Run “old” algorithms, from version 3.1.1 and before. Included to allow exact replication of earlier calculations.  |

[`bounds,muinfo`] = `mussv(M,BlockStructure)` returns `muinfo`, a structure containing more detailed information. The information within `muinfo` must be extracted using `mussvextract`.

## Generalized Structured Singular Value

`ubound = mussv(M,F,BlockStructure)` calculates an upper bound on the generalized structured singular value (generalized  $\mu$ ) for a given block structure. `M` is a double or `frd` object. `M` and `BlockStructure` are as before. `F` is an additional (double or `frd`).

`ubound = mussv(M,F,BlockStructure, 's')` adds an option to run silently. Other options are ignored for generalized  $\mu$  problems.

Note that in generalized structured singular value computations, only an upper bound is calculated. `ubound` is an upper bound of the generalized structured singular value of the pair  $(M, F)$ , with respect to the block-diagonal uncertainty described by `BlockStructure`. Consequently `ubound` is 1-by-1 (with additional array dependence, depending on `M` and `F`). For all matrices `Delta` with block-diagonal structure defined by `BlockStructure` and  $\text{norm}(\Delta) < 1/\text{ubound}$ , the matrix  $[I - \Delta * M; F]$  is guaranteed not to lose column rank. This is verified by the matrix `Q`, which satisfies  $\text{mussv}(M + Q * F, \text{BlockStructure}, 'a') \leq \text{ubound}$ .

## Examples

See `mussvextract` for a detailed example of the structured singular value.

A simple example for generalized structured singular value can be done with random complex matrices, illustrating the relationship between the upper bound for  $\mu$  and generalized  $\mu$ , as well as the fact that the upper bound for generalized  $\mu$  comes from an optimized  $\mu$  upper bound.

`M` is a complex 5-by-5 matrix and `F` is a complex 2-by-5 matrix. The block structure `BlockStructure` is an uncertain real parameter  $\delta_1$ , an uncertain real parameter  $\delta_2$ , an uncertain complex parameter  $\delta_3$  and a twice-repeated uncertain complex parameter  $\delta_4$ .

```
rng(929, 'twister')
M = randn(5,5) + sqrt(-1)*randn(5,5);
F = randn(2,5) + sqrt(-1)*randn(2,5);
BlockStructure = [-1 0; -1 0; 1 1; 2 0];
[ubound,Q] = mussv(M,F,BlockStructure);
bounds = mussv(M,BlockStructure);
optbounds = mussv(M+Q*F,BlockStructure);
```

The quantities `optbounds(1)` and `ubound` should be extremely close, and significantly lower than `bounds(1)` and `bounds(2)`.

```
[optbounds(1) ubound]
```

```
ans =
```

```
    2.2070    2.1749
```

```
[bounds(1) bounds(2)]
```

```
ans =
```

```
    4.4049    4.1960
```

## Algorithms

The lower bound is computed using a power method, Young and Doyle, 1990, and Packard *et al.* 1988, and the upper bound is computed using the balanced/AMI technique, Young *et al.*, 1992, for computing the upper bound from Fan *et al.*, 1991.

Peter Young and Matt Newlin wrote the original function.

The lower-bound power algorithm is from Young and Doyle, 1990, and Packard *et al.* 1988.

The upper-bound is an implementation of the bound from Fan *et al.*, 1991, and is described in detail in Young *et al.*, 1992. In the upper bound computation, the matrix is first balanced using either a variation of Osborne's method (Osborne, 1960) generalized to handle *repeated scalar* and *full* blocks, or a Perron approach. This generates the standard upper bound for the associated complex  $\mu$  problem. The Perron eigenvector method is based on an idea of Safonov, (Safonov, 1982). It gives the exact computation of  $\mu$  for positive matrices with scalar blocks, but is comparable to Osborne on general matrices. Both the Perron and Osborne methods have been modified to handle *repeated scalar* and *full* blocks. Perron is faster for small matrices but has a growth rate of  $n^3$ , compared with less than  $n^2$  for Osborne. This is partly due to the MATLAB implementation, which greatly favors Perron. The default is to use Perron for simple block structures and Osborne for more complicated block structures. A sequence of improvements to the upper bound is then made based on various equivalent forms of the upper bound. A number of descent techniques are used that exploit the structure of the problem, concluding with general purpose LMI optimization (Boyd *et al.*), 1993, to obtain the final answer.

The optimal choice of  $Q$  (to minimize the upper bound) in the generalized  $\mu$  problem is solved by reformulating the optimization into a semidefinite program (Packard *et al.*, 1991).

## References

- [1] Boyd, S. and L. El Ghaoui, "Methods of centers for minimizing generalized eigenvalues," *Linear Algebra and Its Applications*, Vol. 188-189, 1993, pp. 63-111.
- [2] Fan, M., A. Tits, and J. Doyle, "Robustness in the presence of mixed parametric uncertainty and unmodeled dynamics," *IEEE Transactions on Automatic Control*, Vol. AC-36, 1991, pp. 25-38.
- [3] Osborne, E., "On preconditioning of matrices," *Journal of Associated Computer Machines*, Vol. 7, 1960, pp. 338-345.
- [4] Packard, A.K., M. Fan and J. Doyle, "A power method for the structured singular value," *Proc. of 1988 IEEE Conference on Control and Decision*, December 1988, pp. 2132-2137.
- [5] Safonov, M., "Stability margins for diagonally perturbed multivariable feedback systems," *IEEE Proc.*, Vol. 129, Part D, 1992, pp. 251-256.
- [6] Young, P. and J. Doyle, "Computation of with real and complex uncertainties," *Proceedings of the 29th IEEE Conference on Decision and Control*, 1990, pp. 1230-1235.
- [7] Young, P., M. Newlin, and J. Doyle, "Practical computation of the mixed problem," *Proceedings of the American Control Conference*, 1992, pp. 2190-2194.

## See Also

mussvextract | robgain | robstab | wcgain | wcmargin | wcsens

**Introduced before R2006a**

# mussvextract

Extract muinfo structure returned by mussv

## Syntax

```
[VDelta, VSigma, VLmi] = mussvextract(muinfo)
```

## Description

A structured singular value computation of the form

```
[bounds, muinfo] = mussv(M, BlockStructure)
```

returns detailed information in the structure muinfo. mussvextract is used to extract the compressed information within muinfo into a readable form.

The most general call to mussvextract extracts three usable quantities: VDelta, VSigma, and VLmi. VDelta is used to verify the lower bound. VSigma is used to verify the Newlin/Young upper bound and has fields DLeft, DRight, GLeft, GMiddle, and GRight. VLmi is used to verify the LMI upper bound and has fields Dr, Dc, Grc, and Gcr. The relation/interpretation of these quantities with the numerical results in bounds is described below.

## Upper Bound Information

The upper bound is based on a proof that  $\det(I - M \cdot \text{Delta})$  is nonzero for all block-structured matrices Delta with norm smaller than  $1/\text{bounds}(1)$ . The Newlin/Young method consists of finding a scalar  $\beta$  and matrices  $D$  and  $G$ , consistent with BlockStructure, such that

$$\bar{\sigma} \left( (I + G_L^2)^{-\frac{1}{4}} \left( \frac{D_L M D_R^{-1}}{\beta} - jG_M \right) (I + G_R^2)^{-\frac{1}{4}} \right) \leq 1$$

Here  $D_L$ ,  $D_R$ ,  $G_L$ ,  $G_M$ , and  $G_R$  correspond to the DLeft, DRight, GLeft, GMiddle, and GRight fields respectively.

Because some uncertainty blocks and  $M$  need not be square, the matrices  $D$  and  $G$  have a few different manifestations. In fact, in the formula above, there are a left and right  $D$  and  $G$ , as well as a middle  $G$ . Any such  $\beta$  is an upper bound of `mussv(M,BlockStructure)`.

It is true that if `BlockStructure` consists only of complex blocks, then all  $G$  matrices will be zero, and the expression above simplifies to

$$\bar{\sigma}(D_L M D_R^{-1}) \leq \beta.$$

The LMI method consists of finding a scalar  $\beta$  and matrices  $D$  and  $G$ , consistent with `BlockStructure`, such that

$$M D_r M - \beta^2 D_c + j(G_{cr} M - M' G_{rc}) \leq 0$$

is negative semidefinite. Again,  $D$  and  $G$  have a few different manifestations to match the row and column dimensions of  $M$ . Any such  $\beta$  is an upper bound of `mussv(M,BlockStructure)`. If `BlockStructure` consists only of complex blocks, then all  $G$  matrices will be zero, and negative semidefiniteness of  $M' D_r M - \beta^2 D_c$  is sufficient to derive an upper bound.

## Lower Bound Information

The lower bound of `mussv(M,BlockStructure)` is based on finding a “small” (hopefully the smallest) block-structured matrix `VDelta` that causes `det(I - M*VDelta)` to equal 0. Equivalently, the matrix `M*VDelta` has an eigenvalue equal to 1. It will always be true that the lower bound (`bounds(2)`) will be the reciprocal of `norm(VDelta)`.

## Examples

Suppose  $M$  is a 4-by-4 complex matrix. Take the block structure to be two 1-by-1 complex blocks and one 2-by-2 complex block.

```
rng(0, 'twister')
M = randn(4,4) + sqrt(-1)*randn(4,4);
BlockStructure = [1 1; 1 1; 2 2];
```

You can calculate bounds on the structured singular value using the `mussv` command and extract the scaling matrices using `mussvextract`.

```
[bounds,muinfo] = mussv(M,BlockStructure);
[VDelta,VSigma,VLmi] = mussvextract(muinfo);
```

You can first verify the Newlin/Young upper bound with the information extracted from muinfo. The corresponding scalings are D<sub>l</sub> and D<sub>r</sub>.

```
Dl = VSigma.DLeft
```

```
Dl =
```

```
1.0000    0    0    0
    0    0.7437    0    0
    0    0    1.0393    0
    0    0    0    1.0393
```

```
Dr = VSigma.DRight
```

```
Dr =
```

```
1.0000    0    0    0
    0    0.7437    0    0
    0    0    1.0393    0
    0    0    0    1.0393
```

```
[norm(Dl*M/Dr) bounds(1)]
```

```
ans =
```

```
6.2950    6.2950
```

You can first verify the LMI upper bound with the information extracted from muinfo. The corresponding scalings are D<sub>r</sub> and D<sub>c</sub>.

```
Dr = VLmi.Dr;
Dc = VLmi.Dc;
eig(M'*Dr*M - bounds(1)^2*Dc)
```

```
ans =
```

```
-0.0000 - 0.0000i
-17.7242 - 0.0000i
```

```
-33.8550 + 0.0000i
-41.2013 - 0.0000i
```

Note that `VDelta` matches the structure defined by `BlockStructure`, and the norm of `VDelta` agrees with the lower bound,

```
VDelta
```

```
VDelta =
```

```
    0.1301 - 0.0922i    0    0    0
         0          -0.0121 - 0.1590i    0    0
         0           0          -0.0496 - 0.0708i    0.1272 - 0.0075i
         0           0           0.0166 - 0.0163i    0.0076 + 0.0334i
```

```
[norm(VDelta) 1/bounds(2)]
```

```
ans =
```

```
    0.1595    0.1595
```

and that `M*VDelta` has an eigenvalue exactly at 1.

```
eig(M*VDelta)
```

```
ans =
```

```
    1.0000 - 0.0000i
   -0.2501 - 0.1109i
    0.0000 + 0.0000i
   -0.3022 + 0.2535i
```

Keep the matrix the same, but change `BlockStructure` to be a 2-by-2 repeated, real scalar block and two complex 1-by-1 blocks. Run `mussv` with the 'C' option to tighten the upper bound.

```
BlockStructure2 = [-2 0; 1 0; 1 0];
[bounds2,muinfo2] = mussv(M,BlockStructure2,'C');
```

You can compare the computed bounds. Note that `bounds2` should be smaller than `bounds`, because the uncertainty set defined by `BlockStructure2` is a proper subset of that defined by `BlockStructure`.

```
[bounds; bounds2]
```



```
ans =

    6.2950    6.2704
    5.1840    5.1750
```

You can extract the  $D$ ,  $G$  and  $\Delta$  from `muinfo2` using `mussvextract`.

```
[VDelta2, VSigma2, VLmi2] = mussvextract(muinfo2);
```

As before, you can first verify the Newlin/Young upper bound with the information extracted from `muinfo`. The corresponding scalings are  $D_l$ ,  $D_r$ ,  $G_l$ ,  $G_m$  and  $G_r$ .

```
Dl = VSigma2.DLeft;
Dr = VSigma2.DRight;
Gl = VSigma2.GLeft;
Gm = VSigma2.GMiddle;
Gr = VSigma2.GRight;
dmd = Dl*M/Dr/bounds2(1) - sqrt(-1)*Gm;
SL = (eye(4)+Gl*Gl)^-0.25;
SR = (eye(4)+Gr*Gr)^-0.25;
norm(SL*dmd*SR)
```

```
ans =

    1.0000
```

You can first verify the LMI upper bound with the information extracted from `muinfo`. The corresponding scalings are  $D_r$ ,  $D_c$ ,  $G_{rc}$  and  $G_{cr}$ .

```
Dr = VLmi2.Dr;
Dc = VLmi2.Dc;
Grc = VLmi2.Grc;
Gcr = VLmi2.Gcr;
eig(M'*Dr*M - bounds(1)^2 *Dc + j*(Gcr*M-M'*Grc))
```

```
ans =

   -69.9757 + 0.0000i
   -11.2139 - 0.0000i
   -19.2766 - 0.0000i
   -40.2869 - 0.0000i
```

`VDelta2` matches the structure defined by `BlockStructure`, and the norm of `VDelta2` agrees with the lower bound,

```
VDelta2
```

VDelta2 =

```
0.1932          0          0          0
      0          0.1932      0          0
      0          0          -0.1781 - 0.0750i  0
      0          0          0          0.0941 + 0.1688i
```

[norm(VDelta2) 1/bounds2(2)]

ans =

```
0.1932    0.1932
```

and that M\*VDelta2 has an eigenvalue exactly at 1.

eig(M\*VDelta2)

ans =

```
1.0000 + 0.0000i
-0.4328 + 0.1586i
 0.1220 - 0.2648i
-0.3688 - 0.3219i
```

## See Also

muSSV

**Introduced before R2006a**

## ncfmargin

Calculate normalized coprime stability margin of plant-controller feedback loop

### Syntax

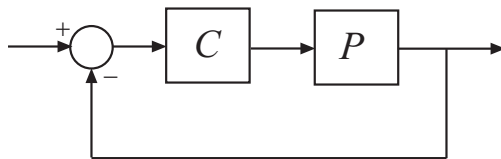
```
[marg,freq] = ncfmargin(P,C)
[marg,freq] = ncfmargin(P,C,sign)
[marg,freq] = ncfmargin(P,C,sign,tol)
```

### Description

`[marg,freq] = ncfmargin(P,C)` returns the normalized coprime stability margin (also called the gap metric stability margin) of the multivariable feedback loop consisting of a controller,  $C$ , in negative feedback with a plant,  $P$ . This margin, `marg`, is achieved at the frequency `freq`. The normalized coprime stability margin is defined as:

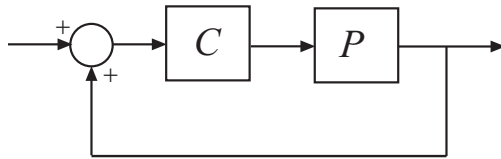
$$\left\| \begin{bmatrix} I \\ C \end{bmatrix} (I - PC)^{-1} \begin{bmatrix} P & I \end{bmatrix} \right\|_{\infty}^{-1}.$$

The calculation assumes the feedback structure of the following diagram:



The normalized coprime robust stability margin is an indication of robustness to unstructured perturbations. The value of the margin lies between 0 and 1. Values greater than 0.3 generally indicate good robustness margins.

`[marg,freq] = ncfmargin(P,C,sign)` specifies the sign of the feedback connection assumed for the margin calculation. The default value, `sign = -1`, specifies negative feedback. Setting `sign = +1` assumes a positive feedback connection for the margin calculation, as in the following diagram:



`[marg, freq] = ncfmargin(P,C,sign,tol)` calculates the normalized coprime factor metric with the specified relative accuracy. `tol` is a scalar value between  $10^{-5}$  and  $10^{-2}$ . The default value is `tol = 0.001` (0.1% accuracy).

## Examples

### Normalized Coprime Stability Margin

Consider an unstable first-order plant, `p`, stabilized by high-gain and low-gain controllers, `cL` and `cH`.

```
p = tf(4,[1 -0.001]);  
cL = 1;  
cH = 10;
```

Compute the stability margin of the closed-loop system with the low-gain controller.

```
[margL,~] = ncfmargin(p,cL)  
  
margL = 0.7069
```

Similarly, compute the stability margin of the closed-loop system with the high-gain controller.

```
[margH,~] = ncfmargin(p,cH)  
  
margH = 0.0995
```

The closed-loop systems with low-gain and high-gain controllers have normalized coprime stability margins of about 0.71 and 0.1, respectively. This result indicates that the closed-loop system with low-gain controller is more robust to unstructured perturbations than the system with the high-gain controller.

To observe this difference in robustness, construct an uncertain plant, `punc`, that has an additional 11% unmodeled dynamics compared to the nominal plant.

```
punc = p + ultidyn('uncstruc',[1 1], 'Bound',0.11);
```

Calculate the robust stability of the closed-loop systems formed by the uncertain plant and each controller.

```
[stabmargL,~] = robstab(feedback(punc,cL))
```

```
stabmargL = struct with fields:
    LowerBound: 9.0728
    UpperBound: 9.0909
    CriticalFrequency: Inf
```

```
[stabmargH,~] = robstab(feedback(punc,cH))
```

```
stabmargH = struct with fields:
    LowerBound: 0.9073
    UpperBound: 0.9091
    CriticalFrequency: Inf
```

As expected, the robust stability analysis shows that the closed-loop system with low-gain controller is more robustly stable in the presence of the unmodeled LTI dynamics. In fact, this closed-loop system can tolerate 90.9% (or  $9.09 \times 11\%$ ) of the unmodeled dynamics. In contrast, closed-loop system with the high-gain controller is not robustly stable. That closed-loop system can only tolerate 90.9% (or  $0.909 \times 11\%$ ) of the unmodeled dynamics.

## Algorithms

The computation of the normalized coprime stability margin amounts to solving 2-block  $H_\infty$  problems. [1] The function, `ncfmargin`, is based on [2].

## References

- [1] Georgiou, T.T., "On the computation of the gap metric," *Systems & Control Letters*, Vol. 11, No. 4, 1988, pp. 253-257.
- [2] Green, M., Glover, K., D. Limebeer, and J.C. Doyle, "A J-spectral factorization approach to  $H_\infty$  control," *SIAM Journal on Control and Optimization*, Vol. 28, No. 6, 1990, pp. 1350-1371.

## **See Also**

gapmetric | loopmargin | ncfsyn | robstab | wcmargin

**Introduced before R2006a**

## ncfmr

Balanced model truncation for normalized coprime factors

### Syntax

```
GRED = ncfmr(G)
```

```
GRED = ncfmr(G,order)
```

```
[GRED,redinfo] = ncfmr(G,key1,value1,...)
```

```
[GRED,redinfo] = ncfmr(G,order,key1,value1,...)
```

### Description

ncfmr returns a reduced order model GRED formed by a set of balanced normalized coprime factors and a struct array redinfo containing the left and right coprime factors of G and their coprime Hankel singular values.

Hankel singular values of coprime factors of such a stable system indicate the respective “state energy” of the system. Hence, reduced order can be directly determined by examining the system Hankel SV's.

With only one input argument G, the function will show a Hankel singular value plot of the original model and prompt for model order number to reduce.

The *left and right normalized coprime factors* are defined as [1]

- *Left Coprime Factorization:*  $G = M_l^{-1}(s)N_l(s)$
- *Right Coprime Factorization:*  $G = N_r(s)M_r^{-1}(s)$

where there exist stable  $U_r(s)$ ,  $V_r(s)$ ,  $U_l(s)$  and  $V_l(s)$  such that

$$U_r N_r + V_r M_r = I$$

$$N_l U_l + M_l V_l = I$$

The left/right coprime factors are stable, hence implies  $M_r(s)$  should contain as RHP-zeros all the RHP-poles of  $G(s)$ . The coprimeness also implies that there should be no common

RHP-zeros in  $N_r(s)$  and  $M_r(s)$ , i.e., when forming  $G = N_r(s)M_r^{-1}(s)$ , there should be no pole-zero cancellations.

This table describes input arguments for `ncmfr`.

| Argument | Description   |
|----------|---|
| G        | LTI model to be reduced (without any other inputs will plot its Hankel singular values and prompt for reduced order)            |
| ORDER    | (Optional) Integer for the desired order of the reduced model, or optionally a vector packed with desired orders for batch runs |

A batch run of a serial of different reduced order models can be generated by specifying `order = x:y`, or a vector of integers. By default, all the anti-stable part of a system is kept, because from control stability point of view, getting rid of unstable state(s) is dangerous to model a system. The `ncmfr` method allows the original model to have  $j\omega$ -axis singularities.

'*MaxError*' can be specified in the same fashion as an alternative for 'ORDER'. In this case, reduced order will be determined when the sum of the tails of the Hankel singular values reaches the '*MaxError*'.

| Argument            | Value   | Description   |
|---------------------|---|---|
| ' <i>MaxError</i> ' | A real number or a vector of different errors | Reduce to achieve $H_\infty$ error.<br>When present, ' <i>MaxError</i> ' overrides ORDER input. |
| ' <i>Display</i> '  | ' <i>on</i> ' or ' <i>off</i> '               | Display Hankel singular plots (default ' <i>off</i> ').   |
| ' <i>Order</i> '    | integer, vector or cell array                 | Order of reduced model. Use only if not specified as 2nd argument.                              |

Weights on the original model input and/or output can make the model reduction algorithm focus on some frequency range of interests. But weights have to be stable, minimum phase, and invertible.

This table describes output arguments.



| Argument | Description   |
|----------|---|
| GRED     | LTI reduced order model, that becomes multi-dimensional array when input is a serial of different model order array.  |
| REDINFO  | A STRUCT array with 3 fields: <ul style="list-style-type: none"> <li>• REDINFO.GL (left coprime factor)</li> <li>• REDINFO.GR (right coprime factor)</li> <li>• REDINFO.hsv (Hankel singular values)</li> </ul> |

$G$  can be stable or unstable, continuous or discrete.

## Examples

Given a continuous or discrete, stable or unstable system,  $G$ , the following commands can get a set of reduced order models based on your selections:

```
rng(1234,'twister');
G = rss(30,5,4);
G.D = zeros(5,4);
[g1, redinfo1] = ncfmr(G); % display Hankel SV plot
                        % and prompt for order (try 15:20)
[g2, redinfo2] = ncfmr(G,20);
[g3, redinfo3] = ncfmr(G,[10:2:18]);
[g4, redinfo4] = ncfmr(G,'MaxError',[0.01, 0.05]);
for i = 1:4
    figure(i)
    eval(['sigma(G,g' num2str(i) ');']);
end
```

## Algorithms

Given a state space  $(A,B,C,D)$  of a system and  $k$ , the desired reduced order, the following steps will produce a similarity transformation to truncate the original state-space system to the  $k^{\text{th}}$  order reduced model.

- 1 Find the normalized coprime factors of  $G$  by solving Hamiltonian described in [1].

$$G_l = [N_l \quad M_l]$$

$$G_r = \begin{bmatrix} N_r \\ M_r \end{bmatrix}$$

- 2 Perform  $k^{\text{th}}$  order square root balanced model truncation on  $G_l$  (or  $G_r$ ) [2].
- 3 The reduced model GRED is:

$$\left[ \begin{array}{c|c} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{array} \right] = \left[ \begin{array}{c|c} A_c - B_m C_l & B_n - B_m D_l \\ \hline C_l & D_l \end{array} \right]$$

where

$$N_l (:= A_c, B_n, C_c, D_n)$$

$$M_l := (A_c, B_m, C_c, D_m)$$

$$C_l = (D_m)^{-1} C_c$$

$$D_l = (D_m)^{-1} D_n$$

## References

- [1] M. Vidyasagar. *Control System Synthesis - A Factorization Approach*. London: The MIT Press, 1985.
- [2] M. G. Safonov and R. Y. Chiang, "A Schur Method for Balanced Model Reduction," *IEEE Trans. on Automat. Contr.*, vol. AC-2, no. 7, July 1989, pp. 729-733.

## See Also

balancmr | bstmr | hankelmr | hankelsv | reduce | schurmr

**Introduced before R2006a**

# ncfsyn

Loop shaping design using Glover-McFarlane method

## Syntax

```
[K,CL,GAM,INFO]=ncfsyn(G)
[K,CL,GAM,INFO]=ncfsyn(G,W1)
[K,CL,GAM,INFO]=ncfsyn(G,W1,W2)
[K,CL,GAM,INFO]=ncfsyn(G,W1,W2,'ref')
```

## Description

ncfsyn is a method for designing controllers that uses a combination of loop shaping and robust stabilization as proposed in McFarlane and Glover [1]-[2]. The first step is for you to select a pre- and post-compensator  $W_1$  and  $W_2$ , so that the gain of the 'shaped plant'  $G_s = W_2GW_1$  is sufficiently high at frequencies where good disturbance attenuation is required and is sufficiently low at frequencies where good robust stability is required. The second step is to use ncfsyn to compute an optimal *positive* feedback controllers K.

The optimal  $K_s$  has the property that the  $\sigma$  plot of the shaped loop

$$L_s = W_2 * G * W_1 * K_s$$

matches the target loop shape  $G_s$  optimally, roughly to within plus or minus  $20 * \log_{10}(GAM)$  db. The number margin  $GAM = 1 / \text{ncfmargin}(G_s, K)$  and is always greater than 1. GAM gives a good indication of robustness of stability to a wide class of unstructured plant variations, with values in the range  $1 < GAM < 3$  corresponding to satisfactory stability margins for most typical control system designs.

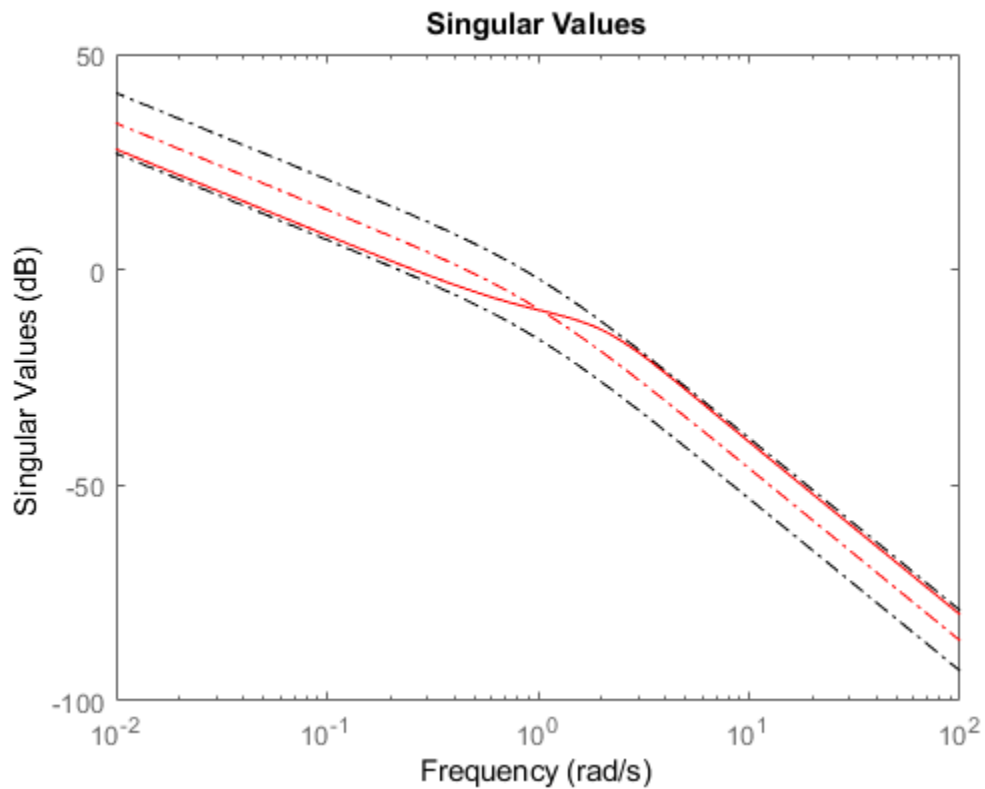
`[K,CL,GAM,INFO]=ncfsyn(G,W1,W2,'ref')` computes the Glover-McFarlane  $H_\infty$  normalized coprime factor loop-shaping controller K, with a reference command, for lti plant G, weights W1 and W2 if the 'ref' option is included. The closed-loop system CL represents the transfer matrix from the reference and disturbance to the feedback error and output of W1.

## Examples

### Loop Shaping With `ncfsyn`

The following code shows how `ncfsyn` can be used for loop-shaping.

```
s = zpk('s');  
G = (s-1)/(s+1)^2;  
W1 = 0.5/s;  
[K,CL,GAM] = ncfsyn(G,W1);  
sigma(G*K, 'r',G*W1, 'r-.',G*W1*GAM, 'k-.',G*W1/GAM, 'k-.')
```



The singular value plot of the achieved loop  $G*K$  is equal to that of the target loop  $G*W_1$  to within plus or minus  $\gamma$  (in dB).

## Algorithms

$K=W_2*K_s*W_1$ , where  $K_s =K_\infty$  is an optimal  $H_\infty$  controller that simultaneously minimizes the two  $H_\infty$  cost functions

$$\gamma := \min_K \left\| \begin{bmatrix} I \\ K \end{bmatrix} (I - G_s K)^{-1} [G_s, I] \right\|_\infty$$

$$\gamma := \min_K \left\| \begin{bmatrix} I \\ G_s \end{bmatrix} (I - K G_s)^{-1} [K, I] \right\|_\infty$$

Roughly speaking, this means for most plants that

$$\sigma(W_2 G W_1 K_\infty), \text{ db} = \sigma(W_2 G W_1), \text{ db} \pm \gamma, \text{ db}$$

$$\sigma(K_\infty W_2 G W_1), \text{ db} = \sigma(W_2 G W_1), \text{ db} \pm \gamma, \text{ db},$$

so you can use the weights  $W_1$  and  $W_2$  for loopshaping. For a more precise bounds on loopshaping accuracy, see Theorem 16.12 of Zhou and Glover.

Theory ensures that if  $G_s = N M^{-1}$  is a normalized coprime factorization (NCF) of the weighted plant model  $G_s$  satisfying

$$G_s = N(j\omega) * N(j\omega) + M(j\omega) * M(j\omega) = I,$$

then the control system will remain robustly stable for any perturbation  $\tilde{G}_s$  to the weighted plant model  $G_s$  that can be written

$$\tilde{G}_s = (N + \Delta_1)(M + \Delta_2)^{-1}$$

for some stable pair  $\Delta_1, \Delta_2$  satisfying

$$\left\| \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} \right\|_{\infty} < \text{MARG} := 1 / \text{GAM}$$

The closed-loop  $H_{\infty}$ -norm objective has the standard signal gain interpretation. Finally it can be shown that the controller,  $K_{\infty}$ , does not substantially affect the loop shape in frequencies where the gain of  $W_2GW_1$  is either high or low, and will guarantee satisfactory stability margins in the frequency region of gain cross-over. In the regulator set-up, the final controller to be implemented is  $K=W_1K_{\infty}W_2$ .

### Input Arguments

|        |   |
|--------|---|
| G      | LTI plant to be controlled  |
| W1, W2 | Stable minimum-phase LTI weights, either SISO or MIMO.<br>Default is $W_1=I, W_2=I$ |
| 'ref'  | Reference input to controller. Default is no reference input is included.           |

### Output Arguments

|      |   |
|------|---|
| K    | LTI controller $K= W_1*K_s*W_2$   |
| CL   | $\begin{bmatrix} I \\ K_{\infty} \end{bmatrix} (I - W_2GW_1K_{\infty})^{-1} [W_2GW_1, I]$<br>, LTI $H_{\infty}$ optimal closed loop |
| GAM  | $H_{\infty}$ optimal cost $\frac{1}{b(W_2GW_1, K_{\infty})} = \text{hinfnorm}(CL) \geq 1$   |
| INFO | Structure array containing additional information   |

Additional output INFO fields

|           |  |
|-----------|--|
| INFO.emax | nugap robustness $\text{emax}=1/\text{GAM}=\text{ncfmargin}(G_s, -K_s)=b(W_2GW_1, K_{\infty})$ |
| INFO.Gs   | 'shaped plant' $G_s=W_2*G*W_1$   |

|         |   |
|---------|---|
| INFO.Ks | Ks = K[[BULLET]] = NCFSYN(Gs) = NCFSYN(W2*G*W1) |
|---------|---|

[MARG, FREQ] = `ncfmargin(G, K, TOL)` calculates the normalized coprime factor/gap metric robust stability margin assuming *negative* feedback.

$$\text{MARG} = b(G, -K) = 1 / \left\| \begin{bmatrix} I \\ -K \end{bmatrix} (I + GK)^{-1} [G, I] \right\|_{\infty}$$

where  $G$  and  $K$  are LTI plant and controller, and  $TOL$  (default=.001) is the tolerance used to compute the  $H_{\infty}$  norm.  $FREQ$  is the peak frequency. That is, the frequency at which the infinity norm is reached to within  $TOL$ .

## Algorithms

See McFarlane and Glover [1]-[2] for details.

## References

- [1] McFarlane, D.C., and K. Glover, Robust Controller Design using Normalised Coprime Factor Plant Descriptions, Springer Verlag, *Lecture Notes in Control and Information Sciences*, vol. 138, 1989.
- [2] McFarlane, D.C., and K. Glover, "A Loop Shaping Design Procedure using Synthesis," *IEEE Transactions on Automatic Control*, vol. 37, no. 6, pp. 759- 769, June 1992.
- [3] Vinnicombe, G., "Measuring Robustness of Feedback Systems," PhD dissertation, Department of Engineering, University of Cambridge, 1993.
- [4] Zhou, K., and J.C. Doyle, *Essentials of Robust Control*. NY: Prentice-Hall, 1998.

## See Also

`gapmetric` | `hinfsyn` | `loopsyn` | `ncfmargin`

**Introduced before R2006a**

## **newlmi**

Attach identifying tag to LMIs

### **Syntax**

```
tag = newlmi
```

### **Description**

`newlmi` adds a new LMI to the LMI system currently described and returns an identifier tag for this LMI. This identifier can be used in `lmiterm`, `showlmi`, or `dellmi` commands to refer to the newly declared LMI. Tagging LMIs is *optional* and only meant to facilitate code development and readability.

Identifiers can be given mnemonic names to help keep track of the various LMIs. Their value is simply the ranking of each LMI in the system (in the order of declaration). They prove useful when some LMIs are deleted from the LMI system. In such cases, the identifiers are the safest means of referring to the remaining LMIs.

### **See Also**

`dellmi` | `getlmis` | `lmiedit` | `lmiterm` | `lmivar` | `setlmis`

**Introduced before R2006a**



# normalized2actual

Convert value for atom in normalized coordinates to corresponding actual value

## Syntax

```
avalue = normalized2actual(A,NV)
```

## Description

Converts a normalized value NV of an atom to its corresponding actual (unnormalized) value.

If NV is an array of values, then `avalue` will be an array of the same dimension.

## Examples

Create uncertain real parameters with a range that is symmetric about the nominal value, where each endpoint is 1 unit from the nominal. Points that lie inside the range are less than 1 unit from the nominal, while points that lie outside the range are greater than 1 unit from the nominal.

```
a = ureal('a',3,'range',[1 5]);
actual2normalized(a,[1 3 5])
ans =
    -1.0000    -0.0000     1.0000
normalized2actual(a,[-1 1])
ans =
     1.0000     5.0000
normalized2actual(a,[-1.5 1.5])
ans =
     0.0000     6.0000
```

## See Also

`actual2normalized` | `getLimits` | `robgain` | `robstab`

**Introduced before R2006a**

## pdlstab

Assess robust stability of polytopic or parameter-dependent system

### Syntax

`[tau,Q0,Q1,...] = pdlstab(pds,options)`

### Description

`pdlstab` uses parameter-dependent Lyapunov functions to establish the stability of uncertain state-space models over some parameter range or polytope of systems. Only sufficient conditions for the existence of such Lyapunov functions are available in general. Nevertheless, the resulting robust stability tests are always less conservative than quadratic stability tests when the parameters are either time-invariant or slowly varying.

For an affine parameter-dependent system

$$E(p)x' = A(p)x + B(p)u$$

$$y = C(p)x + D(p)u$$

with  $p = (p_1, \dots, p_n) \in R^n$ , `pdlstab` seeks a Lyapunov function of the form

$$V(x,p) = x^T Q(p) x, \quad Q(p) = Q_0 + p_1 Q_1 + \dots + p_n Q_n$$

such that  $dV(x, p)/dt < 0$  along all admissible parameter trajectories. The system description `pds` is specified with `psys` and contains information about the range of values and rate of variation of each parameter  $p_i$ .

For a *time-invariant* polytopic system

$$Ex' = Ax + Bu$$

$$y = Cx + Du$$

with

$$\begin{pmatrix} A + jE & B \\ C & D \end{pmatrix} = \sum_{i=1}^n \alpha_i \begin{pmatrix} A + jE_i & B_i \\ C_i & D_i \end{pmatrix}, \alpha_i \geq 0, \sum_{i=1}^n \alpha_i = 1$$

`pdlstab` seeks a Lyapunov function of the form

$$V(x, \alpha) = x^T Q(\alpha) x, \quad Q(\alpha) = \alpha_1 Q_1 + \dots + \alpha_n Q_n$$

such that  $dV(x, \alpha)/dt < 0$  for all polytopic decompositions of the form “Equation 1-14” on page 1-310.

Several options and control parameters are accessible through the optional argument `options`:

- Setting `options(1)=0` tests robust stability (default)
- When `options(2)=0`, `pdlstab` uses simplified sufficient conditions for faster running times. Set `options(2)=1` to use the least conservative conditions

## Tips

For affine parameter-dependent systems with *time-invariant* parameters, there is equivalence between the robust stability of

$$E(p)\dot{x} = A(p)x$$

and that of the dual system

$$E(p)^T \dot{z} = A(p)^T z$$

However, the second system may admit an affine parameter-dependent Lyapunov function while the first does not.

In such case, `pdlstab` automatically restarts and tests stability on the dual system “Equation 1-16” on page 1-310 when it fails on “Equation 1-15” on page 1-310.

## See Also

`quadstab`

**Introduced before R2006a**

## **pdsimul**

Time response of parameter-dependent system along given parameter trajectory

### **Syntax**

```
pdsimul(pds, 'traj', tf, 'ut', xi, options)
```

```
[t, x, y] = pdsimul(pds, pv, 'traj', tf, 'ut', xi, options)
```

### **Description**

`pdsimul` simulates the time response of an affine parameter-dependent system

$$E(p)x' = A(p)x + B(p)u$$

$$y = C(p)x + D(p)u$$

along a parameter trajectory  $p(t)$  and for an input signal  $u(t)$ . The parameter trajectory and input signals are specified by two time functions  $p=\text{traj}(t)$  and  $u=\text{ut}(t)$ . If 'ut' is omitted, the response to a step input is computed by default.

The affine system `pds` is specified with `psys`. The function `pdsimul` also accepts the polytopic representation of such systems as returned by `aff2pol(pds)` or `hifgs`. The final time and initial state vector can be reset through `tf` and `xi` (their respective default values are 5 seconds and 0). Finally, `options` gives access to the parameters controlling the ODE integration (type `help gear` for details).

When invoked without output arguments, `pdsimul` plots the output trajectories  $y(t)$ . Otherwise, it returns the vector of integration time points `t` as well as the state and output trajectories `x, y`.

### **See Also**

`psys` | `pvec`

**Introduced before R2006a**

## polydec

Compute polytopic coordinates with respect to box corners

### Syntax

```
vertx = polydec(PV)
```

```
[C,vertx] = polydec(PV,P)
```

### Description

`vertx = polydec(PV)` takes an uncertain parameter vector `PV` taking values ranging in a box, and returns the corners or vertices of the box as columns of the matrix `vertx`.

`[C,vertx] = polydec(PV,P)` takes an uncertain parameter vector `PV` and a value `P` of the parameter vector `PV`, and returns the convex decomposition `C` of `P` over the set `VERTX` of box corners:

$$P = c_1 \text{VERTX}(:,1) + \dots + c_n \text{VERTX}(:,n)$$
$$c_j \geq 0, \quad c_1 + \dots + c_n = 1$$

The list `vertx` of corners can be obtained directly by typing

```
vertx = polydec(PV)
```

### See Also

`aff2pol` | `hinfo` | `pvec` | `pvinfos`

**Introduced before R2006a**



## popov

Perform Popov robust stability test

### Syntax

```
[t,P,S,N] = popov(sys,delta,flag)
```

### Description

popov uses the Popov criterion to test the robust stability of dynamical systems with possibly nonlinear and/or time-varying uncertainty. The uncertain system must be described as the interconnection of a nominal LTI system `sys` and some uncertainty `delta`.

The command

```
[t,P,S,N] = popov(sys,delta)
```

tests the robust stability of this interconnection. Robust stability is guaranteed if  $t < 0$ . Then `P` determines the quadratic part  $x^T P x$  of the Lyapunov function and `D` and `S` are the Popov multipliers.

If the uncertainty `delta` contains real parameter blocks, the conservatism of the Popov criterion can be reduced by first performing a simple loop transformation. To use this refined test, call `popov` with the syntax

```
[t,P,S,N] = popov(sys,delta,1)
```

### See Also

`pdlstab` | `quadstab`

**Introduced before R2006a**

## psinfo

Inquire about polytopic or parameter-dependent systems created with `psys`

### Syntax

```
psinfo(ps)
[type,k,ns,ni,no] = psinfo(ps)
pv = psinfo(ps,'par')
sk = psinfo(ps,'sys',k)
sys = psinfo(ps,'eval',p)
```

### Description

`psinfo` is a multi-usage function for queries about a polytopic or parameter-dependent system `ps` created with `psys`. It performs the following operations depending on the calling sequence:

- `psinfo(ps)` displays the type of system (affine or polytopic); the number `k` of SYSTEM matrices involved in its definition; and the numbers of `ns`, `ni`, `no` of states, inputs, and outputs of the system. This information can be optionally stored in MATLAB variables by providing output arguments.
- `pv = psinfo(ps,'par')` returns the parameter vector description (for parameter-dependent systems only).
- `sk = psinfo(ps,'sys',k)` returns the  $k$ -th SYSTEM matrix involved in the definition of `ps`. The ranking `k` is relative to the list of systems `syslist` used in `psys`.
- `sys = psinfo(ps,'eval',p)` instantiates the system for a given vector `p` of parameter values or polytopic coordinates.

For *affine parameter-dependent* systems defined by the SYSTEM matrices  $S_0, S_1, \dots, S_n$ , the entries of `p` should be real parameter values  $p_1, \dots, p_n$  and the result is the LTI system of SYSTEM matrix

$$S(p) = S_0 + p_1 S_1 + \dots + p_n S_n$$

For *polytopic* systems with SYSTEM matrix ranging in

$\text{Co}\{S_1, \dots, S_n\}$ ,

the entries of  $\mathbf{p}$  should be polytopic coordinates  $p_1, \dots, p_n$  satisfying  $p_j \geq 0$  and the result is the interpolated LTI system of SYSTEM matrix

$$S = \frac{p_1 S_1 + \dots + p_n S_n}{p_1 + \dots + p_n}$$

## See Also

psys

**Introduced before R2006a**

## psys

Specify polytopic or parameter-dependent linear systems

## Syntax

`pols = psys(syslist)`

`affs = psys(pv,syslist)`

## Description

`psys` specifies state-space models where the state-space matrices can be uncertain, time-varying, or parameter-dependent.

`psys` supports two types of uncertain state-space models:

- *Polytopic* systems

$$E(t) \dot{x} = A(t)x + B(t)u$$

$$y = C(t)x + D(t)u$$

whose SYSTEM matrix takes values in a fixed polytope:

$$\underbrace{\begin{bmatrix} A(t) + jE(t) & B(t) \\ C(t) & D(t) \end{bmatrix}}_{S(t)} \in \text{Co} \left\{ \underbrace{\begin{bmatrix} A_1 + jE_1 & B_1 \\ C_1 & D_1 \end{bmatrix}}_{S_1}, \dots, \underbrace{\begin{bmatrix} A_k + jE_k & B_k \\ C_k & D_k \end{bmatrix}}_{S_k} \right\}$$

where  $S_1, \dots, S_k$  are given “vertex” systems and

$$\text{Co}\{S_1, \dots, S_k\} = \left\{ \sum_{i=1}^k \alpha_i S_i : \alpha_i \geq 0, \sum_{i=1}^k \alpha_i = 1 \right\}$$

denotes the convex hull of  $S_1, \dots, S_k$  (polytope of matrices with vertices  $S_1, \dots, S_k$ )

- *Affine parameter-dependent systems*

$$E(p)x' = A(p)x + B(p)u$$

$$y = C(p)x + D(p)u$$

where  $A(\cdot)$ ;  $B(\cdot)$ , . . . ,  $E(\cdot)$  are fixed affine functions of some vector  $p = (p_1, \dots, p_n)$  of real parameters, i.e.,

$$\underbrace{\begin{bmatrix} A(p) + jE(p) & B(p) \\ C(p) & D(p) \end{bmatrix}}_{S(p)} = \underbrace{\begin{bmatrix} A_0 + jE_0 & B_0 \\ C_0 & D_0 \end{bmatrix}}_{S_0} + p_1 \underbrace{\begin{bmatrix} A_1 + jE_1 & B_1 \\ C_1 & D_1 \end{bmatrix}}_{S_1} + \dots + p_n \underbrace{\begin{bmatrix} A_n + jE_n & B_n \\ C_n & D_n \end{bmatrix}}_{S_n}$$

where  $S_0, S_1, \dots, S_n$  are given SYSTEM matrices. The parameters  $p_i$  can be time-varying or constant but uncertain.

The argument `syslist` lists the SYSTEM matrices  $S_i$  characterizing the polytopic value set or parameter dependence. In addition, the description `pv` of the parameter vector (range of values and rate of variation) is required for affine parameter-dependent models (see `pvec` for details). Thus, a polytopic model with vertex systems  $S_1, \dots, S_4$  is created by

```
polS = psys([s1,s2,s3,s4])
```

while an affine parameter-dependent model with 4 real parameters is defined by

```
affS = psys(pv,[s0,s1,s2,s3,s4])
```

The output is a structured matrix storing all the relevant information.

## See Also

`aff2pol` | `psinfo` | `pvec`

**Introduced before R2006a**

## **pvec**

Specify range and rate of variation of uncertain or time-varying parameters

### **Syntax**

```
pv = pvec('box', range, rates)
```

```
pv = pvec('pol', vertices)
```

### **Description**

`pvec` is used in conjunction with `psys` to specify parameter-dependent systems. Such systems are parametrized by a vector  $p = (p_1, \dots, p_n)$  of uncertain or time-varying real parameters  $p_i$ . The function `pvec` defines the range of values and the rates of variation of these parameters.

The type 'box' corresponds to independent parameters ranging in intervals

$$\underline{p}_j \leq p_j \leq \bar{p}_j$$

The parameter vector  $p$  then takes values in a hyperrectangle of  $R^n$  called the parameter box. The second argument `range` is an  $n$ -by-2 matrix that stacks up the extremal values

$\underline{p}_j$  and  $\bar{p}_j$  of each  $p_j$ . If the third argument `rates` is omitted, all parameters are assumed time-invariant. Otherwise, `rates` is also an  $n$ -by-2 matrix and its  $j$ -th row

specifies lower and upper bounds  $\underline{v}_j$  and  $\bar{v}_j$  on  $\frac{dp_j}{dt}$ :

$$\underline{v}_j \leq \frac{dp_j}{dt} \leq \bar{v}_j$$

Set  $\underline{v}_j = -\text{Inf}$  and  $\bar{v}_j = \text{Inf}$  if  $p_j(t)$  can vary arbitrarily fast or discontinuously.

The type 'pol' corresponds to parameter vectors  $p$  ranging in a polytope of the parameter space  $R^n$ . This polytope is defined by a set of vertices  $V_1, \dots, V_n$  corresponding

to “extremal” values of the vector  $p$ . Such parameter vectors are declared by the command

```
pV = pvec('pol',[v1,v2, . . . , vn])
```

where the second argument is the concatenation of the vectors  $v_1, \dots, v_n$ .

The output argument  $pV$  is a structured matrix storing the parameter vector description. Use `pVinfo` to read the contents of  $pV$ .

## Examples

Consider a problem with two time-invariant parameters

$$p_1 \in [-1, 2], p_2 \in [20, 50]$$

The corresponding parameter vector  $p = (p_1, p_2)$  is specified by

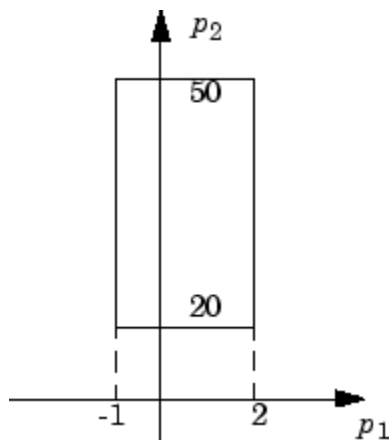
```
pV = pvec('box',[-1 2;20 50])
```

Alternatively, this vector can be regarded as taking values in the rectangle drawn in the following figure. The four corners of this rectangle are the four vectors

$$v_1 = \begin{pmatrix} -1 \\ 20 \end{pmatrix}, v_2 = \begin{pmatrix} -1 \\ 50 \end{pmatrix}, v_3 = \begin{pmatrix} 2 \\ 20 \end{pmatrix}, v_4 = \begin{pmatrix} 2 \\ 50 \end{pmatrix}$$

Hence, you could also specify  $p$  by

```
pV = pvec('pol',[v1,v2,v3,v4])
```



**Parameter box**

## See Also

`psys` | `pvinfos`

**Introduced before R2006a**



## pvinfo

Describe parameter vector specified with `pvec`

### Syntax

```
[typ,k,nv] = pvinfo(pv)
[pmin,pmax,dpmin,dpmax] = pvinfo(pv,'par',j)
vj = pvinfo(pv,'par',j)
p = pvinfo(pv,'eval',c)
```

### Description

`pvinfo` retrieves information about a vector  $p = (p_1, \dots, p_n)$  of real parameters declared with `pvec` and stored in `pv`. The command `pvinfo(pv)` displays the type of parameter vector ('box' or 'pol'), the number  $n$  of scalar parameters, and for the type 'pol', the number of vertices used to specify the parameter range.

For the type 'box':

```
[pmin,pmax,dpmin,dpmax] = pvinfo(pv,'par',j)
```

returns the bounds on the value and rate of variations of the  $j$ -th real parameter  $p_j$ . Specifically,

$$p_{\min} \leq p_j(t) \leq p_{\max}, dp_{\min} \leq \frac{dp_j}{dt} \leq dp_{\max}$$

For the type 'pol':

```
pvinfo(pv,'par',j)
```

returns the  $j$ -th vertex of the polytope of  $R^n$  in which  $p$  ranges, while

```
pvinfo(pv,'eval',c)
```

returns the value of the parameter vector  $p$  given its barycentric coordinates  $c$  with respect to the polytope vertices  $(V_1, \dots, V_k)$ . The vector  $c$  must be of length  $k$  and have nonnegative entries. The corresponding value of  $p$  is then given by

$$p = \frac{\sum_{i=1}^k c_i V_i}{\sum_{i=1}^k c_i}$$

## See Also

psys | pvec

**Introduced before R2006a**

## quadperf

Compute quadratic  $H_\infty$  performance of polytopic or parameter-dependent system

### Syntax

`[perf,P] = quadperf(ps,g,options)`

### Description

The RMS gain of the time-varying system

$$E(t)\dot{x} = A(t)x + B(t)u, \quad y = C(t)x + D(t)u$$

is the smallest  $\gamma > 0$  such that

$$\|y\|_{L_2} \leq \gamma \|u\|_{L_2}$$

for all input  $u(t)$  with bounded energy. A sufficient condition for “Equation 1-18” on page 1-325 is the existence of a quadratic Lyapunov function

$$V(x) = x^T P x, \quad P > 0$$

such that

$$\forall u \in L_2, \quad \frac{dV}{dt} + y^T y - \gamma^2 u^T u < 0$$

Minimizing  $\gamma$  over such quadratic Lyapunov functions yields the quadratic  $H_\infty$  performance, an upper bound on the true RMS gain.

The command

`[perf,P] = quadperf(ps)`

computes the quadratic  $H_\infty$  performance `perf` when “Equation 1-17” on page 1-325 is a polytopic or affine parameter-dependent system `ps` (see `psys`). The Lyapunov matrix  $P$  yielding the performance `perf` is returned in `P`.

The optional input `options` gives access to the following task and control parameters:

- If `options(1)=1`, `perf` is the largest portion of the parameter box where the quadratic RMS gain remains smaller than the positive value `g` (for affine parameter-dependent systems only). The default value is 0.
- If `options(2)=1`, `quadperf` uses the least conservative quadratic performance test. The default is `options(2)=0` (fast mode)
- `options(3)` is a user-specified upper bound on the condition number of  $P$  (the default is 109).

## See Also

`psys` | `quadstab`

**Introduced before R2006a**

# quadstab

Quadratic stability of polytopic or affine parameter-dependent systems

## Syntax

```
[tau,P] = quadstab(ps,options)
```

## Description

For affine parameter-dependent systems

$$E(p)x' = A(p)x, \quad p(t) = (p_1(t), \dots, p_n(t))$$

or polytopic systems

$$E(t)x' = A(t)x, \quad (A, E) \in \text{Co}\{(A_1, E_1), \dots, (A_n, E_n)\},$$

`quadstab` seeks a fixed Lyapunov function  $V(x) = x^T P x$  with  $P > 0$  that establishes quadratic stability. The affine or polytopic model is described by `ps` (see `psys`).

The task performed by `quadstab` is selected by `options(1)`:

- if `options(1)=0` (default), `quadstab` assesses quadratic stability by solving the LMI problem

Minimize  $\tau$  over  $Q = Q^T$  such that

$$A^T Q E + E Q A^T < \tau I \text{ for all admissible values of } (A, E)$$

$$Q > I$$

The global minimum of this problem is returned in `tau` and the system is quadratically stable if `tau < 0`.

- if `options(1)=1`, `quadstab` computes the largest portion of the specified parameter range where quadratic stability holds (only available for affine models). Specifically, if each parameter  $p_i$  varies in the interval

$$p_i \in [p_{i0} - \delta_i, p_{i0} + \delta_i],$$

`quadstab` computes the largest  $\Theta > 0$  such that quadratic stability holds over the parameter box

$$p_i \in [p_{i0} - \Theta\delta_i, p_{i0} + \Theta\delta_i]$$

This “quadratic stability margin” is returned in `tau` and `ps` is quadratically stable if `tau`  $\geq 1$ .

Given the solution  $Q_{\text{opt}}$  of the LMI optimization, the Lyapunov matrix  $P$  is given by  $P = Q_{\text{opt}}^{-1}$ . This matrix is returned in `P`.

Other control parameters can be accessed through `options(2)` and `options(3)`:

- if `options(2)=0` (default), `quadstab` runs in fast mode, using the least expensive sufficient conditions. Set `options(2)=1` to use the least conservative conditions
- `options(3)` is a bound on the condition number of the Lyapunov matrix  $P$ . The default is  $10^9$ .

## See Also

`decay` | `pdlstab` | `psys` | `quadperf`

**Introduced before R2006a**

# randatom

Generate random uncertain atom objects

## Syntax

```
A = randatom(Type)
```

```
A = randatom(Type, sz)
```

```
A = randatom
```

## Description

`A = randatom(Type)` generates a 1-by-1 type uncertain object. Valid values for `Type` include 'ureal', 'ultidyn', 'ucomplex', and 'ucomplexm'.

`A = randatom(Type, sz)` generates an `sz(1)`-by-`sz(2)` uncertain object. Valid values for `Type` include 'ultidyn' or 'ucomplexm'. If `Type` is set to 'ureal' or 'ucomplex', the size variable is ignored and `A` is a 1-by-1 uncertain object.

`A = randatom`, where `randatom` has no input arguments, results in a 1-by-1 uncertain object. The class of this object is randomly selected between 'ureal', 'ultidyn' and 'ucomplex'.

In general, both `rand` and `randn` are used internally. You can control the result of `randatom` by setting seeds for both random number generators before calling the function.

## Examples

The following statement creates the ureal uncertain object `xr`. Note that your display can differ because a random seed is used.

```
xr = randatom('ureal')
```

`xr =`

Uncertain real parameter "NMGXC" with nominal value 5.34 and variability [-2.99,1.92].

The following statement creates the variable `ultidyn` uncertain object `xlti` with three inputs and four outputs. You will get the results shown below if you set the random variable seed to 29.

```
rng(29,'twister');  
xlti = randatom('ultidyn',[4 3])
```

`xlti =`

Uncertain LTI dynamics "LOSWT" with 4 outputs, 3 inputs, and gain less than 0.293.

## See Also

`rand` | `randn` | `randumat` | `randuss` | `ucomplex` | `ucomplexm` | `ultidyn`

**Introduced before R2006a**



# randumat

Generate random uncertain umat objects

## Syntax

```
um = randumat(ny,nu)
```

```
um = randumat
```

## Description

`um = randumat(ny,nu)` generates an uncertain matrix of size `ny-by-nu`. `randumat` randomly selects from uncertain atoms of type `'ureal'`, `'ultidyn'`, and `'ucomplex'`.

`um = randumat` results in a 1-by-1 `umat` uncertain object, including up to four uncertain objects.

## Examples

The following statement creates the `umat` uncertain object `x1` of size 2-by-3. Note that your result can differ because a random seed is used.

```
x1 = randumat(2,3)
```

```
x1 =
```

```
Uncertain matrix with 2 rows and 3 columns.
```

```
The uncertainty consists of the following blocks:
```

```
AWYRT: Uncertain real, nominal = 7.09, variability = [-7.84,16.4]%, 2 occurrences
```

```
HRRED: Uncertain complex, nominal = 3.14+5.47i, radius = 1.92, 1 occurrences
```

```
VSIYA: Uncertain real, nominal = -4.05, variability = [-1.53,3.83], 3 occurrences
```

```
YZEZY: Uncertain complex, nominal = -6.54-2.17i, variability = 24%, 1 occurrences
```

```
Type "x1.NominalValue" to see the nominal value, "get(x1)" to see all properties, and "x1.Uncertainty" to interact with the uncertain elements.
```

The following statement creates the `umat` uncertain object `x2` of size 4-by-2 with the seed 91.

```
rng(91,'twister');  
x2 = randumat(4,2)
```

x2 =

Uncertain matrix with 4 rows and 2 columns.

The uncertainty consists of the following blocks:

YQZBI: Uncertain complex, nominal = 3.61+1.88i, radius = 1.42, 1 occurrences

Type "x2.NominalValue" to see the nominal value, "get(x2)" to see all properties, and "x2.Uncertainty" to interact with the uncertain elements.

## See Also

rand | randatom | randn | randuss | ucomplex | ultidyn

**Introduced before R2006a**

# randuss

Generate stable, random `uss` objects

## Syntax

```
usys = randuss(n)
```

```
usys = randuss(n,p)
```

```
usys = randuss(n,p,m)
```

```
usys = randuss(n,p,m,Ts)
```

```
usys = randuss
```

## Description

`usys = randuss(n)` generates an  $n$ th order single-input/single-output uncertain continuous-time system. `randuss` randomly selects from uncertain atoms of type `'ureal'`, `'ultidyn'`, and `'ucomplex'`.

`usys = randuss(n,p)` generates an  $n$ th order single-input uncertain continuous-time system with  $p$  outputs.

`usys = randuss(n,p,m)` generates an  $n$ th order uncertain continuous-time system with  $p$  outputs and  $m$  inputs.

`usys = randuss(n,p,m,Ts)` generates an  $n$ th order uncertain discrete-time system with  $p$  outputs and  $m$  inputs. The sample time is  $T_s$ .

`usys = randuss` (without arguments) results in a 1-by-1 uncertain continuous-time `uss` object with up to four uncertain objects.

In general, both `rand` and `randn` are used internally. You can control the result of `randuss` by setting seeds for both random number generators before calling the function.

## Examples

The statement creates a fifth order, continuous-time uncertain system `s1` of size 2-by-3. Note your display can differ because a random seed is used.

```
s1 = randuss(5,2,3)
USS: 5 States, 2 Outputs, 3 Inputs, Continuous System
  CTPQV: 1x1 LTI, max. gain = 2.2, 1 occurrence
  IGDHN: real, nominal = -4.03, variability =
[-3.74667 22.7816]%, 1 occurrence
  MLGCD: complex, nominal = 8.36+3.09i, +/- 7.07%, 1 occurrence
  OEDJK: complex, nominal = -0.346-0.296i, radius = 0.895,
1 occurrence
```

## See Also

`rand` | `randatom` | `randn` | `randumat` | `ucomplex` | `ultidyn`

**Introduced before R2006a**

## reduce

Simplified access to Hankel singular value based model reduction functions

### Syntax

```
GRED = reduce(G)
```

```
GRED = reduce(G,order)
```

```
[GRED,redinfo] = reduce(G,'key1','value1',...)
```

```
[GRED,redinfo] = reduce(G,order,'key1','value1',...)
```

### Description

`reduce` returns a reduced order model `GRED` of `G` and a struct array `redinfo` containing the error bound of the reduced model, Hankel singular values of the original system and some other relevant model reduction information.

An error bound is a measure of how close `GRED` is to `G` and is computed based on either *additive error*,  $\|G - GRED\|_{\infty}$ , *multiplicative error*,  $\|G^{-1}(G - GRED)\|_{\infty}$ , or *nugap error* (ref.: `ncfmr`) [1],[4],[5].

Hankel singular values of a stable system indicate the respective state energy of the system. Hence, reduced order can be directly determined by examining the system Hankel SV's. Model reduction routines, which based on Hankel singular values are grouped by their error bound types. In many cases, the additive error method `GRED=reduce(G,ORDER)` is adequate to provide a good reduced order model. But for systems with lightly damped poles and/or zeros, a multiplicative error method (namely, `GRED=reduce(G,ORDER,'ErrorType','mult')`) that minimizes the relative error between `G` and `GRED` tends to produce a better fit.

This table describes input arguments for `reduce`.

| Argument | Description  |
|----------|--|
| G        | LTI model to be reduced (without any other inputs will plot its Hankel singular values and prompt for reduced order).            |
| ORDER    | (Optional) Integer for the desired order of the reduced model, or optionally a vector packed with desired orders for batch runs. |

A batch run of a serial of different reduced order models can be generated by specifying `order = x:y`, or a vector of integers. By default, all the anti-stable part of a physical system is kept, because from control stability point of view, getting rid of unstable state(s) is dangerous to model a system.

'*MaxError*' can be specified in the same fashion as an alternative for 'ORDER' after an '*ErrorType*' is selected. In this case, reduced order will be determined when the sum of the tails of the Hankel SV's reaches the '*MaxError*'.

| Argument             | Value   | Description   |
|----------------------|---|---|
| ' <i>Algorithm</i> ' | ' <i>balance</i> '                            | Default for 'add' (balancmr)  |
|                      | ' <i>schur</i> '                              | Option for 'add' (schurmr)  |
|                      | ' <i>hankel</i> '                             | Option for 'add' (hankelmr)   |
|                      | ' <i>bst</i> '                                | Default for 'mult' (bstmr)  |
|                      | ' <i>ncf</i> '                                | Default for 'ncf' (ncfmr)   |
| ' <i>ErrorType</i> ' | ' <i>add</i> '                                | Additive error (default)  |
|                      | ' <i>mult</i> '                               | Multiplicative error at model output  |
|                      | ' <i>ncf</i> '                                | NCF nugap error   |
| ' <i>MaxError</i> '  | A real number or a vector of different errors | Reduce to achieve $H_{\infty}$ error.<br>When present, ' <i>MaxError</i> ' overrides ORDER input.   |
| ' <i>Weights</i> '   | {Wout,Win} cell array                         | Optimal 1x2 cell array of LTI weights Wout (output) and Win (input); default is both identity; used only with ' <i>ErrorType</i> ', ' <i>add</i> '. Weights must be invertible. |
| ' <i>Display</i> '   | ' <i>on</i> ' or ' <i>off</i> '               | Display Hankel singular plots (default ' <i>off</i> ').   |

| Argument | Value                         | Description  |
|----------|-------------------------------|--|
| 'Order'  | Integer, vector or cell array | Order of reduced model. Use only if not specified as 2nd argument. |

Weights on the original model input and/or output can make the model reduction algorithm focus on some frequency range of interests. But weights have to be stable, minimum phase and invertible.

This table describes output arguments.

| Argument | Description   |
|----------|---|
| GRED     | LTI reduced order model. Becomes multi-dimensional array when input is a serial of different model order array.   |
| REDINFO  | <p>A STRUCT array with 3 fields:</p> <ul style="list-style-type: none"> <li>• REDINFO.ErrorBound</li> <li>• REDINFO.StabSV</li> <li>• REDINFO.UnstabSV</li> </ul> <p>For 'hankel' algorithm, STRUCT array becomes:</p> <ul style="list-style-type: none"> <li>• REDINFO.ErrorBound</li> <li>• REDINFO.StabSV</li> <li>• REDINFO.UnstabSV</li> <li>• REDINFO.Ganticausal</li> </ul> <p>For 'ncf' option, STRUCT array becomes:</p> <ul style="list-style-type: none"> <li>• REDINFO.GL</li> <li>• REDINFO.GR</li> <li>• REDINFO.hsv</li> </ul> |

G can be stable or unstable. G and GRED can be either continuous or discrete.

A successful model reduction with a well-conditioned original model G will ensure that the reduced model GRED satisfies the infinity norm error bound.

## Examples

### Reduce Model Order

Given a continuous or discrete, stable or unstable system,  $G$ , create a set of reduced-order models based on your selections.

```
rng(1234, 'twister'); % For reproducibility
G = rss(30,5,4);
```

If you call `reduce` without specifying an order for the reduced model, the software displays a Hankel singular-value plot and prompts you to select an order.

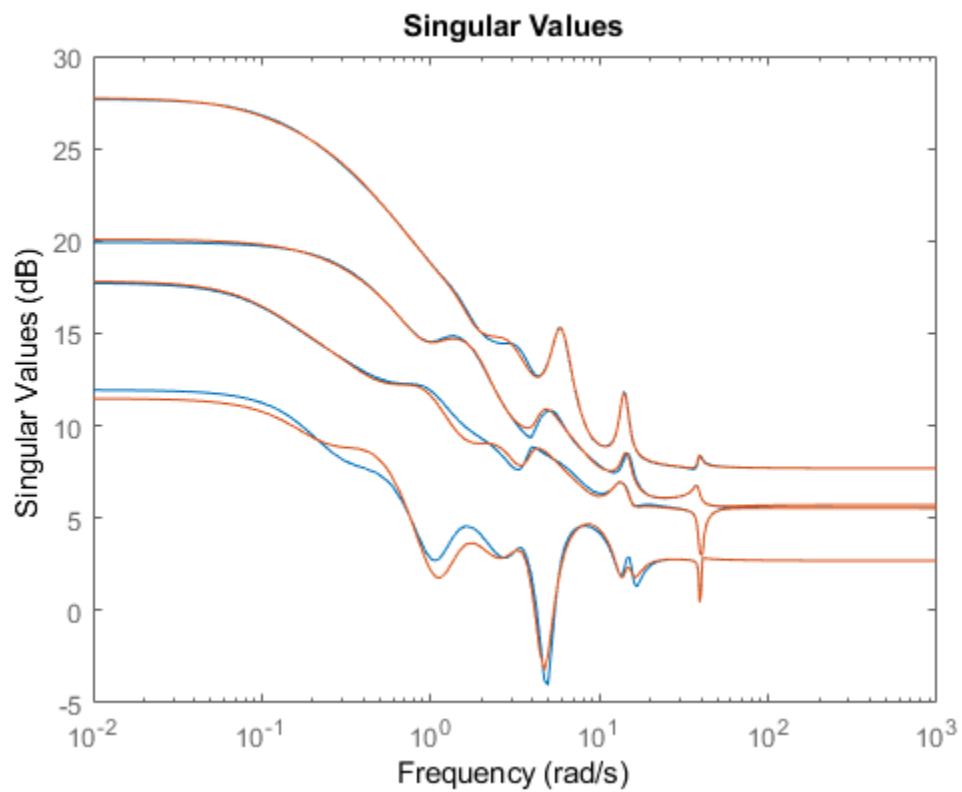
If you specify a reduced-model order, `reduce` defaults to the `balancmr` algorithm for model reduction.

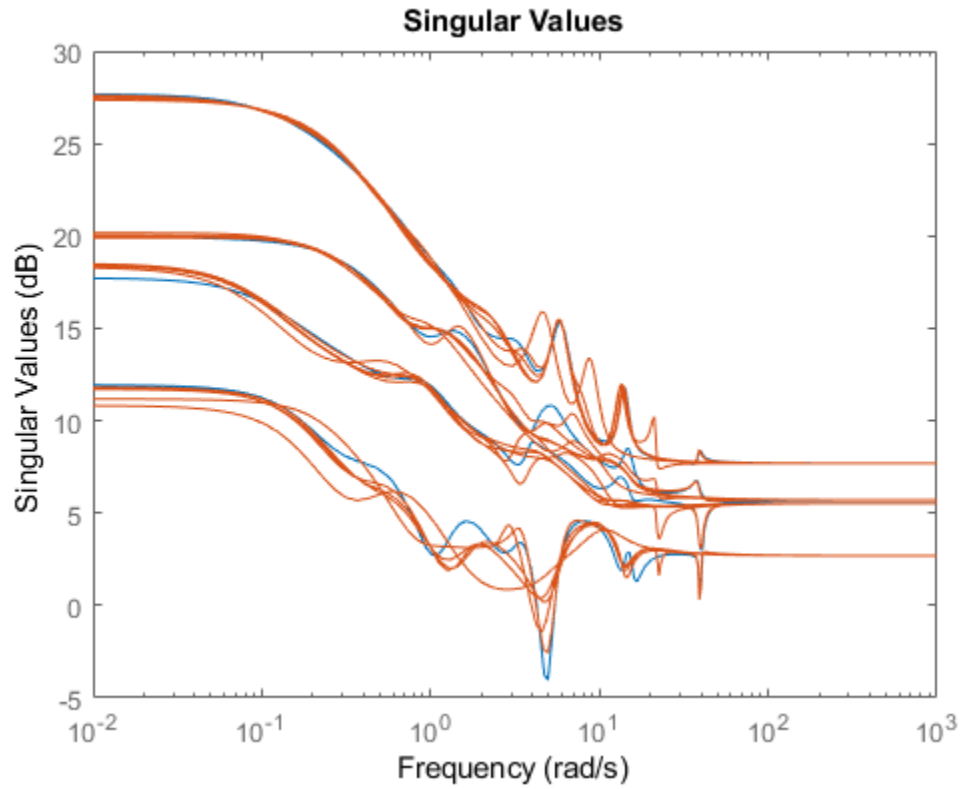
```
[g1,redinfo1] = reduce(G,20);
```

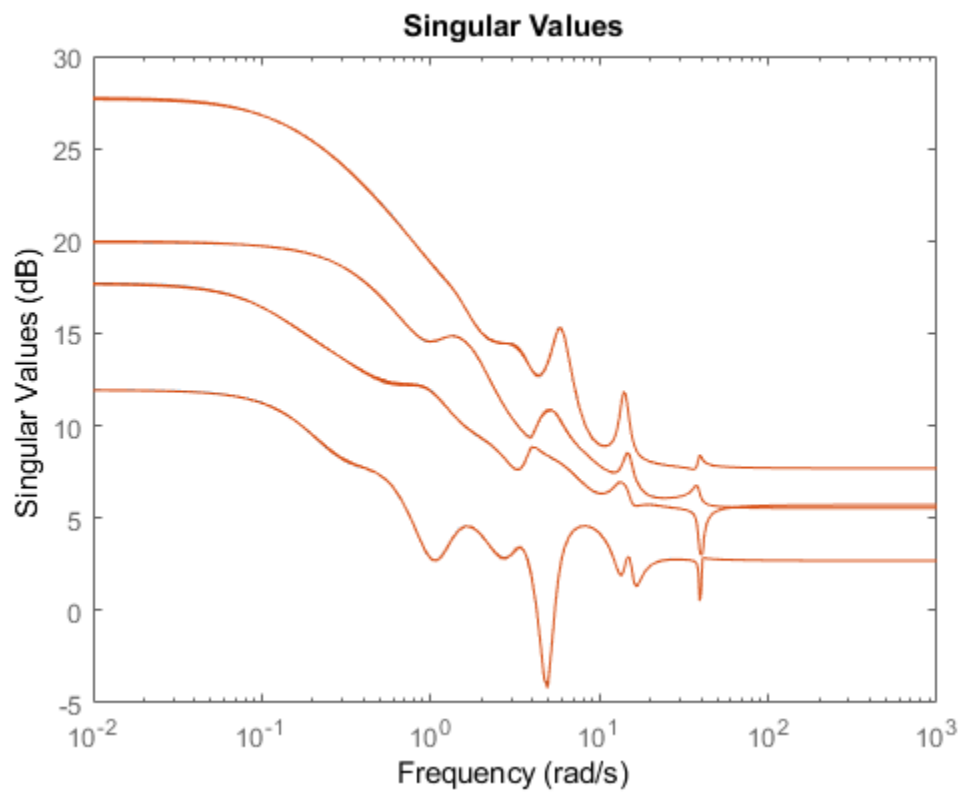
Specify other algorithms using the `Algorithm` argument. Use the `ErrorType` argument to specify whether the algorithm uses multiplicative or additive error, and the maximum permissible error in the reduced model.

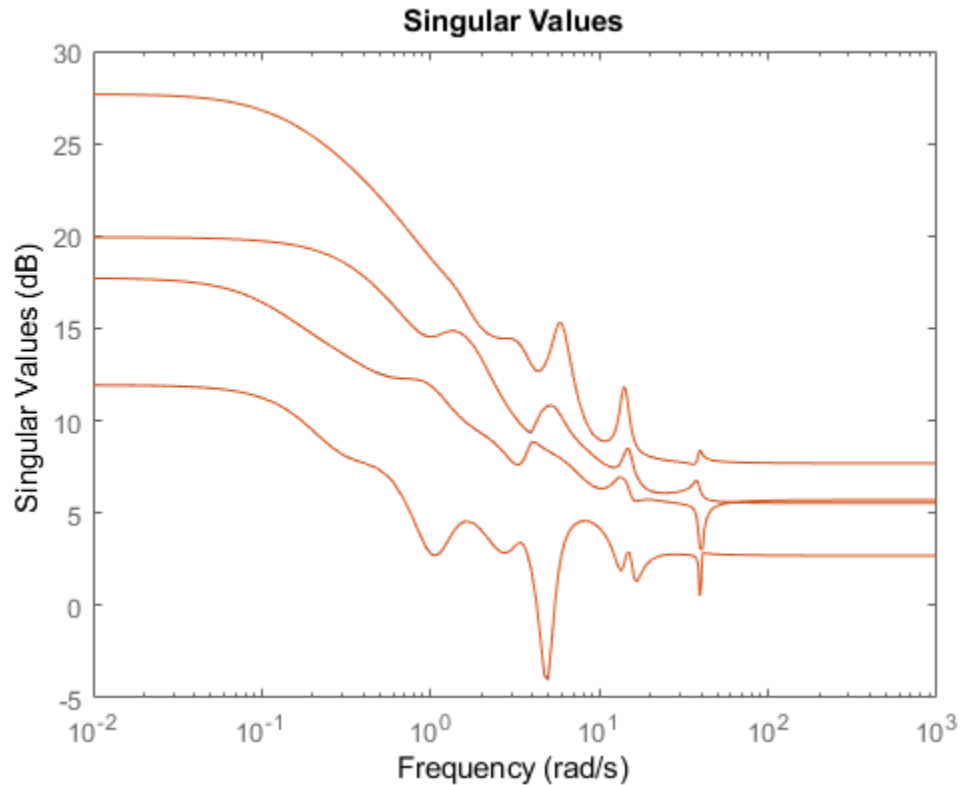
```
[g2,redinfo2] = reduce(G,[10:2:18], 'Algorithm','schur');
[g3,redinfo3] = reduce(G,'ErrorType','mult','MaxError',[0.01 0.05]);
[g4,redinfo4] = reduce(G,'ErrorType','add','Algorithm','hankel','MaxError',[0.01]);
for i = 1:4
    figure(i); eval(['sigma(G,g' num2str(i) ');']);
end
```











## References

- [1] K. Glover, "All Optimal Hankel Norm Approximation of Linear Multivariable Systems, and Their  $L_\infty$ - error Bounds," *Int. J. Control*, vol. 39, no. 6, pp. 1145-1193, 1984.
- [2] M. G. Safonov and R. Y. Chiang, "A Schur Method for Balanced Model Reduction," *IEEE Trans. on Automat. Contr.*, vol. AC-2, no. 7, July 1989, pp. 729-733.
- [3] M. G. Safonov, R. Y. Chiang and D. J. N. Limebeer, "Optimal Hankel Model Reduction for Nonminimal Systems," *IEEE Trans. on Automat. Contr.*, vol. 35, No. 4, April, 1990, pp. 496-502.

[4] M. G. Safonov and R. Y. Chiang, "Model Reduction for Robust Control: A Schur Relative-Error Method," *International Journal of Adaptive Control and Signal Processing*, vol. 2, pp. 259-272, 1988.

[5] K. Zhou, "Frequency weighted L[[BULLET]] error bounds," *Syst. Contr. Lett.*, Vol. 21, 115-125, 1993.

## **See Also**

balancmr | bstmr | hankelmr | hankelsv | ncfmr | schurmr

**Introduced before R2006a**

## repmat

Replicate and tile array

### Syntax

`B = repmat(A,M,N)`

### Description

`B = repmat(A,M,N)` creates a large matrix `B` consisting of an `M`-by-`N` tiling of copies of `A`.

`B = repmat(A, [M N])` accomplishes the same result as `repmat(A,M,N)`.

`B = repmat(A, [M N P ...])` tiles the array `A` to produce an `M`-by-`N`-by-`P`-by-... block array. `A` can be `N`-D.

`repmat(A,M,N)` for scalar `A` is commonly used to produce an `M`-by-`N` matrix filled with values of `A`.

### Examples

Simple examples of using `repmat` are

```
repmat(randumat(2,2),2,3)
repmat(ureal('A',6),[4 2])
```

**Introduced before R2006a**

# robgain

Robust performance of uncertain system

## Syntax

```
[perfmarg,wcu] = robgain(usys,gamma)
[perfmarg,wcu] = robgain(usys,gamma,w)
[perfmarg,wcu] = robgain(____,opts)
[perfmarg,wcu,info] = robgain(____)
```

## Description

`[perfmarg,wcu] = robgain(usys,gamma)` calculates the robust performance margin for an uncertain system and the performance level `gamma`. The performance of `usys` is measured by its peak gain or peak singular value (see “Robustness and Worst-Case Analysis”). The performance margin is relative to the uncertainty level specified in `usys`. A margin greater than 1 means that the gain of `usys` remains below `gamma` for all values of the uncertainty modeled in `usys`. A margin less than 1 means that at some frequency, the gain of `usys` exceeds `gamma` for some values of the uncertain elements within their specified ranges. For example, a margin of 0.5 implies the following:

- The gain of `usys` remains below `gamma` as long as the uncertain element values stay within 0.5 normalized units of their nominal values.
- There is a perturbation of size 0.5 normalized units that drives the peak gain to the level `gamma`.

The structure `perfmarg` contains upper and lower bounds on the actual performance margin and the critical frequency at which the margin upper bound is smallest. The structure `wcu` contains the uncertain-element values that drive the peak gain to the level `gamma`.

`[perfmarg,wcu] = robgain(usys,gamma,w)` assesses the robust performance margin for the frequencies specified by `w`.

- If `w` is a cell array of the form `{wmin,wmax}`, then `robgain` restricts the performance margin computation to the interval between `wmin` and `wmax`.

- If `w` is a vector of frequencies, then `robgain` computes the performance margin at the specified frequencies only.

`[perfmarg,wcu] = robgain( ____,opts)` specifies additional options for the computation. Use `robOptions` to create `opts`. You can use this syntax with any of the previous input-argument combinations.

`[perfmarg,wcu,info] = robgain( ____)` returns a structure with additional information about the performance margins and the perturbations that drive the gain to gamma. See `info` for details about this structure. You can use this syntax with any of the previous input-argument combinations.

## Examples

### Robust Performance of Closed-Loop System

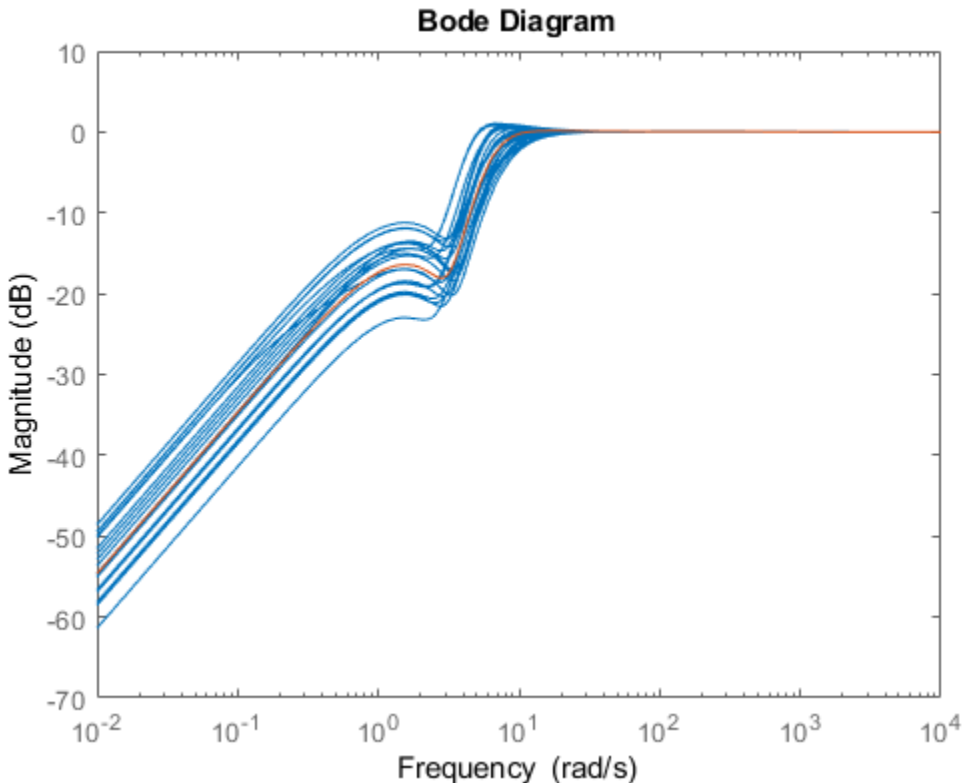
Consider a control system whose plant contains both parametric uncertainty and dynamic uncertainty. Create a model of the plant using uncertain elements.

```
k = ureal('k',10,'Percent',40);  
delta = ultidyn('delta',[1 1]);  
G = tf(18,[1 1.8 k]) * (1 + 0.5*delta);
```

Create a model of the controller, and build the closed-loop sensitivity function, `S`. The sensitivity measures the closed-loop response at the plant output to a disturbance at the plant input.

```
C = pid(2.3,3,0.38,0.001);  
S = feedback(1,G*C);  
bodemag(S,S.NominalValue)
```





The peak gain of the nominal response is very nearly 1, but some of the sampled systems within the uncertainty range exceed that level. Suppose that you can tolerate some ringdown in the response but do not want the peak gain to exceed 1.5. Use `robgain` to find out how much uncertainty the system can have while the peak gain remains below 1.5.

```
[perfmarg,wcu] = robgain(S,1.5);
perfmarg
```

```
perfmarg = struct with fields:
    LowerBound: 0.7821
    UpperBound: 0.7837
    CriticalFrequency: 7.8688
```

The `LowerBound` and `UpperBound` fields of `perfmarg` show that the robust performance margin is around 0.78. This result means that there is a perturbation of only about 78% of the uncertainty specified in `S` with peak gain exceeding 1.5.

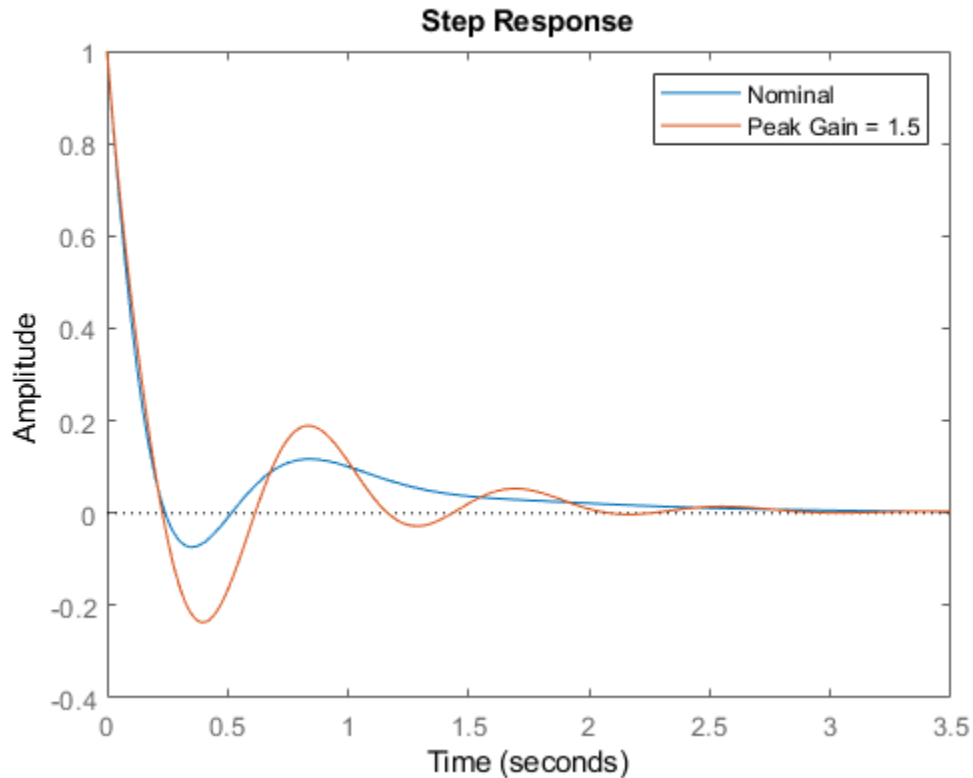
The output `wcu` is a structure that contains the corresponding perturbations to `k` and `delta`. Verify that the values in `wcu` cause `Smax` to achieve the gain level of 1.5 by substituting them into `S`.

```
Smax = usubs(S,wcu);  
getPeakGain(Smax,1e-6)
```

```
ans = 1.5000
```

Examine the disturbance rejection of the system with these values.

```
step(S.NominalValue,Smax)  
legend('Nominal','Peak Gain = 1.5')
```



The `CriticalFrequency` field of `perfmarg` contains the frequency at which the peak gain reaches 1.5.

### Sensitivity of Performance to Uncertain Elements

Examine the relative sensitivity of the robust performance margin to the uncertain elements of the system. Consider a model of a control system containing uncertain elements.

```
k = ureal('k',10,'Percent',50);
delta = ultidyn('delta',[1 1]);
```

```
G = tf(18,[1 1.8 k]) * (1 + 0.15*delta);
C = pid(2.3,3,0.38,0.001);
S = feedback(1,G*C);
```

Create an options set for `robgain` that enables the sensitivity calculation.

```
opts = robOptions('Sensitivity','On');
```

Calculate the robust performance margin of the system relative to a peak gain of 1.5, specifying the `info` output to access additional information about the calculation.

```
[perfmarg,wcu,info] = robgain(S,1.5,opts);
```

Examine the `Sensitivity` field of `info`.

```
info.Sensitivity
```

```
ans = struct with fields:
    delta: 75
    k: 28
```

The values in this field indicate how much a change in the normalized perturbation on each element affects the performance margin. For example, the sensitivity for `k` is 28. This value means that a given change  $dk$  in the normalized uncertainty range of `k` causes a change of about 28% of that, or  $0.28*dk$ , in the performance margin. The margin in this case is more sensitive to `delta`, for which the margin changes by about 75% of the change in the normalized uncertainty range.

### Robust Performance Margin as a Function of Frequency

Consider a model of a control system containing uncertain elements.

```
k = ureal('k',10,'Percent',40);
delta = ultidyn('delta',[1 1]);
G = tf(18,[1 1.8 k]) * (1 + 0.5*delta);
C = pid(2.3,3,0.38,0.001);
S = feedback(1,G*C);
```

By default, `robgain` computes only the weakest performance margin over all frequencies. To see how the margin varies with frequency, use the `'VaryFrequency'`

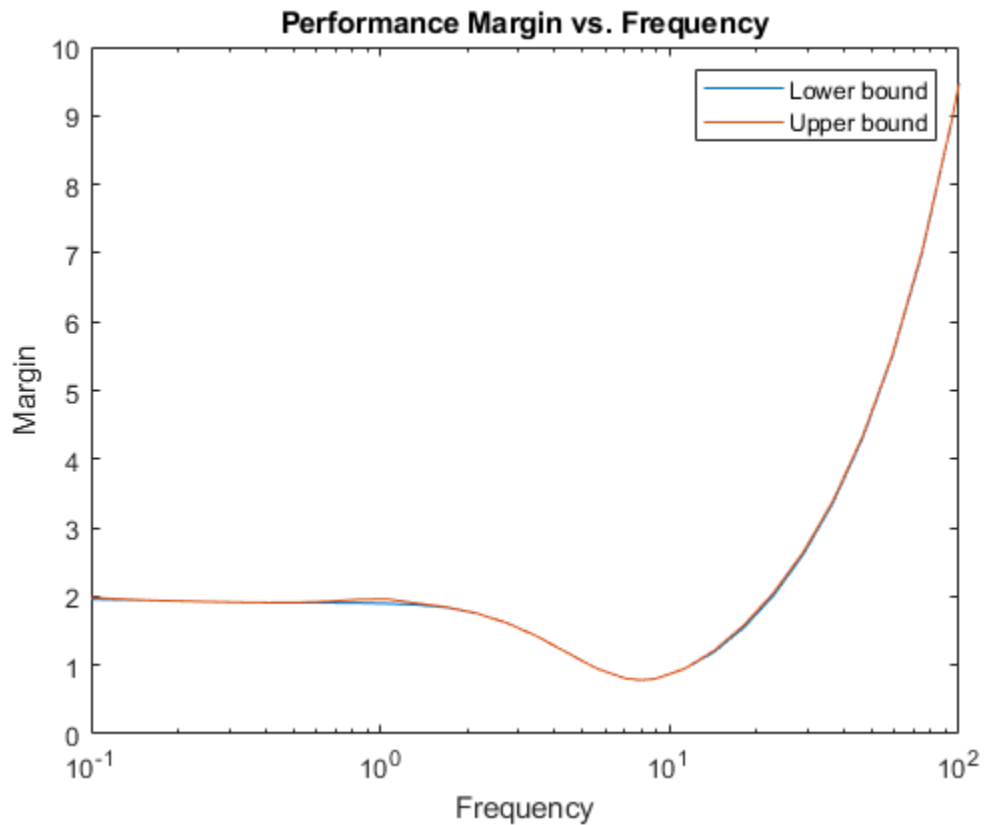
option of `robOptions`. For example, compute the performance margin of the system for a performance level of 1.5, at frequency points between 0.1 and 100 rad/s.

```
opts = robOptions('VaryFrequency','on');  
[perfmarg,wcu,info] = robgain(S,1.5,{0.1,100},opts);  
info
```

```
info = struct with fields:  
    Model: 1  
    Frequency: [32x1 double]  
    Bounds: [32x2 double]  
    WorstPerturbation: [32x1 struct]  
    Sensitivity: [1x1 struct]
```

`robgain` returns the vector of frequencies in the `info` output, in the `Frequencies` field. `info.Bounds` contains the upper and lower bounds on the performance margin at each frequency. Use these values to plot the frequency dependence of the performance margin.

```
semilogx(info.Frequency,info.Bounds)  
title('Performance Margin vs. Frequency')  
ylabel('Margin')  
xlabel('Frequency')  
legend('Lower bound','Upper bound')
```



When you use the 'VaryFrequency' option, robgain chooses frequency points automatically. The frequencies it selects are guaranteed to include the frequency at which the margin is smallest (within the specified range). Display the returned frequency values to confirm that they include the critical frequency.

```
info.Frequency
```

```
ans = 32x1
```

```
0.1000  
0.1266  
0.1604  
0.2031  
0.2572
```

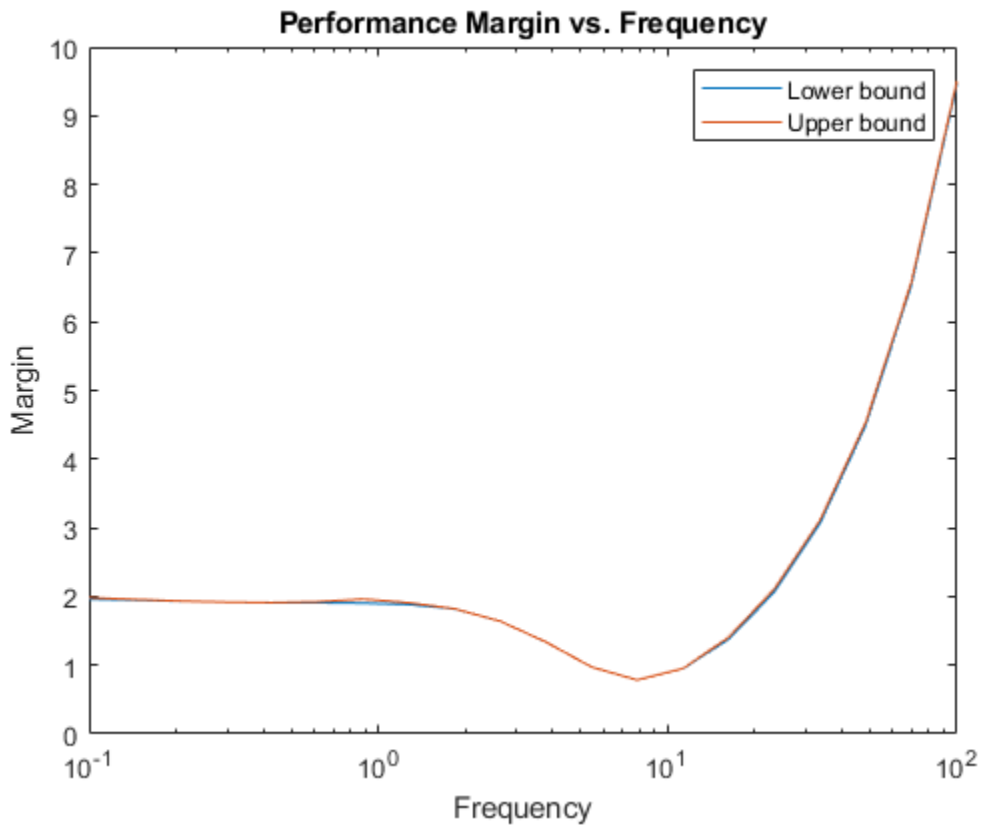
```
0.3257
0.4125
0.5223
0.6615
0.8377
:
```

```
perfmarg.CriticalFrequency
```

```
ans = 7.9642
```

Alternatively, instead of using 'VaryFrequency', you can specify particular frequencies at which to compute the robust performance margins. `info.Bounds` contains the margins at all specified frequencies. However, these results are not guaranteed to include the weakest margin, which might fall between specified frequency points.

```
w = logspace(-1,2,20);
[perfmarg,wcu,info] = robgain(S,1.5,w);
semilogx(w,info.Bounds)
title('Performance Margin vs. Frequency')
ylabel('Margin')
xlabel('Frequency')
legend('Lower bound','Upper bound')
```



- "Robust Stability, Robust Performance and Mu Analysis"

## Input Arguments

### **usys** — Dynamic system with uncertainty

uss | ufrd | genss | genfrd

Dynamic system with uncertainty, specified as a `uss`, `ufrd`, `genss`, or `genfrd` model that contains uncertain elements. For `genss` or `genfrd` models, `robgain` uses the current value of any tunable blocks and folds them into the known (not uncertain) part of the model.



`usys` can also be an array of uncertain models. In that case, `robgain` returns the smallest margin across all models in the array, and the `info` output contains the index of the corresponding model.

### **gamma — Performance level**

positive scalar

Performance level, specified as a positive scalar. The performance level is the peak gain of a system or, for MIMO systems, the peak singular value ( $H_\infty$  norm). Generally, the lower this value, the better the system performance. `robgain` computes the amount of uncertainty the system can tolerate while keeping the peak gain below this level. For more information about this performance measure, see “Robustness and Worst-Case Analysis”.

### **w — Frequencies**

{`wmin`, `wmax`} | vector

Frequencies at which to compute robust performance margins, specified as the cell array `{wmin, wmax}` or as a vector of frequency values.

- If `w` is a cell array of the form `{wmin, wmax}`, then the function computes the margins at frequencies ranging between `wmin` and `wmax`.
- If `w` is a vector of frequencies, then the function computes the margins at each specified frequency. For example, use `logspace` to generate a row vector with logarithmically spaced frequency values.

Specify frequencies in units of `rad/TimeUnit`, where `TimeUnit` is the `TimeUnit` property of the model.

### **opts — Options for margin computation**

`robOptions` object

Options for computation of robust performance margins, specified as an object you create with `robOptions`. The available options include settings that let you:

- Extract frequency-dependent performance margins.
- Examine the sensitivity of the margins to each uncertain element.
- Improve the results of the performance-margin calculation by setting certain options for the underlying `mussv` calculation. In particular, setting the option '`MussvOptions`' to '`mN`' can reduce the gap between the lower bound and upper bound. `N` is the number of restarts.

For more information about all available options, see `robOptions`.

Example: `robOptions('Sensitivity','on','MussvOptions','m3')`

## Output Arguments

### **perfmarg** — Robust performance margin and critical frequency

structure

Robust performance margin and critical frequency, returned as a structure containing the following fields:

| Field             | Description  |
|-------------------|--|
| LowerBound        | Lower bound on the actual robust performance margin of the model with respect to <code>gamma</code> , returned as a scalar value. The exact margin is guaranteed to be no smaller than <code>LowerBound</code> . In other words, for all modeled uncertainty with normalized magnitude up to <code>LowerBound</code> , the system is guaranteed to have peak gain below <code>gamma</code> . |
| UpperBound        | Upper bound on the actual robust performance margin, returned as a scalar value. The exact margin is guaranteed to be no larger than <code>UpperBound</code> . In other words, there exist some uncertain-element values associated with this magnitude that drive the peak gain above <code>gamma</code> . <code>robgain</code> returns one such instance in <code>wcu</code> .             |
| CriticalFrequency | Frequency at which the performance margin is the smallest, in rad/TimeUnit, where <code>TimeUnit</code> is the <code>TimeUnit</code> property of <code>usys</code> .   |

A margin greater than 1 means that the gain of `usys` remains below `gamma` for all values of the uncertainty modeled in `usys`. A margin less than 1 means that at some frequency, the gain of `usys` exceeds `gamma` for some values of the uncertain elements within their specified ranges. For example, a margin of 0.5 implies the following:

- The gain of `usys` remains below `gamma` as long as the uncertain element values stay within 0.5 normalized units of their nominal values.

- There is a perturbation of size 0.5 normalized units that drives the peak gain above `gamma`.

Use `normalized2actual` to convert the normalized uncertainty value expressed in the performance margin to actual deviations from nominal values.

If the nominal value of `usys` has peak gain greater than `gamma`, the performance margin is 0.

If `usys` is an array of uncertain models, `perfmargin` contains the smallest margin across all models in the array. In that case, the `info` output contains the index of the corresponding model in its `Model` field.

### **wcu — Perturbations driving system gain to gamma**

structure

Smallest perturbations of uncertain elements that drive the peak gain of `usys` to the level `gamma`, returned as a structure whose fields are the names of the uncertain elements of `usys`. Each field contains the actual value of the corresponding uncertain element. For example, if `usys` includes an uncertain matrix `M` and SISO uncertain dynamics `delta`, then `wcu.M` is a numeric matrix and `wcu.delta` is a SISO state-space model.

Use `usubs(usys,wcu)` to substitute these values for the uncertain elements in `usys` and obtain the corresponding dynamic system. This system has peak gain `gamma`. Use `actual2normalized` to convert these actual uncertainty values to the normalized units in which the performance margin is expressed.

For `ureal` parameters in `usys` whose range is not centered around the nominal value, `robgain` makes the following adjustments for the purposes of its analysis:

- When the worst perturbation (the smallest perturbation achieving target gain) lies outside the range of validity of the actual-to-normalized transformation (see `getLimits`), then `robgain` sets the corresponding entry of `wcu` to the nearest valid value. In other words, if `actpert` is the worst perturbation in actual units, `robgain` sets `wcu` to the nearest value inside the interval `ActLims` returned by `getLimits`.
- When there is no perturbation causing the system to exceed the target gain, then `robgain` sets the corresponding entry of `wcu` to the nominal value of the `ureal` parameter.

### **info — Additional information about performance margins**

structure

Additional information about the performance margins, returned as a structure with the following fields:

| Field     | Description   |
|-----------|---|
| Model     | Index of the model that has the weakest performance margin, when <code>usys</code> is an array of models.   |
| Frequency | <p>Frequency points at which <code>robgain</code> returns the robust performance margin, returned as a vector.</p> <ul style="list-style-type: none"> <li>• If the 'VaryFrequency' option of <code>robOptions</code> is 'off', then <code>info.Frequency</code> is the critical frequency, the frequency at which the smallest margin occurs. If the smallest lower bound and the smallest upper bound on the performance margin occur at different frequencies, then <code>info.Frequency</code> is a vector containing these two frequencies.</li> <li>• If the 'VaryFrequency' option of <code>robOptions</code> is 'on', then <code>info.Frequency</code> contains the frequencies selected by <code>robgain</code>. These frequencies are guaranteed to include the frequency at which the performance margin is smallest.</li> <li>• If you specify a vector of frequencies <code>w</code> at which to compute the performance margins, then <code>info.Frequency = w</code>. When you specify a frequency vector, these frequencies are not guaranteed to include the frequency at which the margin is smallest.</li> </ul> <p>The 'VaryFrequency' option is meaningful only for <code>uss</code> and <code>genss</code> models. <code>robgain</code> ignores the option for <code>ufrd</code> and <code>genfrd</code> models.</p> |

| Field             | Description   |
|-------------------|---|
| Bounds            | Lower and upper bounds on the actual robust performance margin of the model, returned as an array. <code>info.Bounds(:,1)</code> contains the lower bound at each corresponding frequency in <code>info.Frequency</code> , and <code>info.Bounds(:,2)</code> contains the corresponding upper bounds.   |
| WorstPerturbation | Smallest perturbations at each frequency point in <code>info.Frequency</code> , returned as a structure array. The fields of <code>info.WorstPerturbation</code> are the names of the uncertain elements in <code>usys</code> . Each field contains the value of the corresponding element that drives the peak gain above <code>gamma</code> at each frequency. For example, if <code>usys</code> includes an uncertain parameter <code>p</code> and SISO uncertain dynamics <code>delta</code> , then <code>info.WorstPerturbation.p</code> is a collection of numeric values and <code>info.WorstPerturbation.delta</code> is a collection of SISO state-space models.   |
| Sensitivity       | Sensitivity of the performance margin to each uncertain element, returned as a structure when the 'Sensitivity' option of <code>robOptions</code> is 'on'. The fields of <code>info.Sensitivity</code> are the names of the uncertain elements in <code>usys</code> . Each field contains a percentage that measures how much the uncertainty in the corresponding element affects the performance margin. For example, if <code>info.Sensitivity.p</code> is 50, then a given fractional change in the uncertainty range of <code>p</code> causes half as much fractional change in the performance margin.<br><br>If the 'Sensitivity' option of <code>robOptions</code> is off (the default setting), then <code>info.Sensitivity</code> is NaN. |

## Algorithms

Computing the robustness margin at a particular frequency is equivalent to computing the structured singular value,  $\mu$ , for some appropriate block structure ( $\mu$ -analysis).

For `uss` and `genss` models, `robgain(usys)` and `robgain(usys, {wmin, wmax})` use an algorithm that finds the smallest margin across frequency. This algorithm does not rely on frequency gridding and is not adversely affected by discontinuities of the  $\mu$  structured singular value. See “Getting Reliable Estimates of Robustness Margins” for more information.

For `ufrd` and `genfrd` models, `robgain` computes the  $\mu$  lower and upper bounds at each frequency point. This computation offers no guarantee between frequency points and can be inaccurate if there are discontinuities or sharp peaks in  $\mu$ . The syntax `robgain(uss, w)`, where `w` is a vector of frequency points, is the same as `robgain(ufrd(uss, w))` and also relies on frequency gridding to compute the margin.

In general, the algorithm for state-space models is faster and safer than the frequency-gridding approach. In some cases, however, the state-space algorithm requires many  $\mu$  calculations. In those cases, specifying a frequency grid as a vector `w` can be faster, provided that the robustness margin varies smoothly with frequency. Such smooth variation is typical for systems with dynamic uncertainty.

## See Also

`robOptions` | `robstab` | `wcgain`

## Topics

“Robust Stability, Robust Performance and Mu Analysis”

“Robustness and Worst-Case Analysis”

**Introduced in R2016b**

# robOptions

Option set for robustness analysis

## Syntax

```
opts = robOptions
opts = robOptions(Name,Value,...)
```

## Description

`opts = robOptions` returns the default option set for robustness analysis commands `robstab` and `robgain`, and for `dksynperf`.

`opts = robOptions(Name,Value,...)` creates an option set with the options specified by one or more `Name,Value` pair arguments.

## Examples

### Options for Robustness Margin Calculation

Create an options set for a `robstab` or `robgain` calculation that displays the progress of the underlying `mussv` calculation. Also, turn on the element-by-element sensitivity calculation.

```
opts = robOptions('Display','on','Sensitivity','on');
```

Alternatively, create a default option set, and use dot notation to set the values of particular options.

```
opts = robOptions;
opts.Display = 'on';
opts.Sensitivity = 'on';
```

Use `opts` as an input argument to `robstab` or `robgain`.

- “Robust Stability, Robust Performance and Mu Analysis”

## Input Arguments

### Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes ( `' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

Example: `'Display','on','Sensitivity','on'`

#### **Display — Display progress of computation and summary report**

`'off'` (default) | `'on'`

Display progress and summary of the robustness computation, specified as the comma-separated pair consisting of `'Display'` and one of these values:

- `'off'` — Do not display progress and report.
- `'on'` — Display progress and report. When you use this option, a progress indicator and summary of results is displayed in the command window, similar to the following.

```
points completed (of 28) ... 28
System is robustly stable for the modeled uncertainty.
-- It can tolerate up to 116% of the modeled uncertainty.
-- There is a destabilizing perturbation amounting to 117% of the modeled uncertainty.
-- This perturbation causes an instability at the frequency 5.9 rad/seconds.
```

This setting overrides the silent (`'s'`) option in the `MussvOptions` option.

#### **VaryFrequency — Compute robustness margin as function of frequency**

`'off'` (default) | `'on'`

Return robustness margin as a function of frequency, specified as the comma-separated pair consisting of `'VaryFrequency'` and one of these values:

- `'off'` — Only return margins at frequencies where robustness is weakest.
- `'on'` — Compute robustness margins over a frequency grid suitable for plotting. The frequency grid is chosen automatically based on system dynamics. This calculation is



done in addition to identifying the critical frequency where the margin is weakest. Access the frequency values and corresponding margins in the `info` output of `robstab` and `robgain`.

This option is ignored for `ufrd` and `genfrd` models.

### **Sensitivity** — Calculate sensitivity of robustness margin

'off' (default) | 'on'

Calculate the sensitivity of the robustness margin to each uncertain element in the model, specified as the comma-separated pair consisting of 'Sensitivity' and either 'off' or 'on'.

Each uncertain element contributes to the overall stability margin in a coupled manner. Set this option to 'on' to estimate the sensitivity of the margin to each element. This element-by-element sensitivity provides an indication of which elements are most problematic for robustness. Access the sensitivity estimates in the `info` output of `robstab` and `robgain`.

### **SensitivityPercent** — Percentage variation of uncertainty for computing sensitivity

25 (default) | positive scalar value

Percentage variation of uncertainty level for computing sensitivity, specified as the comma-separated pair consisting of 'SensitivityPercent' and a positive scalar value. The sensitivity to a particular uncertain element is estimated using a finite difference calculation. This calculation increases the (normalized) amount of uncertainty on this element by some percentage, computes the resulting robustness, and computes the ratio of percent variations. This option specifies the percentage increase in uncertainty level applied to each element. The default value is 25%.

### **MussvOptions** — Options for mussv calculation

' ' (default) | character vector

Options for the underlying `mussv` calculation that `robstab` and `robgain` perform, specified as the comma-separated pair consisting of 'MussvOptions' and a character vector such as 'sm3' or 'ad'.

Some `MussvOptions` values that are especially useful for improving robustness-margin calculations include:

- 'a' — Force the use of LMI optimization to compute the  $\mu$  upper bound, which yields better results in general but can be expensive when some `ureal` elements are repeated multiple times.
- 'mN' — Use multiple restarts when computing the  $\mu$  lower bound, which corresponds to the upper bound for robustness margins. This option can reduce the gap between the lower bound and upper bound on the robustness margins. N is the number of restarts. For example, setting 'MussvOptions' to 'm3' causes three restarts.

See `mussv` for the remaining available options and corresponding characters. The default, '', uses the default options for `mussv`.

## Output Arguments

### **opts** — Options for robustness commands

`robOptions` object

Options for robustness commands `robstab`, `robgain`, and `dksynperf`, returned as a `robOptions` object. Use the options as an input argument to `robstab` or `robgain`. For example:

```
[stabmarg,wcu,info] = robstab(usys,opts)
```

## See Also

`dksynperf` | `robgain` | `robstab`

## Topics

“Robust Stability, Robust Performance and Mu Analysis”

“Robustness and Worst-Case Analysis”

**Introduced in R2016b**

# robstab

Robust stability of uncertain system

## Syntax

```
[stabmarg,wcu] = robstab(usys)
[stabmarg,wcu] = robstab(usys,w)
[stabmarg,wcu] = robstab(____,opts)
[stabmarg,wcu,info] = robstab(____)
```

## Description

`[stabmarg,wcu] = robstab(usys)` calculates the robust stability margin for an uncertain system. This stability margin is relative to the uncertainty level specified in `usys`. A robust stability margin greater than 1 means that the system is stable for all values of its modeled uncertainty. A robust stability margin less than 1 means that the system becomes unstable for some values of the uncertain elements within their specified ranges. For example, a margin of 0.5 implies the following:

- `usys` remains stable as long as the uncertain element values stay within 0.5 normalized units of their nominal values.
- There is a destabilizing perturbation of size 0.5 normalized units.

The structure `stabmarg` contains upper and lower bounds on the actual stability margin and the critical frequency at which the stability margin is smallest. The structure `wcu` contains the destabilizing values of the uncertain elements.

`[stabmarg,wcu] = robstab(usys,w)` restricts the robust stability margin computation to the frequencies specified by `w`.

- If `w` is a cell array of the form `{wmin,wmax}`, then `robstab` restricts the stability margin computation to the interval between `wmin` and `wmax`.
- If `w` is a vector of frequencies, then `robstab` computes the robust stability margin at the specified frequencies only.

`[stabmarg,wcu] = robstab( ____,opts)` specifies additional options for the computation. Use `robOptions` to create `opts`. You can use this syntax with any of the previous input-argument combinations.

`[stabmarg,wcu,info] = robstab( ____)` returns a structure with additional information about the stability margins and destabilizing perturbations. See `info` for details about this structure. You can use this syntax with any of the previous input-argument combinations.

## Examples

### Robust Stability Margin of Closed-Loop System

Consider a control system whose plant contains both parametric uncertainty and dynamic uncertainty. Create a model of the plant using uncertain elements.

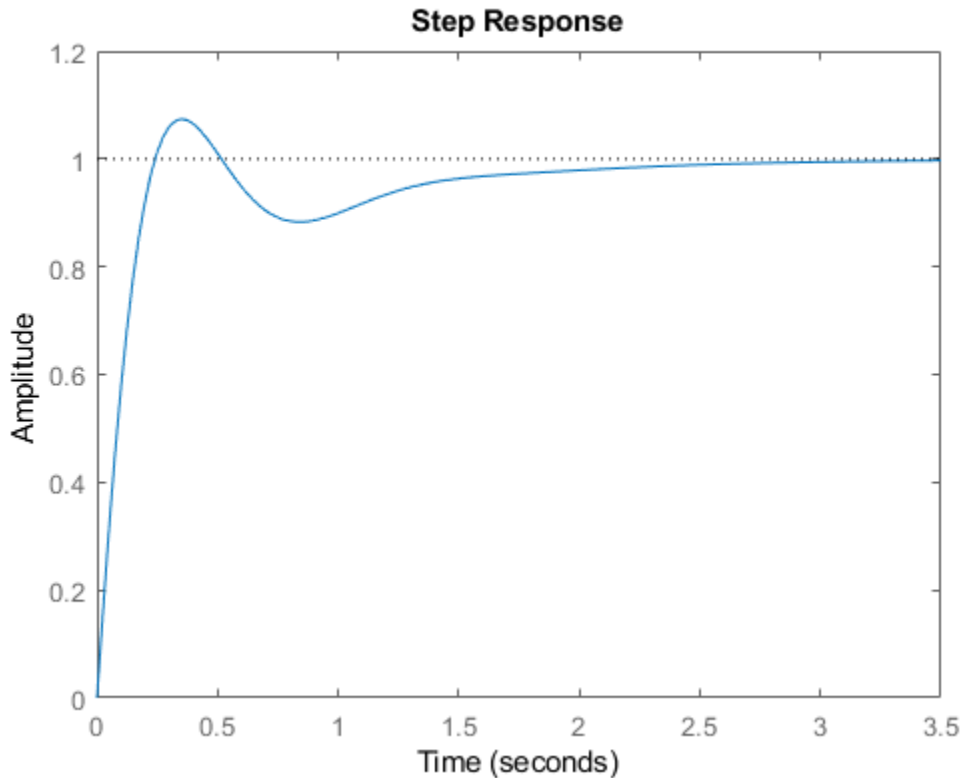
```
k = ureal('k',10,'Percent',40);  
delta = ultidyn('delta',[1 1]);  
G = tf(18,[1 1.8 k]) * (1 + 0.5*delta);
```

Create a model of the controller, and build the closed-loop transfer function.

```
C = pid(2.3,3,0.38,0.001);  
CL = feedback(G*C,1);
```

A step response plot shows that the closed-loop system is nominally stable.

```
step(CL.NominalValue)
```



Examine the robust stability of the closed-loop system.

```
[stabmarg,wcu] = robstab(CL);  
stabmarg  
  
stabmarg = struct with fields:  
    LowerBound: 1.5960  
    UpperBound: 1.5993  
    CriticalFrequency: 4.8947
```

The `LowerBound` and `UpperBound` fields of `stabmarg` show the robust stability margin of the closed-loop system is around 1.6. This result means that the system can withstand

about 60% more uncertainty than is specified in the uncertain elements without going unstable.

The output `wcu` is a structure that contains the smallest perturbation to `k` and `delta` that make the system unstable. Confirm the instability by substituting these values into the closed-loop model and examining the pole locations.

```
CLunst = usubs(CL,wcu);  
pole(CLunst)
```

```
ans = 8×1 complex  
102 ×
```

```
-9.9314 + 0.0000i  
-0.1035 + 0.1013i  
-0.1035 - 0.1013i  
-0.0000 + 0.0489i  
-0.0000 - 0.0489i  
-0.0116 + 0.0000i  
-0.0216 + 0.0000i  
-0.0403 + 0.0000i
```

The resulting system has an undamped pair of complex poles with natural frequency 4.89, which renders it unstable. The `CriticalFrequency` field of `stabmargin` contains the same value, which is the frequency at which the CL is closest to instability.

## Sensitivity to Uncertain Elements

Examine the relative sensitivity of the robust stability margin to the uncertain elements of the system. Consider a model of a control system containing uncertain elements.

```
k = ureal('k',10,'Percent',40);  
delta = ultidyn('delta',[1 1]);  
G = tf(18,[1 1.8 k]) * (1 + 0.25*delta);  
C = pid(2.3,3,0.38,0.001);  
CL = feedback(G*C,1);
```

Create an options set for `robstab` that enables the sensitivity calculation.

```
opts = robOptions('Sensitivity','On');
```

Calculate the robust stability margin, specifying the `info` output to access additional information about the calculation.

```
[stabmarg,wcu,info] = robstab(CL,opts);
```

Examine the `Sensitivity` field of `info`.

```
info.Sensitivity
```

```
ans = struct with fields:
    delta: 81
    k: 21
```

The values in this field indicate how much a change in the normalized perturbation on each element affects the stability margin. For example, the sensitivity for `k` is 21. This value means that a given change  $dk$  in the normalized uncertainty range of `k` causes a change of about 21% percent of that, or  $0.21*dk$ , in the stability margin. The margin in this case is much more sensitive to `delta`, for which the margin changes by about 81% of the change in the normalized uncertainty range.

## Robust Stability Margin as a Function of Frequency

Consider a model of a control system containing uncertain elements.

```
k = ureal('k',10,'Percent',40);
delta = ultidyn('delta',[1 1]);
G = tf(18,[1 1.8 k]) * (1 + 0.5*delta);
C = pid(2.3,3,0.38,0.001);
CL = feedback(G*C,1);
```

By default, `robstab` computes only the weakest stability margin over all frequencies. To see how the stability margin varies with frequency, use the `'VaryFrequency'` option of `robOptions`. For example, compute the stability margin of the system at frequency points between 0.1 and 10 rad/s.

```
opts = robOptions('VaryFrequency','on');
[stabmarg,wcu,info] = robstab(CL,{0.1,10},opts);
info
```

```
info =
```

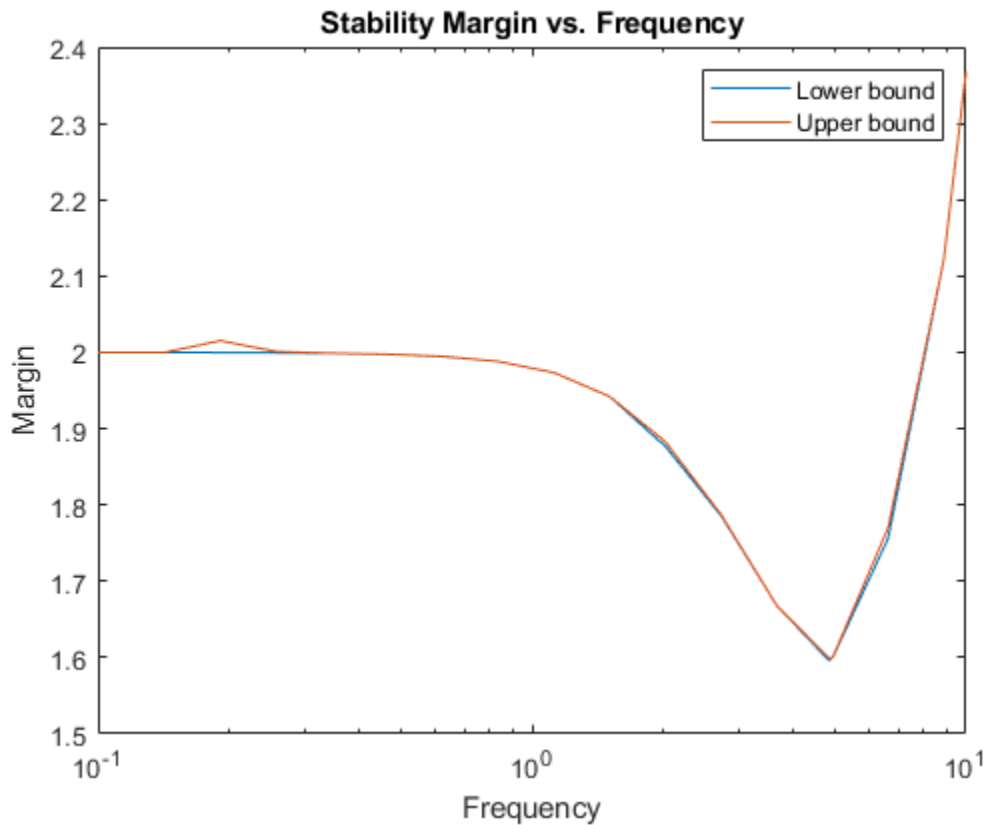
struct with fields:

```
    Model: 1
    Frequency: [19x1 double]
    Bounds: [19x2 double]
    WorstPerturbation: [19x1 struct]
    Sensitivity: [1x1 struct]
```

`robstab` returns the vector of frequencies in the `info` output, in the `Frequencies` field. `info.Bounds` contains the upper and lower bounds on the stability margin at each frequency. Use these values to plot the frequency dependence of the stability margin.

```
semilogx(info.Frequency,info.Bounds)
title('Stability Margin vs. Frequency')
ylabel('Margin')
xlabel('Frequency')
legend('Lower bound','Upper bound')
```





When you use the 'VaryFrequency' option, robstab chooses frequency points automatically. The frequencies it selects are guaranteed to include the frequency at which the stability margin is weakest (within the specified range). Display the returned frequency values to confirm that they include the critical frequency.

```
info.Frequency
stabmarg.CriticalFrequency
```

```
ans =
```

```
0.1000
0.1061
0.1425
```

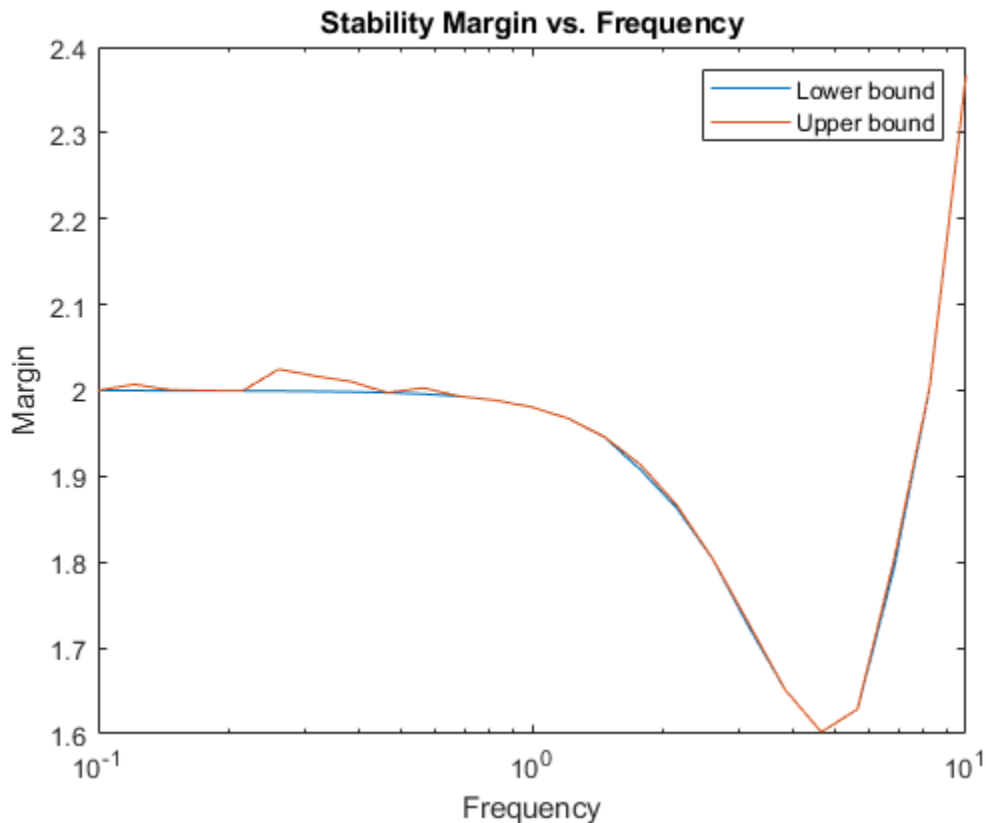
```
0.1914
0.2572
0.3455
0.4642
0.6236
0.8377
1.1253
1.5118
2.0309
2.7283
3.6652
4.8358
4.9239
6.6147
8.8862
10.0000
```

```
ans =
```

```
4.8358
```

Alternatively, instead of using 'VaryFrequency', you can specify particular frequencies at which to compute the robust stability margins. `info.Bounds` contains the margins at all specified frequencies. However, these results are not guaranteed to include the weakest margin, which might fall between specified frequency points.

```
w = logspace(-1,1,25);
[stabmarg,wcu,info] = robstab(CL,w);
semilogx(w,info.Bounds)
title('Stability Margin vs. Frequency')
ylabel('Margin')
xlabel('Frequency')
legend('Lower bound', 'Upper bound')
```



- “Robust Stability and Worst-Case Gain of Uncertain System”
- “Robust Stability, Robust Performance and Mu Analysis”

## Input Arguments

### **usys** — Dynamic system with uncertainty

uss | ufrd | genss | genfrd

Dynamic system with uncertainty, specified as a `uss`, `ufrd`, `genss`, or `genfrd` model that contains uncertain elements. For `genss` or `genfrd` models, `robstab` uses the current value of any tunable blocks and folds them into the known (not uncertain) part of the model.

For frequency-response models, `ufrd` or `genfrd`, `robstab` assumes that the system is nominally stable.

`usys` can also be an array of uncertain models. In that case, `robstab` returns the smallest margin across all models in the array, and the `info` output contains the index of the corresponding model.

## **w — Frequencies**

`{wmin,wmax} | vector`

Frequencies at which to compute robust stability margins, specified as the cell array `{wmin,wmax}` or as a vector of frequency values.

- If `w` is a cell array of the form `{wmin,wmax}`, then the function computes the margins at frequencies ranging between `wmin` and `wmax`.
- If `w` is a vector of frequencies, then the function computes the margins at each specified frequency. For example, use `logspace` to generate a row vector with logarithmically spaced frequency values.

For `uss` and `genss` models, when `w` is a vector, `robstab(usys,w)` is equivalent to `robstab(ufrd(usys,w))`. Therefore, `usys` must be nominally stable.

Specify frequencies in units of `rad/TimeUnit`, where `TimeUnit` is the `TimeUnit` property of the model.

## **opts — Options for margin computation**

`robOptions` object

Options for computation of robust stability margins, specified as an object you create with `robOptions`. The available options include settings that let you:

- Extract frequency-dependent stability margins.
- Examine the sensitivity of the margins to each uncertain element.
- Improve the results of the stability-margin calculation by setting certain options for the underlying `mussv` calculation. In particular, setting the option `'MussvOptions'` to `'mN'` can reduce the gap between the lower bound and upper bound. `N` is the number of restarts.

For more information about all available options, see `robOptions`.

Example: `robOptions('Sensitivity','on','MussvOptions','m3')`

## Output Arguments

### **stabmarg** — Robust stability margin and critical frequency

structure

Robust stability margin and critical frequency, returned as a structure containing the following fields:

| Field             | Description  |
|-------------------|--|
| LowerBound        | Lower bound on the actual robust stability margin of the model, returned as a scalar value. The exact stability margin is guaranteed to be no smaller than LowerBound. In other words, for all modeled uncertainty with normalized magnitude up to LowerBound, the system is guaranteed stable.                                  |
| UpperBound        | Upper bound on the actual robust stability margin of the model, returned as a scalar value. The exact stability margin is guaranteed to be no larger than UpperBound. In other words, there exist some uncertain-element values associated with this magnitude that cause instability. robstab returns one such instance in wcu. |
| CriticalFrequency | Frequency at which the stability margin is the smallest, in rad/TimeUnit, where TimeUnit is the TimeUnit property of usys.   |

A robust stability margin greater than 1 means that `usys` is stable for all values of its modeled uncertainty. A robust stability margin less than 1 implies that `usys` becomes unstable for some values of its uncertain elements within their specified ranges. For example, a margin of 0.5 implies the following:

- `usys` remains stable as long as the uncertain element values stay within 0.5 normalized units of their nominal values.
- There is a destabilizing perturbation of size 0.5 normalized units.

Use `normalized2actual` to convert the normalized uncertainty value expressed in the stability margin to actual deviations from nominal values.

If the nominal value of `usys` is unstable, the stability margin is 0. If `usys` is a `ufrd` or `genfrd` model, `robstab` assumes it is nominally stable.

If `usys` is an array of uncertain models, `stabmarg` contains the smallest margin across all models in the array. In that case, the `info` output contains the index of the corresponding model in its `Model` field.

## **wcu – Perturbations causing instability**

structure

Smallest perturbations of uncertain elements that cause instability in `usys`, returned as a structure whose fields are the names of the uncertain elements of `usys`. Each field contains the actual destabilizing value for each uncertain element of `usys`. For example, if `usys` includes an uncertain matrix `M` and SISO uncertain dynamics `delta`, then `wcu.M` is a numeric matrix and `wcu.delta` is a SISO state-space model.

Use `usubs(usys, wcu)` to substitute these values for the uncertain elements in `usys`, to obtain the unstable dynamic system that deviates the least from the nominal system. Use `actual2normalized` to convert these actual uncertainty values to the normalized units in which the stability margin is expressed.

For `ureal` parameters in `usys` whose range is not centered around the nominal value, `robstab` makes the following adjustments for the purposes of its analysis:

- When the worst perturbation (the smallest destabilizing perturbation) lies outside the range of validity of the actual-to-normalized transformation (see `getLimits`), then `robstab` sets the corresponding entry of `wcu` to the nearest valid value. In other words, if `actpert` is the worst perturbation in actual units, `robgain` sets `wcu` to the nearest value inside the interval `ActLims` returned by `getLimits`.
- When there is no destabilizing perturbation, then `robstab` sets the corresponding entry of `wcu` to the nominal value of the `ureal` parameter.

## **info – Additional information about stability margins**

structure

Additional information about the robust stability margins, returned as a structure with the following fields:

| Field              | Description   |
|--------------------|---|
| <code>Model</code> | Index of the model that has the weakest stability margin, when <code>usys</code> is an array of models. |

| Field     | Description   |
|-----------|---|
| Frequency | <p>Frequency points at which <code>robstab</code> returns the robust stability margin, returned as a vector.</p> <ul style="list-style-type: none"> <li>• If the 'VaryFrequency' option of <code>robOptions</code> is 'off', then <code>info.Frequency</code> is the critical frequency, the frequency at which the smallest margin occurs. If the smallest lower bound and the smallest upper bound on the stability margin occur at different frequencies, then <code>info.Frequency</code> is a vector containing these two frequencies.</li> <li>• If the 'VaryFrequency' option of <code>robOptions</code> is 'on', then <code>info.Frequency</code> contains the frequencies selected by <code>robstab</code>. These frequencies are guaranteed to include the frequency at which the stability margin is smallest.</li> <li>• If you specify a vector of frequencies <code>w</code> at which to compute the stability margins, then <code>info.Frequency = w</code>. When you specify a frequency vector, these frequencies are not guaranteed to include the frequency at which the stability margin is smallest.</li> </ul> <p>The 'VaryFrequency' option is meaningful only for <code>uss</code> and <code>genss</code> models. <code>robstab</code> ignores the option for <code>ufrd</code> and <code>genfrd</code> models.</p> |
| Bounds    | <p>Lower and upper bounds on the actual robust stability margin of the model, returned as an array. <code>info.Bounds(:,1)</code> contains the lower bound at each corresponding frequency in <code>info.Frequency</code>, and <code>info.Bounds(:,2)</code> contains the corresponding upper bounds.</p>   |

| Field             | Description   |
|-------------------|---|
| WorstPerturbation | <p>Smallest destabilizing perturbations at each frequency point in <code>info.Frequency</code>, returned as a structure array. The fields of <code>info.WorstPerturbation</code> are the names of the uncertain elements in <code>usys</code>, and each field contains the destabilizing value of the corresponding element at each frequency. For example, if <code>usys</code> includes an uncertain parameter <code>p</code> and SISO uncertain dynamics <code>delta</code>, then <code>info.WorstPerturbation.p</code> is a collection of numeric values and <code>info.WorstPerturbation.delta</code> is a collection of SISO state-space models.</p>  |
| Sensitivity       | <p>Sensitivity of the stability margin to each uncertain element, returned as a structure when the 'Sensitivity' option of <code>robOptions</code> is 'on'. The fields of <code>info.Sensitivity</code> are the names of the uncertain elements in <code>usys</code>. Each field contains a percentage that measures how much the uncertainty in the corresponding element affects the stability margin. For example, if <code>info.Sensitivity.p</code> is 50, then a given fractional change in the uncertainty range of <code>p</code> causes half as much fractional change in the stability margin.</p> <p>If the 'Sensitivity' option of <code>robOptions</code> is off (the default setting), then <code>info.Sensitivity</code> is NaN.</p> |

## Algorithms

Computing the robustness margin at a particular frequency is equivalent to computing the structured singular value,  $\mu$ , for some appropriate block structure ( $\mu$ -analysis).

For `uss` and `genss` models, `robstab(usys)` and `robstab(usys, {wmin, wmax})` use an algorithm that finds the smallest margin across frequency. This algorithm does not rely on frequency gridding and is not adversely affected by discontinuities of the  $\mu$  structured



singular value. See “Getting Reliable Estimates of Robustness Margins” for more information.

For `ufrd` and `genfrd` models, `robstab` computes the  $\mu$  lower and upper bounds at each frequency point. This computation offers no guarantee between frequency points and can be inaccurate if there are discontinuities or sharp peaks in  $\mu$ . The syntax `robstab(uss,w)`, where `w` is a vector of frequency points, is the same as `robstab(ufrd(uss,w))` and also relies on frequency gridding to compute the margin.

In general, the algorithm for state-space models is faster and safer than the frequency-gridding approach. In some cases, however, the state-space algorithm requires many  $\mu$  calculations. In those cases, specifying a frequency grid as a vector `w` can be faster, provided that the robustness margin varies smoothly with frequency. Such smooth variation is typical for systems with dynamic uncertainty.

## See Also

`actual2normalized` | `mussv` | `normalized2actual` | `robOptions` | `robgain` | `wcgain`

## Topics

“Robust Stability and Worst-Case Gain of Uncertain System”

“Robust Stability, Robust Performance and Mu Analysis”

“Robustness and Worst-Case Analysis”

**Introduced in R2016b**

## robustperf

(Not recommended) Robust performance margin of uncertain multivariable system

---

**Note** `robustperf` is not recommended. Use `robgain` instead.

---

### Syntax

```
perfmarg = robustperf(usys)
```

```
[perfmarg,wcu,report,info] = robustperf(usys)
```

```
[perfmarg,wcu,report,info] = robustperf(usys,opt)
```

### Description

The performance of a nominally stable uncertain system model will generally degrade for specific values of its uncertain elements. `robustperf`, largely included for historical purposes, computes the robust performance margin, which is one measure of the level of degradation brought on by the modeled uncertainty.

As with other *uncertain-system* analysis tools, only bounds on the performance margin are computed. The exact robust performance margin is guaranteed to lie between these upper and lower bounds.

The computation used in `robustperf` is a frequency-domain calculation. Coupled with stability of the nominal system, this frequency domain calculation determines robust performance of `usys`. If the input system `usys` is a `ufrd`, then the analysis is performed on the frequency grid within the `ufrd`. Note that the stability of the nominal system is not verified by the computation. If the input system `sys` is a `uss`, then the stability of the nominal system is first checked, an appropriate frequency grid is generated (automatically), and the analysis performed on that frequency grid. In all discussion that follows,  $N$  denotes the number of points in the frequency grid.

### Basic Syntax

Suppose `usys` is a `ufrd` or `uss` with  $M$  uncertain elements. The results of

```
[perfmarg,perfmargunc,Report] = robustperf(usys)
```

are such that `perfmarg` is a structure with the following fields:

| Field             | Description  |
|-------------------|--|
| LowerBound        | Lower bound on robust performance margin, positive scalar.   |
| UpperBound        | Upper bound on robust performance margin, positive scalar.   |
| CriticalFrequency | The value of frequency at which the performance degradation curve crosses the $y = 1/x$ curve. See “Robustness and Worst-Case Analysis”. |

`perfmargunc` is a `struct` of values of uncertain elements associated with the intersection of the performance degradation curve and the  $y = 1/x$  curve. See “Robustness and Worst-Case Analysis”. There are  $M$  field names, which are the names of uncertain elements of `usys`.

`Report` is a text description of the robust performance analysis results.

If `usys` is an array of uncertain models, the outputs are `struct` arrays whose entries correspond to each model in the array.

## Examples

Create a plant with a nominal model of an integrator, and include additive unmodeled dynamics uncertainty of a level of 0.4 (this corresponds to 100% model uncertainty at 2.5 rads/s).

```
P = tf(1,[1 0]) + ultidyn('delta',[1 1],'bound',0.4);
```

Design a “proportional” controller  $K$  that puts the nominal closed-loop bandwidth at 0.8 rad/s. Roll off  $K$  at a frequency 25 times the nominal closed-loop bandwidth. Form the closed-loop sensitivity function.

```
BW = 0.8;
K = tf(BW,[1/(25*BW) 1]);
S = feedback(1,P*K);
```

Assess the performance margin of the closed-loop sensitivity function. Because the nominal gain of the sensitivity function is 1, and the performance degradation curve is

monotonically increasing (see “Robustness and Worst-Case Analysis”), the performance margin should be less than 1.

```
[perfmargin,punc] = robustperf(S);  
perfmargin  
perfmargin =  
    UpperBound: 7.4305e-001  
    LowerBound: 7.4305e-001  
    CriticalFrequency: 5.3096e+000
```

You can verify that the upper bound of the performance margin corresponds to a point on or above the  $y=1/x$  curve. First, compute the normalized size of the value of the uncertain element, and check that this agrees with the upper bound.

```
nsiz = actual2normalized(S.Uncertainty.delta, punc.delta)  
nsiz =  
perfmargin.UpperBound  
ans =  
    7.4305e-001
```

Compute the system gain with that value substituted, and verify that the product of the normalized size and the system gain is greater than or equal to 1.

```
gain = norm(usubs(S,punc),inf,.00001);  
nsiz*gain  
ans =  
    1.0000e+000
```

Finally, as a sanity check, verify that the robust performance margin is less than the robust stability margin.

```
[stabmargin] = robuststab(S);  
stabmargin  
stabmargin =  
    UpperBound: 3.1251e+000  
    LowerBound: 3.1251e+000  
    DestabilizingFrequency: 4.0862e+000
```

While the robust stability margin is easy to describe (poles migrating from stable region into unstable region), describing the robust performance margin is less elementary. See the diagrams and figures in “Robustness and Worst-Case Analysis”. Rather than finding values for uncertain elements that lead to instability, the analysis finds values of uncertain elements “corresponding to the intersection point of the performance degradation curve with a  $y=1/x$  hyperbola.” This characterization, mentioned above in the description of

`perfmarg.CriticalFrequency` and `perfmargunc`, is used often in the descriptions below.

## Basic Syntax with Fourth Output Argument

A fourth output argument yields more specialized information, including sensitivities and frequency-by-frequency information.

```
[perfmarg,perfmargunc,Report,Info] = robustperf(usys)
```

In addition to the first 3 output arguments, described previously, `Info` is a structure with the following fields:

| Field              | Description   |
|--------------------|---|
| Sensitivity        | A struct with $M$ fields, field names are names of uncertain elements of <code>usys</code> . Values of fields are positive and contain the local sensitivity of the overall Stability Margin to that element's uncertainty range. For instance, a value of 25 indicates that if the uncertainty range is enlarged by 8%, then the stability margin should drop by about 2% (25% of 8). If the <code>Sensitivity</code> property of the <code>robustperfOptions</code> object is 'off', the values are set to NaN. |
| Frequency          | $N$ -by-1 frequency vector associated with analysis.  |
| BadUncertainValues | $N$ -by-1 struct array containing the worst uncertain element values at each frequency.   |
| MussvBnds          | A 1-by-2 frd, with upper and lower bounds from <code>mussv</code> . The (1,1) entry is the $\mu$ -upper bound (corresponds to <code>perfmarg.LowerBound</code> ) and the (1,2) entry is the $\mu$ -lower bound (for <code>perfmarg.UpperBound</code> ).   |
| MussvInfo          | Structure of compressed data from <code>mussv</code> .  |

## Specifying Additional Options

Use `robustperfOptions` to specify additional options for the `robustperf` computation. For example, you can control what is displayed during the computation, turn the sensitivity computation on or off, set the step size in the sensitivity computation, or

control the option argument used in the underlying call to `mussv`. For example, you can turn the display on and turn off the sensitivity by executing

```
opt = robustperfOptions('Sensitivity','off','Display','on');  
[PerfMarg, Destabunc, Report, Info] = robustperf(usys, opt)
```

See the `robustperfOptions` reference page for more information about available options.

## Limitations

Because the calculation is carried out with a frequency gridding, it is possible (likely) that the true critical frequency is missing from the frequency vector used in the analysis. This is similar to the problem in `robuststab`. However, in comparing to `robuststab`, the problem in `robustperf` is less acute. The robust performance margin, considered a function of problem data and frequency, is typically a continuous function (unlike the robust stability margin, described in “Getting Reliable Estimates of Robustness Margins”). Hence, in robust performance margin calculations, increasing the density of the frequency grid will always increase the accuracy of the answers, and in the limit, answers arbitrarily close to the actual answers are obtainable with finite frequency grids.

## Algorithms

A rigorous robust performance analysis consists of two steps:

- 1 Verify that the nominal system is stable.
- 2 Robust performance analysis on an augmented system.

The algorithm in `robustperf` follows this in spirit, with the following limitations:

- If `usys` is a `uss` object, then `robustperf` explicitly checks the stability of the nominal value. However, if `usys` is a `ufrd` model, `robustperf` instead assumes that the nominal value is stable, and does not perform this check.
- The exact performance margin is guaranteed to be no larger than `UpperBound` (some uncertain elements associated with this magnitude cause instability - one instance is returned in the structure `perfmargunc`). The instability created by `perfmargunc` occurs at the frequency value in `CriticalFrequency`.

- Similarly, the exact performance margin is guaranteed to be no smaller than LowerBound.

## **See Also**

actual2normalized | mussv | norm | robgain | robstab | wcgain | wcmargin | wcsens

**Introduced before R2006a**

## robustperfOptions

(Not recommended) Option set for robustperf

### Syntax

```
options = robustperfOptions  
options = robustperfOptions(Name,Value,...)
```

---

**Note** robustperfOptions is not recommended. Use robOptions instead.

---

### Description

options = robustperfOptions returns the default option set for the robustperf command.

options = robustperfOptions(Name,Value,...) creates an option set with the options specified by one or more Name,Value pair arguments.

### Input Arguments

#### Name-Value Pair Arguments

Specify optional comma-separated pairs of Name,Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes ( ' '). You can specify several name and value pair arguments in any order as Name1,Value1,...,NameN,ValueN.

robustperfOptions takes the following Name arguments:

#### Display

Specifies whether robustperf displays progress of mussv computations.



- 'off' — Do not display progress.
- 'on' — Display progress. This setting overrides the silent ('s') option in the `Mussv` option.

**Default:** 'off'

### **Sensitivity**

Specifies whether `robustperf` computes the sensitivity of the performance margin with respect to each individual uncertain element. This element-by-element sensitivity provides an indication of which elements the performance margin is most sensitive to. Turning off the element-by-element sensitivity calculation speeds up `robustperf`.

- 'on' — Compute the sensitivity for each uncertain element.
- 'off' — Do not compute the sensitivity for each uncertain element.

**Default:** 'on'

### **VaryUncertainty**

Percentage variation of uncertainty for computing sensitivity. The sensitivity estimate uses a finite difference calculation.

**Default:** 25

### **Mussv**

Options for the `mussv` calculation that `robustperf` performs. See `mussv` for the available options.

**Default:** '' (default behavior of `mussv`)

## **Output Arguments**

### **options**

Option set containing the specified options for the `robustperf` command.

## Examples

Create an options set for a `robustperf` calculation that displays the progress of the `mussv` calculation. Also, turn off the element-by-element sensitivity calculation.

```
options = robustperfOptions('Display','on','Sensitivity','off');
```

Alternatively, use dot notation to set the values of options.

```
options = robustperfOptions;  
options.Display = 'on';  
options.Sensitivity = 'off';
```

## See Also

`robOptions`

**Introduced in R2011b**

## robuststab

(Not recommended) Calculate robust stability margins of uncertain multivariable system

---

**Note** `robuststab` is not recommended. Use `robstab` instead.

---

### Syntax

```
[stabmarg,destabunc,report,info] = robuststab(sys)
```

```
[stabmarg,destabunc,report,info] = robuststab(sys,opt)
```

### Description

A nominally stable uncertain system is generally unstable for specific values of its uncertain elements. Determining the values of the uncertain elements closest to their nominal values for which instability occurs is a *robust stability* calculation.

If the uncertain system is stable for all values of uncertain elements within their allowable ranges (ranges for `ureal`, norm bound or positive-real constraint for `ultidyn`, radius for `ucomplex`, weighted ball for `ucomplexm`), the uncertain system is *robustly stable*. Conversely, if there is a combination of element values that cause instability, and all lie within their allowable ranges, then the uncertain system is not robustly stable.

`robuststab` computes the margin of stability robustness for an uncertain system. A stability robustness margin greater than 1 means that the uncertain system is stable for all values of its modeled uncertainty. A stability robustness margin less than 1 implies that certain allowable values of the uncertain elements, within their specified ranges, lead to instability.

Numerically, a margin of 0.5 (for example) implies two things: the uncertain system remains stable for all values of uncertain elements that are less than 0.5 normalized units away from their nominal values and, there is a collection of uncertain elements that are less than or equal to 0.5 normalized units away from their nominal values that results in instability. Similarly, a margin of 1.3 implies that the uncertain system remains stable for all values of uncertain elements up to 30% outside their modeled uncertain ranges. See

actual2normalized for converting between actual and normalized deviations from the nominal value of an uncertain element.

As with other *uncertain-system* analysis tools, only bounds on the exact stability margin are computed. The exact robust stability margin is guaranteed to lie in between these upper and lower bounds.

The computation used in `robuststab` is a frequency-domain calculation, which determines whether poles can migrate (due to variability of the uncertain atoms) across the stability boundary (imaginary axis for continuous-time, unit circle for discrete-time). Coupled with stability of the nominal system, determining that no migration occurs constitutes robust stability. If the input system `sys` is a `ufrd`, then the analysis is performed on the frequency grid within the `ufrd`. Note that the stability of the nominal system is not verified by the computation. If the input system `sys` is a `uss`, then the stability of the nominal system is first checked, an appropriate frequency grid is generated (automatically), and the analysis performed on that frequency grid. In all discussion that follows,  $N$  denotes the number of points in the frequency grid.

## Basic Syntax

Suppose `sys` is a `ufrd` or `uss` with  $M$  uncertain elements. The results of

```
[stabmarg,destabunc,Report] = robuststab(sys)
```

are that `stabmarg` is a structure with the following fields

| Field      | Description   |
|------------|---|
| LowerBound | Lower bound on stability margin, positive scalar. If greater than 1, then the uncertain system is guaranteed stable for all values of the modeled uncertainty. If the nominal value of the uncertain system is unstable, then <code>stabmarg.UpperBound</code> and <code>stabmarg.LowerBound</code> both equal 0. |
| UpperBound | Upper bound on stability margin, positive scalar. If less than 1, the uncertain system is not stable for all values of the modeled uncertainty.   |

| Field                  | Description   |
|------------------------|---|
| DestabilizingFrequency | The critical value of frequency at which instability occurs, with uncertain elements closest to their nominal values. At a particular value of uncertain elements (see <code>destabunc</code> below), the poles migrate across the stability boundary (imaginary-axis in continuous-time systems, unit-disk in discrete-time systems) at the frequency given by <code>DestabilizingFrequency</code> . |

`destabunc` is a structure of values of uncertain elements, closest to nominal, that cause instability. There are  $M$  field names, which are the names of uncertain elements of `sys`. The value of each field is the corresponding value of the uncertain element, such that when jointly combined, lead to instability. The command `pole(usubs(sys,destabunc))` shows the instability. If  $A$  is an uncertain element of `sys`, then

```
actual2normalized(destabunc.A,sys.Uncertainty.A)
```

will be less than or equal to `UpperBound`, and for at least one uncertain element of `sys`, this normalized distance will be equal to `UpperBound`, proving that `UpperBound` is indeed an upper bound on the robust stability margin.

`Report` is a text description of the arguments returned by `robuststab`.

If `sys` is an array of uncertain models, the outputs are struct arrays whose entries correspond to each model in the array.

## Examples

Construct a feedback loop with a second-order plant and a PID controller with approximate differentiation. The second-order plant has frequency-dependent uncertainty, in the form of additive unmodeled dynamics, introduced with an `ultidyn` object and a shaping filter.

`robuststab` is used to compute the stability margins of the closed-loop system with respect to the plant model uncertainty.

```
P = tf(4,[1 .8 4]);
delta = ultidyn('delta',[1 1],'SampleStateDimension',5);
Pu = P + 0.25*tf([1],[.15 1])*delta;
C = tf([1 1],[.1 1]) + tf(2,[1 0]);
```

```
S = feedback(1,Pu*C);  
[stabmarg,destabunc,report,info] = robuststab(S);
```

You can view the `stabmarg` variable.

```
stabmarg  
stabmarg =  
           UpperBound: 0.8181  
           LowerBound: 0.8181  
DestabilizingFrequency: 9.1321
```

As the margin is less than 1, the closed-loop system is not stable for plant models covered by the uncertain model `Pu`. There is a specific plant within the uncertain behavior modeled by `Pu` (actually about 82% of the modeled uncertainty) that leads to closed-loop instability, with the poles migrating across the stability boundary at 9.1 rads/s.

The `report` variable is specific, giving a plain-language version of the conclusion.

```
report  
report =  
Uncertain System is NOT robustly stable to modeled uncertainty.  
-- It can tolerate up to 81.8% of modeled uncertainty.  
-- A destabilizing combination of 81.8% the modeled uncertainty  
exists, causing an instability at 9.13 rad/s.  
-- Sensitivity with respect to uncertain element ...  
   'delta' is 100%. Increasing 'delta' by 25% leads to a  
25% decrease in the margin.
```

Because the problem has only one uncertain element, the stability margin is completely determined by this element, and hence the margin exhibits 100% sensitivity to this uncertain element.

You can verify that the destabilizing value of `delta` is indeed about 0.82 normalized units from its nominal value.

```
actual2normalized(S.Uncertainty.delta,destabunc.delta)  
ans =  
    0.8181
```

Use `usubs` to substitute the specific value into the closed-loop system. Verify that there is a closed-loop pole near  $j9.1$ , and plot the unit-step response of the nominal closed-loop system, as well as the unstable closed-loop system.

```
Sbad = usubs(S,destabunc);  
pole(Sbad)
```

```

ans =
  1.0e+002 *
  -3.2318
  -0.2539
  -0.0000 + 0.0913i
  -0.0000 - 0.0913i
  -0.0203 + 0.0211i
  -0.0203 - 0.0211i
  -0.0106 + 0.0116i
  -0.0106 - 0.0116i
step(S.NominalValue, 'r--', Sbad, 'g', 4);

```

Finally, as an ad-hoc test, set the gain bound on the uncertain `delta` to 0.81 (slightly less than the stability margin). Sample the closed-loop system at 100 values, and compute the poles of all these systems.

```

S.Uncertainty.delta.Bound = 0.81;
S100 = usample(S,100);
p100 = pole(S100);
max(real(p100(:)))
ans =
  -6.4647e-007

```

As expected, all poles have negative real parts.

## Basic Syntax with Fourth Output Argument

A fourth output argument yields more specialized information, including sensitivities and frequency-by-frequency information.

```
[StabMarg, Destabunc, Report, Info] = robuststab(sys)
```

In addition to the first 3 output arguments, described previously, `Info` is a structure with the following fields

| Field              | Description  |
|--------------------|--|
| Sensitivity        | A struct with $M$ fields, Field names are names of uncertain elements of <code>sys</code> . Values of fields are positive, each the local sensitivity of the overall stability margin to that element's uncertainty range. For instance, a value of 25 indicates that if the uncertainty range is enlarged by 8%, then the stability margin should drop by about 2% (25% of 8). If the <code>Sensitivity</code> property of the <code>robuststabOptions</code> object is 'off', the values are set to NaN. |
| Frequency          | $N$ -by-1 frequency vector associated with analysis.   |
| BadUncertainValues | $N$ -by-1 struct array containing the destabilizing uncertain element values at each frequency.  |
| MussvBnds          | A 1-by-2 <code>frd</code> , with upper and lower bounds from <code>mussv</code> . The (1,1) entry is the $\mu$ -upper bound (corresponds to <code>stabmarg.LowerBound</code> ) and the (1,2) entry is the $\mu$ -lower bound (for <code>stabmarg.UpperBound</code> ).  |
| MussvInfo          | Structure of compressed data from <code>mussv</code> .   |

## Specifying Additional Options

Use `robuststabOptions` to specify additional options for the `robuststab` computation. For example, you can control what is displayed during the computation, turning the sensitivity computation on or off, set the step-size in the sensitivity computation, or control the option argument used in the underlying call to `mussv`. For instance, you can turn the display on, and the sensitivity calculation off by executing

```
opt = robuststabOptions('Sensitivity','off','Display','on');
[StabMarg, Destabunc, Report, Info] = robuststab(sys, opt)
```

See the `robuststabOptions` reference page for more information about available options.

## Limitations

Under most conditions, the robust stability margin at each frequency is a continuous function of the problem data at that frequency. Because the problem data, in turn, is a continuous function of frequency, it follows that finite frequency grids are usually



adequate in correctly assessing robust stability bounds, assuming the frequency grid is dense enough.

Nevertheless, there are simple examples that violate this. In some problems, the migration of poles from stable to unstable *only* occurs at a finite collection of specific frequencies (generally unknown to you). Any frequency grid that excludes these critical frequencies (and almost every grid will exclude them) will result in undetected migration and misleading results, namely stability margins of  $\infty$ .

See “Getting Reliable Estimates of Robustness Margins” for more information about circumventing the problem in an engineering-relevant fashion.

## Algorithms

A rigorous robust stability analysis consists of two steps:

- 1 Verify that the nominal system is stable;
- 2 Verify that no poles cross the stability boundary as the uncertain elements vary within their ranges.

Because the stability boundary is also associated with the frequency response, the second step can be interpreted (and carried out) as a frequency domain calculation. This amounts to a classical  $\mu$ -analysis problem.

The algorithm in `robuststab` follows this in spirit, with the following limitations.

- If `sys` is a `uss` object, then the first requirement of stability of nominal value is explicitly checked within `robuststab`. However, if `sys` is an `ufrd`, then the verification of nominal stability from the nominal frequency response data is not performed, and is instead assumed.
- In the second step (monitoring the stability boundary for the migration of poles), rather than check all points on stability boundary, the algorithm only detects migration of poles across the stability boundary at the frequencies in `info.Frequency`.

See “Limitations” on page 1-394 for information about issues related to migration detection.

The exact stability margin is guaranteed to be no larger than `UpperBound` (some uncertain elements associated with this magnitude cause instability - one instance is

returned in the structure `destabunc`). The instability created by `destabunc` occurs at the frequency value in `DestabilizingFrequency`.

Similarly, the exact stability margin is guaranteed to be no smaller than `LowerBound`. In other words, for all modeled uncertainty with magnitude up to `LowerBound`, the system is guaranteed stable. These bounds are derived using the upper bound for the structured singular value, which is essentially optimally-scaled, small-gain theorem analysis.

## See Also

`loopmargin` | `mussv` | `robgain` | `robstab` | `wcgain` | `wcmargin` | `wcsens`

**Introduced before R2006a**

# robuststabOptions

(Not recommended) Option set for `robuststab`

## Syntax

```
options = robuststabOptions  
options = robuststabOptions(Name,Value,...)
```

---

**Note** `robuststabOptions` is not recommended. Use `robOptions` instead.

---

## Description

`options = robuststabOptions` returns the default option set for the `robuststab` command.

`options = robuststabOptions(Name,Value,...)` creates an option set with the options specified by one or more `Name,Value` pair arguments.

## Input Arguments

### Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

`robuststabOptions` takes the following `Name` arguments:

### Display

Specifies whether `robuststab` displays progress of `mussv` computations.

- 'off' — Do not display progress.
- 'on' — Display progress. This setting overrides the silent ('s') option in the `Mussv` option.

**Default:** 'off'

## Sensitivity

Specifies whether `robuststab` computes the sensitivity of the stability margin with respect to each individual uncertain element. This element-by-element sensitivity provides an indication of which elements the stability margin is most sensitive to. Turning off the element-by-element sensitivity calculation speeds up `robuststab`.

- 'on' — Compute the sensitivity for each uncertain element.
- 'off' — Do not compute the sensitivity for each uncertain element.

**Default:** 'on'

## VaryUncertainty

Percentage variation of uncertainty for computing sensitivity. The sensitivity estimate uses a finite difference calculation.

**Default:** 25

## Mussv

Options for the `mussv` calculation that `robustperf` performs. See `mussv` for the available options.

**Default:** '' (default behavior of `mussv`)

## Output Arguments

### options

Option set containing the specified options for the `robuststab` command.

## Examples

Create an options set for a `robuststab` calculation that displays the progress of the `mu`sv calculation. Also, turn off the element-by-element sensitivity calculation.

```
options = robuststabOptions('Display','on','Sensitivity','off');
```

Alternatively, use dot notation to set the values of `options`.

```
options = robuststabOptions;  
options.Display = 'on';  
options.Sensitivity = 'off';
```

## See Also

`robOptions`

**Introduced in R2011b**

## schurmr

Balanced model truncation via Schur method

### Syntax

```
GRED = schurmr(G)
```

```
GRED = schurmr(G,order)
```

```
[GRED,redinfo] = schurmr(G,key1,value1,...)
```

```
[GRED,redinfo] = schurmr(G,order,key1,value1,...)
```

### Description

`schurmr` returns a reduced order model `GRED` of `G` and a struct array `redinfo` containing the error bound of the reduced model and Hankel singular values of the original system.

The error bound is computed based on Hankel singular values of `G`. For a stable system Hankel singular values indicate the respective state energy of the system. Hence, reduced order can be directly determined by examining the system Hankel SV's,  $\sigma_i$ .

With only one input argument `G`, the function will show a Hankel singular value plot of the original model and prompt for model order number to reduce.

This method guarantees an error bound on the infinity norm of the *additive error*  $\|G - GRED\|_\infty$  for well-conditioned model reduced problems [1]:

$$\|G - Gred\|_\infty \leq 2 \sum_{k=1}^n \sigma_k$$

This table describes input arguments for `schurmr`.

| Argument | Description  |
|----------|--|
| G        | LTI model to be reduced (without any other inputs will plot its Hankel singular values and prompt for reduced order).              |
| ORDER    | (Optional) an integer for the desired order of the reduced model, or optionally a vector packed with desired orders for batch runs |

A batch run of a serial of different reduced order models can be generated by specifying `order = x:y`, or a vector of integers. By default, all the anti-stable part of a system is kept, because from control stability point of view, getting rid of unstable state(s) is dangerous to model a system.

'*MaxError*' can be specified in the same fashion as an alternative for 'ORDER'. In this case, reduced order will be determined when the sum of the tails of the Hankel sv's reaches the '*MaxError*'.

| Argument            | Value   | Description  |
|---------------------|---|--|
| ' <i>MaxError</i> ' | A real number or a vector of different errors | Reduce to achieve $H_{\infty}$ error.<br>When present, ' <i>MaxError</i> ' overrides ORDER input.  |
| ' <i>Weights</i> '  | { <i>Wout</i> , <i>Win</i> } cell array       | Optimal 1x2 cell array of LTI weights <i>Wout</i> (output) and <i>Win</i> (input); default is both identity; Weights must be invertible. |
| ' <i>Display</i> '  | ' <i>on</i> ' or ' <i>off</i> '               | Display Hankel singular plots (default ' <i>off</i> ').  |
| ' <i>Order</i> '    | Integer, vector or cell array                 | Order of reduced model. Use only if not specified as 2nd argument.   |

Weights on the original model input and/or output can make the model reduction algorithm focus on some frequency range of interests. But weights have to be stable, minimum phase and invertible.

This table describes output arguments.

| Argument | Description   |
|----------|---|
| GRED     | LTI reduced order model. Becomes multi-dimensional array when input is a serial of different model order array. |

| Argument | Description  |
|----------|--|
| REDINFO  | A STRUCT array with 3 fields: <ul style="list-style-type: none"><li>• REDINFO.ErrorBound</li><li>• REDINFO.StabSV</li><li>• REDINFO.UnstabSV</li></ul> |

G can be stable or unstable. G and GRED can be either continuous or discrete.

## Examples

Given a continuous or discrete, stable or unstable system, G, the following commands can get a set of reduced order models based on your selections:

```
rng(1234, 'twister');
G = rss(30,5,4);
[g1, redinfo1] = schurmr(G); % display Hankel SV plot
                        % and prompt for order (try 15:20)
[g2, redinfo2] = schurmr(G,20);
[g3, redinfo3] = schurmr(G,[10:2:18]);
[g4, redinfo4] = schurmr(G,'MaxError',[0.01, 0.05]);
for i = 1:4
    figure(i); eval(['sigma(G,g' num2str(i) ');']);
end
```

## Algorithms

Given a state space  $(A,B,C,D)$  of a system and  $k$ , the desired reduced order, the following steps will produce a similarity transformation to truncate the original state-space system to the  $k^{\text{th}}$  order reduced model [16].

- 1 Find the controllability and observability grammians  $P$  and  $Q$ .
- 2 Find the Schur decomposition for  $PQ$  in both ascending and descending order, respectively,



$$V_A^T P Q V_A = \begin{bmatrix} \lambda_1 & \dots & \dots \\ 0 & \dots & \dots \\ 0 & 0 & \lambda_n \end{bmatrix}$$

$$V_D^T P Q V_D = \begin{bmatrix} \lambda_n & \dots & \dots \\ 0 & \dots & \dots \\ 0 & 0 & \lambda_1 \end{bmatrix}$$

- 3 Find the left/right orthonormal eigen-bases of  $PQ$  associated with the  $k^{\text{th}}$  big Hankel singular values.

$$V_A = [V_{R,SMALL}, \overbrace{V_{L,BIG}}]$$

- 4 Find the SVD of  $(V_{L,BIG}^T V_{R,BIG}) = U \Sigma V^T$

$$V_D = [\overbrace{V_{R,BIG}}, V_{L,SMALL}]$$

- 5 Form the left/right transformation for the final  $k^{\text{th}}$  order reduced model

$$S_{L,BIG} = V_{L,BIG} U \Sigma(1:k, 1:k)^{-1/2}$$

$$S_{R,BIG} = V_{R,BIG} V \Sigma(1:k, 1:k)^{-1/2}$$

- 6 Finally,

$$\begin{bmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{bmatrix} = \begin{bmatrix} S_{L,BIG}^T A S_{R,BIG} & S_{L,BIG}^T B \\ C S_{R,BIG} & D \end{bmatrix}$$

The proof of the Schur balance truncation algorithm can be found in [2].

## References

- [1] K. Glover, "All Optimal Hankel Norm Approximation of Linear Multivariable Systems, and Their  $L_\infty$ - error Bounds," Int. J. Control, vol. 39, no. 6, pp. 1145-1193, 1984.

[2] M. G. Safonov and R. Y. Chiang, "A Schur Method for Balanced Model Reduction,"  
*IEEE Trans. on Automat. Contr.*, vol. 34, no. 7, July 1989, pp. 729-733.

## **See Also**

balancmr | bstmr | hankelmr | hankelsv | ncfmr | reduce

**Introduced before R2006a**

# sdhinfnorm

Compute  $L_2$  norm of continuous-time system in feedback with discrete-time system

## Syntax

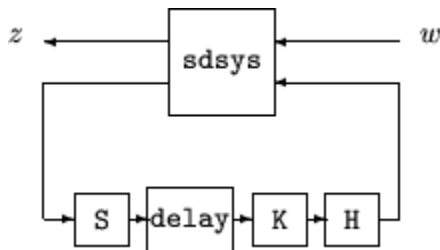
```
[gaml,gamu] = sdhinfnorm(sdsys,k)
```

```
[gaml,gamu] = sdhinfnorm(sdsys,k,delay)
```

```
[gaml,gamu] = sdhinfnorm(sdsys,k,delay,tol)
```

## Description

`[gaml,gamu] = sdhinfnorm(sdsys,k)` computes the  $L_2$  induced norm of a continuous-time LTI plant, `sdsys`, in feedback with a discrete-time controller, `k`, connected through an ideal sampler and a zero-order hold (see figure below). `sdsys` must be strictly proper, such that the constant feedback gain must be zero. The outputs, `gamu` and `gaml`, are upper and lower bounds on the induced  $L_2$  norm of the sampled-data closed-loop system.



`[gaml,gamu] = sdhinfnorm(sdsys,k,h,delay)` includes the input argument `delay`. `delay` is a nonnegative integer associated with the number of computational delays of the controller. The default value of the `delay` is 0.

`[gaml,gamu] = sdhinfnorm(sdsys,k,h,delay,tol)` includes the input argument, `tol`, which defines the difference between upper and lower bounds when search terminates. The default value of `tol` is 0.001.

## Examples

Consider an open-loop, continuous-time transfer function  $p = 30/s(s+30)$  and a continuous-time controller  $k = 4/(s+4)$ . The closed-loop continuous-time system has a peak magnitude across frequency of 1.

```
p = ss(tf(30,[1 30])*tf([1],[1 0]));
k = ss(tf(4,[1 4]));
cl = feedback(p,k);
norm(cl,'inf')
ans =
    1
```

Initially the controller is to be implemented at a sample rate of 1.5 Hz. The sample-data norm of the closed-loop system with the discrete-time controller is 1.0.

```
kd = c2d(k,0.75,'zoh');
[gu,gl] = sdhinfnorm([1; 1]*p*[1 1],-kd);
[gu gl]
ans =
    3.7908    3.7929
```

Because of the large difference in norm between the continuous-time and sampled-data closed-loop system, the sample rate of the controller is increased from 1.5 Hz to 5 Hz. The sample-data norm of the new closed-loop system is 3.79.

```
kd = c2d(k,0.2,'zoh');
[gu,gl] = sdhinfnorm([1; 1]*p*[1 1],-kd);
[gu gl]
ans =
    1.0044    1.0049
```

## Algorithms

`sdhinfnorm` uses variations of the formulas described in the Bamieh and Pearson paper to obtain an equivalent discrete-time system. (These variations are done to improve the numerical conditioning of the algorithms.) A preliminary step is to determine whether the norm of the continuous-time system over one sampling period without control is less than the given value. This requires a search and is, computationally, a relatively expensive step.

## References

Bamieh, B.A., and J.B. Pearson, "A General Framework for Linear Periodic Systems with Applications to Sampled-Data Control," *IEEE Transactions on Automatic Control*, Vol. AC-37, 1992, pp. 418-435.

## See Also

gapmetric | hinfosyn | norm | sdhinfosyn | sdlsim

**Introduced before R2006a**

## sdhinfosyn

Compute  $H_\infty$  controller for sampled-data system

### Syntax

```
[K,GAM]=sdhinfosyn(P,NMEAS,NCON)
```

```
[K,GAM]=sdhinfosyn(P,NMEAS,NCON, KEY1,VALUE1,KEY2,VALUE2,...)
```

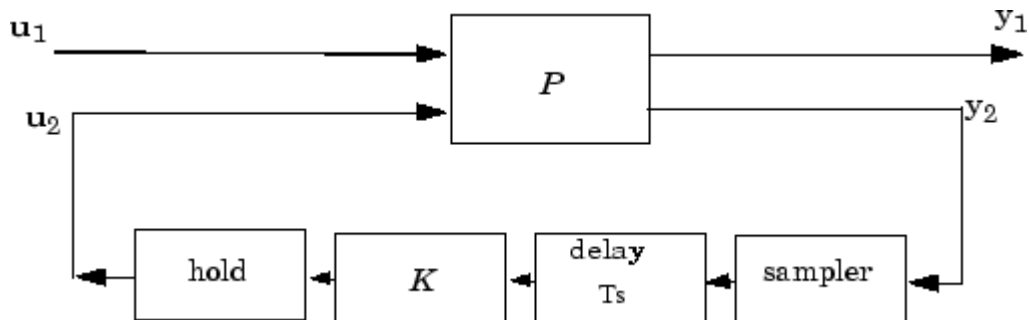
### Description

sdhinfosyn controls a continuous-time LTI system  $P$  with a discrete-time controller  $K$ . The continuous-time LTI plant  $P$  has a state-space realization partitioned as follows:

$$P = \left[ \begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & 0 & 0 \\ C_2 & 0 & 0 \end{array} \right]$$

where the continuous-time disturbance inputs enter through  $B_1$ , the outputs from the controller are held constant between sampling instants and enter through  $B_2$ , the continuous-time errors (to be kept small) correspond to the  $C_1$  partition, and the output measurements that are sampled by the controller correspond to the  $C_2$  partition.  $B_2$  has column size  $ncon$  and  $C_2$  has row size  $nmeas$ . Note that the  $D$  matrix must be zero.

sdhinfosyn synthesizes a discrete-time LTI controller  $K$  to achieve a given norm (if possible) or find the minimum possible norm to within tolerance **TOLGAM**.



Similar to `hinfosyn`, the function `sdhinfosyn` employs a  $\gamma$  iteration. Given a high and low value of  $\gamma$ , `GMAX` and `GMIN`, the bisection method is used to iterate on the value of  $\gamma$  in an effort to approach the optimal  $H_\infty$  control design. If `GMAX` = `GMIN`, only one  $\gamma$  value is tested. The stopping criterion for the bisection algorithm requires that the relative difference between the last  $\gamma$  value that failed and the last  $\gamma$  value that passed be less than `TOLGAM`.

Input arguments

|       |   |
|-------|---|
| P     | LTI plant                                   |
| NMEAS | Number of measurements output to controller |
| NCON  | Number of control inputs                    |

Optional input arguments (KEY, VALUE) pairs are similar to `hinfosyn`, but with additional KEY values 'Ts' and 'DELAY'.

| KEY      | VALUE   | Meaning   |
|----------|---------|---|
| 'GMAX'   | real    | Initial upper bound on GAM (default=Inf)  |
| 'GMIN'   | real    | Initial lower bound on GAM (default=0)  |
| 'TOLGAM' | real    | Relative error tolerance for GAM (default=.01)  |
| 'Ts'     | real    | (Default=1) sample time of the controller to be designed  |
| 'DELAY'  | integer | (Default=0) a nonnegative integer giving the number of sample periods delay for the control computation |

| KEY       | VALUE         | Meaning  |
|-----------|---------------|--|
| 'DISPLAY' | 'off'<br>'on' | (Default) no command window display, or the command window displays synthesis progress information |

Output arguments

|     |  |
|-----|--|
| K   | $H_\infty$ controller                            |
| GAM | Final $\gamma$ value of $H_\infty$ cost achieved |

## Algorithms

sdhinfosyn uses a variation of the formulas described in the Bamieh and Pearson paper [1] to obtain an equivalent discrete-time system. (This is done to improve the numerical conditioning of the algorithms.) A preliminary step is to determine whether the norm of the continuous-time system over one sampling period without control is less than the given  $\gamma$ -value. This requires a search and is computationally a relatively expensive step.

## References

- [1] Bamieh, B.A., and J.B. Pearson, "A General Framework for Linear Periodic Systems with Applications to Sampled-Data Control," *IEEE Transactions on Automatic Control*, Vol. AC-37, 1992, pp. 418-435.

## See Also

hinfosyn | norm | sdhinfoform

**Introduced before R2006a**



## sdlsim

Time response of sampled-data feedback system

### Syntax

```
sdlsim(p,k,w,t,tf)
```

```
sdlsim(p,k,w,t,tf,x0,z0)
```

```
sdlsim(p,k,w,t,tf,x0,z0,int)
```

```
[vt,yt,ut,t] = sdlsim(p,k,w,t,tf)
```

```
[vt,yt,ut,t] = sdlsim(p,k,w,t,tf,x0,z0,int)
```

### Description

`sdlsim(p,k,w,t,tf)` plots the time response of the hybrid feedback system.

`lft(p,k)`, is forced by the continuous input signal described by `w` and `t` (values and times, as in `lsim`). `p` must be a continuous-time LTI system, and `k` must be discrete-time LTI system with a specified sample time (the unspecified sample time `-1` is not allowed). The final time is specified with `tf`.

`sdlsim(p,k,w,t,tf,x0,z0)` specifies the initial state vector `x0` of `p`, and `z0` of `k`, at time `t(1)`.

`sdlsim(p,k,w,t,tf,x0,z0,int)` specifies the continuous-time integration step size `int`. `sdlsim` forces `int = (k.Ts)/N` where `N > 4` is an integer. If any of these optional arguments is omitted, or passed as empty matrices, then default values are used. The default value for `x0` and `z0` is zero. Nonzero initial conditions are allowed for `p` (and/or `k`) only if `p` (and/or `k`) is an `ss` object.

If `p` and/or `k` is an LTI array with consistent array dimensions, then the time simulation is performed pointwise across the array dimensions.

`[vt,yt,ut,t] = sdlsim(p,k,w,t,tf)` computes the continuous-time response of the hybrid feedback system `lft(p,k)` forced by the continuous input signal defined by `w`

and  $t$  (values and times, as in `lsim`).  $p$  must be a continuous-time system, and  $k$  must be discrete-time, with a specified sample time (the unspecified sample time `-1` is not allowed). The final time is specified with `tf`. The outputs  $vt$ ,  $yt$  and  $ut$  are 2-by-1 cell arrays: in each the first entry is a time vector, and the second entry is the signal values. Stored in this manner, the signal  $vt$  is plotted by using one of the following commands:

```
plot(vt{1},vt{2})  
plot(vt{:})
```

Signals  $yt$  and  $ut$  are respectively the input to  $k$  and output of  $k$ .

If  $p$  and/or  $k$  are LTI arrays with consistent array dimensions, then the time simulation is performed pointwise across the array dimensions. The outputs are 2-by-1-by-array dimension cell arrays. All responses can be plotted simultaneously, for example, `plot(vt)`.

`[vt,yt,ut,t] = sdlsim(p,k,w,t,tf,x0,z0,int)` The optional arguments are `int` (integration step size), `x0` (initial condition for  $p$ ), and `z0` (initial condition for  $k$ ). `sdlsim` forces `int = (k.Ts)/N`, where  $N > 4$  is an integer. If any of these arguments is omitted, or passed as empty matrices, then default values are used. The default value for `x0` and `z0` is zero. Nonzero initial conditions are allowed for  $p$  (and/or  $k$ ) only if  $p$  (and/or  $k$ ) is an `ss` object.

## Examples

### Time Response of Continuous Plant with Discrete Controller

To illustrate the use of `sdlsim`, consider the application of a discrete controller to a plant with an integrator and near integrator. A continuous plant and a discrete controller are created. A sample-and-hold equivalent of the plant is formed and the discrete closed-loop system is calculated. Simulating this gives the system response at the sample points. `sdlsim` is then used to calculate the intersample behavior.

```
P = tf(1,[1, 1e-5,0]);  
T = 1.0/20;  
C = ss([-1.5 T/4; -2/T -.5],[.5 2;1/T 1/T],...  
      [-1/T^2 -1.5/T], [1/T^2 0],T);  
Pd = c2d(P,T,'zoh');
```

The closed-loop digital system is now set up. You can use `sysic` to construct the interconnected feedback system.

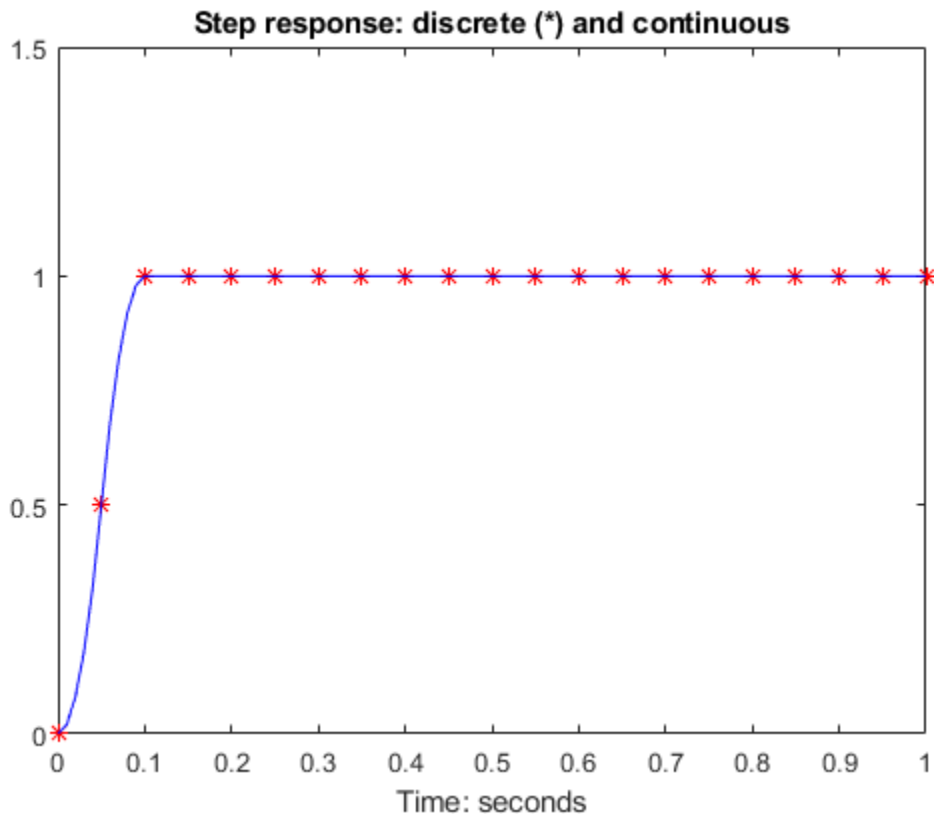
```
systemnames = 'Pd C';  
inputvar = '[ref]';  
outputvar = '[Pd]';  
input_to_Pd = '[C]';  
input_to_C = '[ref ; Pd]';  
sysoutname = 'dclp';  
cleanup_sysic = 'yes';  
sysic;
```

Use `step` to simulate the digital step response.

```
[yd,td] = step(dclp,20*T);
```

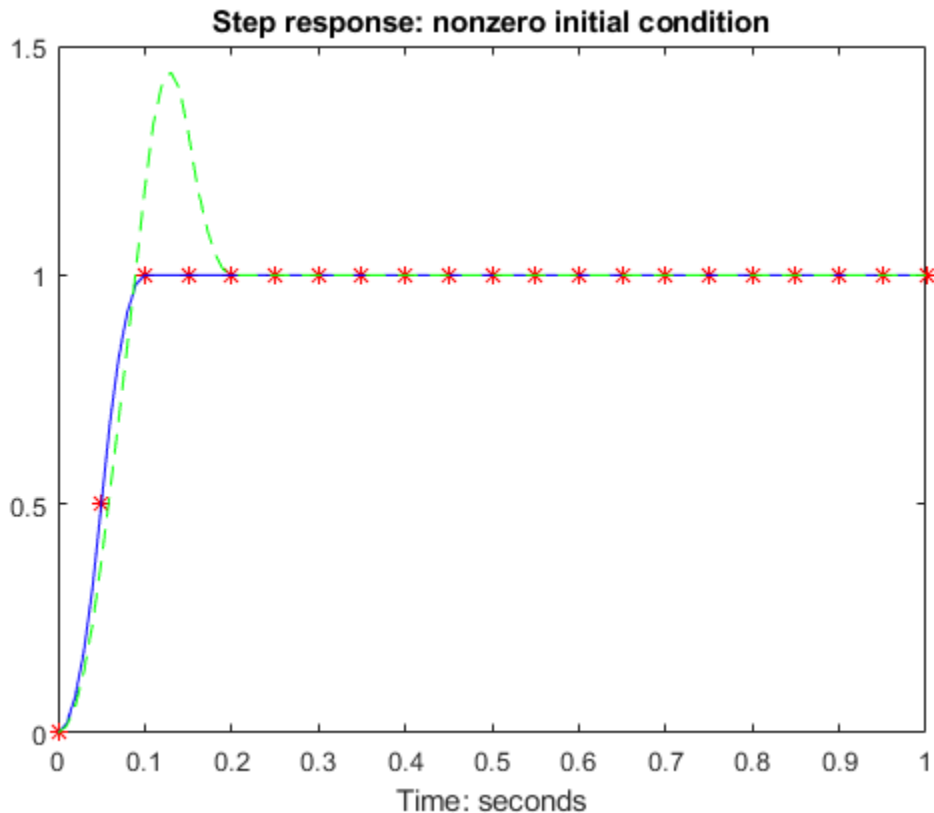
Set up the continuous interconnection and calculate the sampled data response with `sdlsim`.

```
M = [0,1;1,0;0,1]*blkdiag(1,P);  
t = [0:.01:1]';  
u = ones(size(t));  
y1 = sdlsim(M,C,u,t);  
plot(td,yd,'r*',y1{:},'b-')  
axis([0,1,0,1.5])  
xlabel('Time: seconds')  
title('Step response: discrete (*) and continuous')
```



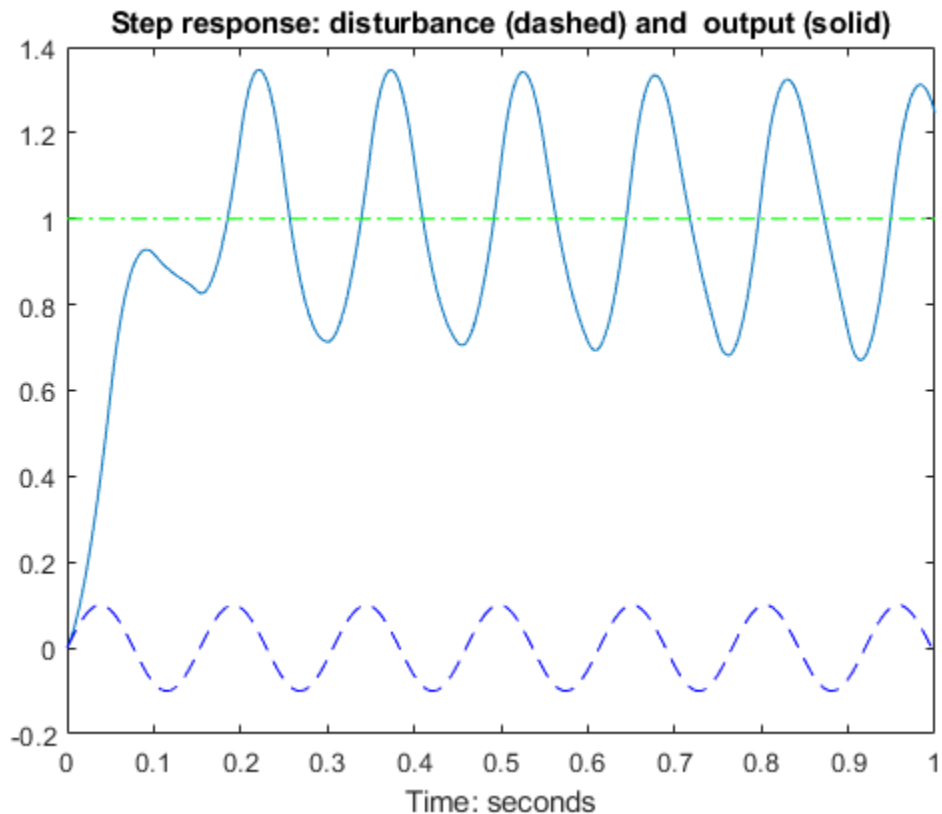
You can see the effect of a nonzero initial condition in the continuous-time system. Note how examining the system at only the sample points will underestimate the amplitude of the overshoot.

```
y2 = sdlsim(M,C,u,t,1,0,[0.25;0]);  
plot(td,yd,'r*',y1{:},'b-',y2{:},'g--')  
axis([0,1,0,1.5])  
xlabel('Time: seconds')  
title('Step response: nonzero initial condition')
```



Finally, you can examine the effect of a sinusoidal disturbance at the continuous-time plant output. This controller is not designed to reject such a disturbance and the system does not contain antialiasing filters. Simulating the effect of antialiasing filters is easily accomplished by including them in the continuous interconnection structure.

```
M2 = [0,1,1;1,0,0;0,1,1]*blkdiag(1,1,P);
t = [0:.001:1]';
dist = 0.1*sin(41*t);
u = ones(size(t));
[y3,meas,act] = sdlsim(M2,C,[u dist],t,1);
plot(y3{:},'-' ,t,dist,'b--',t,u,'g-.')
xlabel('Time: seconds')
title('Step response: disturbance (dashed) and output (solid)')
```



## Algorithms

`sdlsim` oversamples the continuous-time,  $N$  times the sample rate of the controller  $k$ .

## See Also

`gapmetric` | `hinfo` | `norm` | `sdhinfo` | `sdhinfo` | `sysic`

Introduced before R2006a

## sectf

State-space sector bilinear transformation

### Syntax

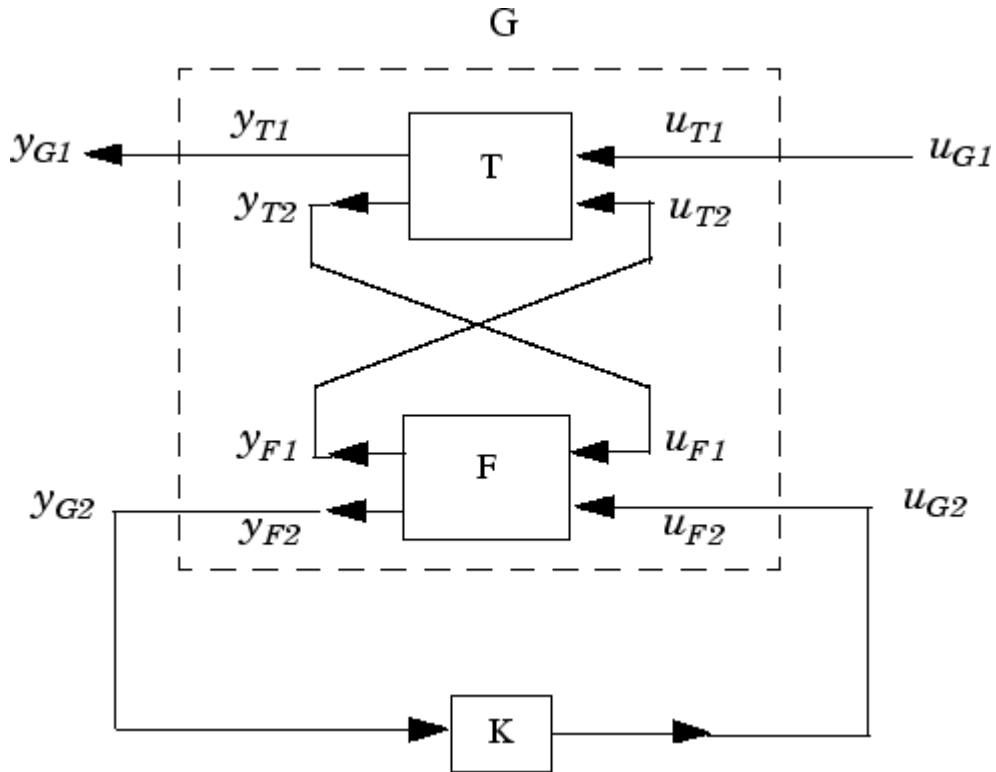
$[G, T] = \text{sectf}(F, \text{SECF}, \text{SECG})$

### Description

$[G, T] = \text{sectf}(F, \text{SECF}, \text{SECG})$  computes a linear fractional transform  $T$  such that the system  $\text{lft}(F, K)$  is in sector  $\text{SECF}$  if and only if the system  $\text{lft}(G, K)$  is in sector  $\text{SECG}$  where

$G = \text{lft}(T, F, NU, NY)$

where  $NU$  and  $NY$  are the dimensions of  $u_{T_2}$  and  $y_{T_2}$ , respectively—see the following figure.



**Sector transform  $G = \text{lft}(T, F, NU, NY)$ .**

sect f are used to transform general conic-sector control system performance specifications into equivalent  $H_\infty$ -norm performance specifications.

| Input Arguments |                        |                        |
|-----------------|------------------------|------------------------|
| F               | LTI state-space plant  |                        |
| SECG, SECF:     | Conic Sector:          |                        |
|                 | $[-1, 1]$ or $[-1; 1]$ | $\ y\ ^2 \leq \ u\ ^2$ |



| Input Arguments |  |   |
|-----------------|--|---|
|                 | $[0, \text{Inf}]$ or $[0; \text{Inf}]$ | $0 \leq \text{Re}[y * u]$   |
|                 | $[A, B]$ or $[A; B]$                   | $0 \geq \text{Re}[(y - Au) * (y - Bu)]$                           |
|                 | $[a, b]$ or $[a; b]$                   | $0 \geq \text{Re}[(y - \text{diag}(a)u) * (y - \text{diag}(b)u)]$ |
|                 | S                                      | $0 \geq \text{Re}[(S_{11}u + S_{12}y) * (S_{21}u + S_{22}y)]$     |
|                 | S                                      | $0 \geq \text{Re}[(S_{11}u + S_{12}y) * (S_{21}u + S_{22}y)]$     |

where A,B are scalars in  $[-\infty, \infty]$  or square matrices; a, b are vectors; S=[S11 S12; S21, S22] is a square matrix whose blocks S11, S12, S21, S22 are either scalars or square matrices; S is a two-port system S=mksys(a,b1,b2,..., 'tss') with transfer function

$$S(s) = \begin{bmatrix} S_{11}(s) & S_{12}(s) \\ S_{21}(s) & S_{22}(s) \end{bmatrix}$$

| Output Arguments | Description   |
|------------------|---|
| G                | Transformed plant $G(s) = \text{lftf}(T, F)$                        |
| T                | LFT sector transform, maps conic sector SECF into conic sector SECG |

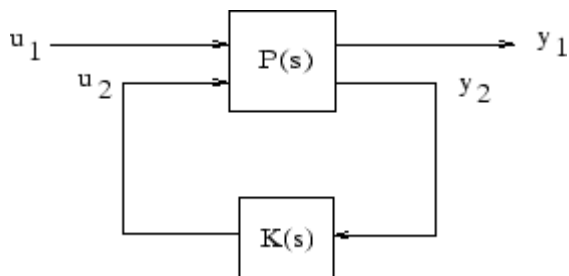
| Output Variables |  |
|------------------|--|
| G                | The transformed plant $G(s) = \text{lftf}(T, F)$ : |
| T                | The linear fractional transformation $T(s) = T$    |

## Examples

The statement  $G(j\omega)$  inside sector $[-1, 1]$  is equivalent to the  $H_\infty$  inequality

$$\sup_{\omega} \bar{\sigma}(G(j\omega)) = \|G\|_{\infty} \leq 1$$

Given a two-port open-loop plant  $P(s) := P$ , the command `P1 = sectf(P, [0, Inf], [-1, 1])` computes a transformed  $P_1(s) := P1$  such that  $\text{lft}(G, K)$  is inside  $\text{sector}[-1, 1]$  if and only if  $\text{lft}(F, K)$  is inside  $\text{sector}[0, \infty]$ . In other words,  $\text{norm}(\text{lft}(G, K), \text{inf}) < 1$  if and only if  $\text{lft}(F, K)$  is strictly positive real. See “Example of Sector Transform” on page 1-421.



### Sector Transform Block Diagram

Here is a simple example of the sector transform.

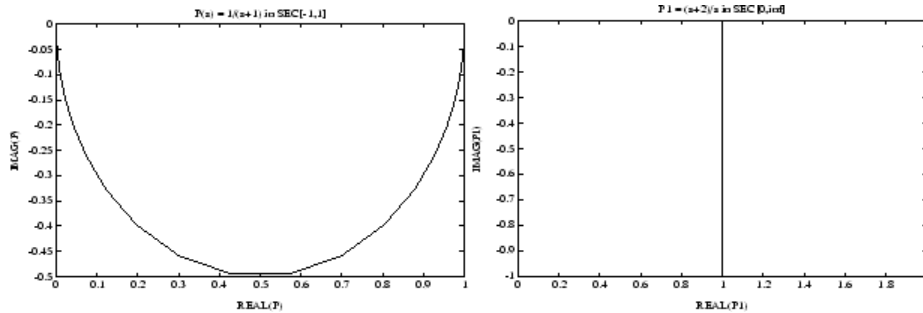
$$P(s) = \frac{1}{s+1} \in \text{sector}[-1, 1] \rightarrow P_1(s) = \frac{s+2}{2} \in \text{sector}[0, \infty].$$

You can compute this by simply executing the following commands:

```
P = ss(tf(1, [1 1]));
P1 = sectf(P, [-1, 1], [0, Inf]);
```

The Nyquist plots for this transformation are depicted in “Example of Sector Transform” on page 1-421. The condition  $P_1(s)$  inside  $[0, \infty]$  implies that  $P_1(s)$  is stable and  $P_1(j\omega)$  is *positive real*, i.e.,

$$P_1^*(j\omega) + P_1(j\omega) \geq 0 \quad \forall \omega$$



### Example of Sector Transform

## Limitations

A well-posed conic sector must have  $\det(B-A) \neq 0$  or

$$\det \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix} \neq 0.$$

Also, you must have  $\dim(u_{F1}) = \dim(y_{F1})$  since sectors are only defined for square systems.

## Algorithms

sectf uses the generalization of the sector concept of [3] described by [1]. First the sector input data  $S_f = \text{SECF}$  and  $S_g = \text{SECG}$  is converted to two-port state-space form; non-dynamical sectors are handled with empty  $a$ ,  $b1$ ,  $b2$ ,  $c1$ ,  $c2$  matrices. Next the equation

$$S_g(s) \begin{bmatrix} u_{g_1} \\ y_{g_1} \end{bmatrix} = S_f(s) \begin{bmatrix} u_{f_1} \\ y_{f_1} \end{bmatrix}$$

is solved for the two-port transfer function  $T(s)$  from  $u_{g_1} y_{f_1}$  to  $u_{f_1} y_{g_1}$ . Finally, the function `lftf` is used to compute  $G(s)$  as  $G = \text{lftf}(T, F)$ .

## References

- [1] Safonov, M.G., *Stability and Robustness of Multivariable Feedback Systems*. Cambridge, MA: MIT Press, 1980.
- [2] Safonov, M.G., E.A. Jonckheere, M. Verma and D.J.N. Limebeer, "Synthesis of Positive Real Multivariable Feedback Systems," *Int. J. Control*, vol. 45, no. 3, pp. 817-842, 1987.
- [3] Zames, G., "On the Input-Output Stability of Time-Varying Nonlinear Feedback Systems  $\geq$ — Part I: Conditions Using Concepts of Loop Gain, Conicity, and Positivity," *IEEE Trans. on Automat. Contr.*, AC-11, pp. 228-238, 1966.

## See Also

hinfsyn | lft

**Introduced before R2006a**

## setlmis

Initialize description of LMI system

### Syntax

```
setlmis(lmi0)
```

### Description

Before starting the description of a new LMI system with `lmivar` and `lmiterm`, type

```
setlmis([])
```

to initialize its internal representation.

To add on to an existing LMI system, use the syntax

```
setlmis(lmi0)
```

where `lmi0` is the internal representation of this LMI system. Subsequent `lmivar` and `lmiterm` commands will then add new variables and terms to the initial LMI system `lmi0`.

### See Also

`getlmis` | `lmiterm` | `lmivar` | `newlmi`

**Introduced before R2006a**

## setmvar

Instantiate matrix variable and evaluate all LMI terms involving this matrix variable

### Syntax

```
mnewsys = setmvar(lmisys,X,Xval)
```

### Description

`setmvar` sets the matrix variable  $X$  with identifier  $X$  to the value  $Xval$ . All terms involving  $X$  are evaluated, the constant terms are updated accordingly, and  $X$  is removed from the list of matrix variables. A description of the resulting LMI system is returned in `newsys`.

The integer  $X$  is the identifier returned by `lmivar` when  $X$  is declared. Instantiating  $X$  with `setmvar` does not alter the identifiers of the remaining matrix variables.

The function `setmvar` is useful to freeze certain matrix variables and optimize with respect to the remaining ones. It saves time by avoiding partial or complete redefinition of the set of LMI constraints.

### Examples

Consider the system

$$\dot{x} = Ax + Bu$$

and the problem of finding a stabilizing state-feedback law  $u = Kx$  where  $K$  is an unknown matrix.

By the Lyapunov Theorem, this is equivalent to finding  $P > 0$  and  $K$  such that

$$(A + BK)P + P(A + BK^T) + I < 0.$$

With the change of variable  $Y := KP$ , this condition reduces to the LMI

$$AP + PA^T + BY + Y^TB^T + I < 0.$$

This LMI is entered by the commands

```
n = size(A,1)           % number of states
ncon = size(B,2)       % number of inputs

setlmis([])
P = lmivar(1,[n 1])    % P full symmetric
Y = lmivar(2,[ncon n]) % Y rectangular

lmiterm([1 1 1 P],A,1,'s') % AP+PA'
lmiterm([1 1 1 Y],B,1,'s') % BY+Y'B'
lmiterm([1 1 1 0],1)    % I
lmis = getlmis
```

To find out whether this problem has a solution  $K$  for the particular Lyapunov matrix  $P = I$ , set  $P$  to  $I$  by typing

```
news = setmvar(lmis,P,1)
```

The resulting LMI system `news` has only one variable  $Y = K$ . Its feasibility is assessed by calling `feasp`:

```
[tmin,xfeas] = feasp(news)
Y = dec2mat(news,xfeas,Y)
```

The computed  $Y$  is feasible whenever `tmin < 0`.

## See Also

delmvar | evalldmi

**Introduced before R2006a**

## **showlmi**

Return left and right sides of LMI after evaluation of all variable terms

### **Syntax**

```
[lhs,rhs] = showlmi(evalsys,n)
```

### **Description**

For given values of the decision variables, the function `evallmi` evaluates all variable terms in a system of LMIs. The left and right sides of the  $n$ -th LMI are then constant matrices that can be displayed with `showlmi`. If `evalsys` is the output of `evallmi`, the values `lhs` and `rhs` of these left and right sides are given by

```
[lhs,rhs] = showlmi(evalsys,n)
```

An error is issued if `evalsys` still contains variable terms.

### **Examples**

See the description of `evallmi`.

### **See Also**

`evallmi` | `setmvar`

**Introduced before R2006a**



# simplify

Simplify representation of uncertain object

## Syntax

```
B = simplify(A)
```

```
B = simplify(A, 'full')
```

```
B = simplify(A, 'basic')
```

```
B = simplify(A, 'class')
```

## Description

`B = simplify(A)` performs model-reduction-like techniques to detect and eliminate redundant copies of uncertain elements. Depending on the result, the class of `B` may be lower than `A`. The `AutoSimplify` property of each uncertain element in `A` governs what reduction methods are used. After reduction, any uncertain element which does not actually affect the result is deleted from the representation.

`B = simplify(A, 'full')` overrides all uncertain element's `AutoSimplify` property, and uses 'full' reduction techniques.

`B = simplify(A, 'basic')` overrides all uncertain element's `AutoSimplify` property, and uses 'basic' reduction techniques.

`B = simplify(A, 'class')` does not perform reduction. However, any uncertain elements in `A` with zero occurrences are eliminated, and the class of `B` may be lower than the class of `A`.

## Examples

Create a simple `umat` with a single uncertain real parameter. Select specific elements, note that result remains in class `umat`. Simplify those same elements, and note that class changes.

```
p1 = ureal('p1',3,'Range',[2 5]);
L = [2 p1];
L(1)
UMAT: 1 Rows, 1 Columns
L(2)
UMAT: 1 Rows, 1 Columns
  p1: real, nominal = 3, range = [2 5], 1 occurrence
simplify(L(1))
ans =
     2
simplify(L(2))
Uncertain Real Parameter: Name p1, NominalValue 3, Range [2 5]
```

Create four uncertain real parameters, with a default value of `AutoSimplify('basic')`, and define a high order polynomial [1].

```
m = ureal('m',125000,'Range',[100000 150000]);
xcg = ureal('xcg',.23,'Range',[.15 .31]);
zcg = ureal('zcg',.105,'Range',[0 .21]);
va = ureal('va',80,'Range',[70 90]);
cw = simplify(m/(va*va)*va,'full')
UMAT: 1 Rows, 1 Columns
  m: real, nominal = 1.25e+005, range = [100000 150000],
1 occurrence
  va: real, nominal = 80, range = [70 90], 1 occurrence
cw = m/va;
fac2 = .16726*xcg*cw*cw*zcg - .17230*xcg*xcg*cw ...
      -3.9*xcg*cw*zcg - .28*xcg*xcg*cw*cw*zcg ...
      -.07*xcg*xcg*zcg + .29*xcg*xcg*cw*zcg ...
      + 4.9*xcg*cw - 2.7*xcg*cw*cw ...
      +.58*cw*cw - 0.25*xcg*xcg - 1.34*cw ...
      +100.1*xcg -14.1*zcg - 1.91*cw*cw*zcg ...
      +1.12*xcg*zcg + 24.6*cw*zcg ...
      +.45*xcg*xcg*cw*cw - 46.85
UMAT: 1 Rows, 1 Columns
  m: real, nominal = 1.25e+005, range = [100000 150000],
18 occurrences
  va: real, nominal = 80, range = [70 90], 8 occurrences
  xcg: real, nominal = 0.23, range = [0.15 0.31], 18 occurrences
  zcg: real, nominal = 0.105, range = [0 0.21], 1 occurrence
```

The result of the high-order polynomial is an inefficient representation involving 18 copies of `m`, 8 copies of `va`, 18 copies of `xcg` and 1 copy of `zcg`. Simplify the expression, using the `'full'` simplification algorithm

```

fac2s = simplify(fac2,'full')
UMAT: 1 Rows, 1 Columns
      m: real, nominal = 1.25e+005, range = [100000 150000],
4 occurrences
      va: real, nominal = 80, range = [70 90], 4 occurrences
      xcg: real, nominal = 0.23, range = [0.15 0.31], 2 occurrences
      zcg: real, nominal = 0.105, range = [0 0.21], 1 occurrence

```

which results in a much more economical representation.

Alternatively, change the AutoSimplify property of each parameter to 'full' before forming the polynomial.

```

m.AutoSimplify = 'full';
xcg.AutoSimplify = 'full';
zcg.AutoSimplify = 'full';
va.AutoSimplify = 'full';

```

You can form the polynomial, which immediately gives a low order representation.

```

cw = m/va;
fac2f = .16726*xcg*cw*cw*zcg - .17230*xcg*xcg*cw ...
        -3.9*xcg*cw*zcg - .28*xcg*xcg*cw*cw*zcg ...
        -.07*xcg*xcg*zcg + .29*xcg*xcg*cw*zcg ...
        + 4.9*xcg*cw - 2.7*xcg*cw*cw ...
        +.58*cw*cw - 0.25*xcg*xcg - 1.34*cw ...
        +100.1*xcg -14.1*zcg - 1.91*cw*cw*zcg ...
        +1.12*xcg*zcg + 24.6*cw*zcg ...
        +.45*xcg*xcg*cw*cw - 46.85
UMAT: 1 Rows, 1 Columns
      m: real, nominal = 1.25e+005, range = [100000 150000],
4 occurrences
      va: real, nominal = 80, range = [70 90], 4 occurrences
      xcg: real, nominal = 0.23, range = [0.15 0.31], 2 occurrences
      zcg: real, nominal = 0.105, range = [0 0.21], 1 occurrence

```

Create two real parameters, da and dx, and a 2-by-3 matrix, ABmat, involving polynomial expressions in the two real parameters .

```

da = ureal('da',0,'Range',[-1 1]);
dx = ureal('dx',0,'Range',[-1 1]);
a11 = -.32 + da*(.8089 + da*(-.987 + 3.39*da)) + .15*dx;
a12 = .934 + da*(.0474 - .302*da);
a21 = -1.15 + da*(4.39 + da*(21.97 - 561*da*da)) ...
      + dx*(9.65 - da*(55.7 + da*177));

```

```
a22 = -.66 + da*(1.2 - da*2.27) + dx*(2.66 - 5.1*da);
b1 = -0.00071 + da*(0.00175 - da*.00308) + .0011*dx;
b2 = -0.031 + da*(.078 + da*(-.464 + 1.37*da)) + .0072*dx;
ABmat = [a11 a12 b1;a21 a22 b2]
UMAT: 2 Rows, 3 Columns
  da: real, nominal = 0, range = [-1 1], 19 occurrences
  dx: real, nominal = 0, range = [-1 1], 2 occurrences
```

Use 'full' simplification to reduce the complexity of the description.

```
ABmatsimp = simplify(ABmat,'full')
UMAT: 2 Rows, 3 Columns
  da: real, nominal = 0, range = [-1 1], 7 occurrences
  dx: real, nominal = 0, range = [-1 1], 2 occurrences
```

Alternatively, you can set the parameter's AutoSimplify property to 'full'.

```
da.AutoSimplify = 'full';
dx.AutoSimplify = 'full';
```

Now you can rebuild the matrix

```
a11 = -.32 + da*(.8089 + da*(-.987 + 3.39*da)) + .15*dx;
a12 = .934 + da*(.0474 - .302*da);
a21 = -1.15 + da*(4.39 + da*(21.97 - 561*da*da)) ...
      + dx*(9.65 - da*(55.7 + da*177));
a22 = -.66 + da*(1.2 - da*2.27) + dx*(2.66 - 5.1*da);
b1 = -0.00071 + da*(0.00175 - da*.00308) + .0011*dx;
b2 = -0.031 + da*(.078 + da*(-.464 + 1.37*da)) + .0072*dx;
ABmatFull = [a11 a12 b1;a21 a22 b2]
UMAT: 2 Rows, 3 Columns
  da: real, nominal = 0, range = [-1 1], 7 occurrences
  dx: real, nominal = 0, range = [-1 1], 2 occurrences
```

## Limitations

Multidimensional model reduction and realization theory are only partially complete theories. The heuristics used by `simplify` are that - heuristics. The order in which expressions involving uncertain elements are built up, eg., distributing across addition and multiplication, can affect the details of the representation (i.e., the number of occurrences of a `ureal` in an uncertain matrix). It is possible that `simplify`'s naive methods cannot completely resolve these differences, so one may be forced to work with “nonminimal” representations of uncertain systems.

## Algorithms

simplify uses heuristics along with one-dimensional model reduction algorithms to partially reduce the dimensionality of the representation of an uncertain matrix or system.

## References

- [1] Varga, A. and G. Looye, "Symbolic and numerical software tools for LFT-based low order uncertainty modeling," *IEEE International Symposium on Computer Aided Control System Design*, 1999, pp. 5-11.
- [2] Belcastro, C.M., K.B. Lim and E.A. Morelli, "Computer aided uncertainty modeling for nonlinear parameter-dependent systems Part II: F-16 example," *IEEE International Symposium on Computer Aided Control System Design*, 1999, pp. 17-23.

## See Also

ucomplex | umat | ureal | uss | uss

**Introduced before R2006a**

## skewdec

Form skew-symmetric matrix

### Syntax

$X = \text{skewdec}(m, n)$

### Description

$X = \text{skewdec}(m, n)$  forms the  $m$ -by- $m$  skew-symmetric matrix

$$\begin{bmatrix} 0 & -(n+1) & -(n+2) & \dots \\ (n+1) & 0 & -(n+3) & \dots \\ (n+2) & (n+3) & 0 & \dots \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix}$$

This function is useful to define skew-symmetric matrix variables. In this case, set  $n$  to the number of decision variables already used.

### Examples

#### Skew-Symmetric Matrix

Create a 3-by-3 skew-symmetric matrix for an LMI problem in which  $n = 2$ . Display the matrix to verify its form.

$X = \text{skewdec}(3, 2)$

$X = 3 \times 3$

$$\begin{bmatrix} 0 & -3 & -4 \\ 3 & 0 & 0 \\ 4 & 0 & 0 \end{bmatrix}$$

|   |   |    |
|---|---|----|
| 3 | 0 | -5 |
| 4 | 5 | 0  |

## See Also

decinfo | lmvivar

**Introduced before R2006a**

## slowfast

Slow and fast modes decomposition

### Syntax

$[G1, G2] = \text{slowfast}(G, ns)$

### Description

`slowfast` computes the slow and fast modes decompositions of a system  $G(s)$  such that

$$G(s) = [G_1(s)] + [G_2(s)]$$

$G(s)$  contains the  $N$  slowest modes (modes with the smallest absolute value) of  $G$ .

$[G_1(s)] := (A_{11}, B_1, C_1, D_1)$  denotes the slow part of  $G(s)$ . The slow poles have low frequency and magnitude values.

$[G_2(s)] := (A_{22}, B_2, C_2, D_2)$  denotes the fast part. The fast poles have high frequency and magnitude values.

The variable `ns` denotes the index where the modes will be split.

Use `freqsep` to separate slow and fast modes at a specified cutoff frequency instead of a specified number of modes.

### References

M.G. Safonov, E.A. Jonckheere, M. Verma and D.J.N. Limebeer, "Synthesis of Positive Real Multivariable Feedback Systems", *Int. J. Control*, vol. 45, no. 3, pp. 817-842, 1987.



## See Also

freqsep | modreal | schur

**Introduced before R2006a**

## **squeeze**

Remove singleton dimensions for `umat` objects

### **Syntax**

`B = squeeze(A)`

### **Description**

`B = squeeze(A)` returns an array `B` with the same elements as `A` but with all the singleton dimensions removed. A singleton is a dimension such that `size(A,dim)==1`. 2-D arrays are unaffected by `squeeze` so that row vectors remain rows.

### **See Also**

`permute` | `reshape`

**Introduced before R2006a**

## uss/ssbal

Scale state/uncertainty while preserving uncertain input/output map of uncertain system

### Syntax

```
usysout = ssbal(usys)
```

```
usysout = ssbal(usys,wc)
```

```
usysout = ssbal(usys,wc,FSflag)
```

```
usysout = ssbal(usys,wc,FSflag,BLTflag)
```

### Description

`usysout = ssbal(usys)` yields a system whose input/output and uncertain properties are the same as `usys`, a `uss` object. The numerical conditioning of `usysout` is usually better than that of `usys`, improving the accuracy of additional computations performed with `usysout`. `usysout` is a `uss` object. The balancing algorithm uses `mussv` to balance the constant uncertain state-space matrices in discrete time. If `usys` is a continuous-time uncertain system, the uncertain state-space is mapped by using a bilinear transformation into discrete time for balancing.

`usysout = ssbal(usys,wc)` defines the critical frequency `wc` for the bilinear prewarp transformation from continuous time to discrete time. The default value of `wc` is 1 when the nominal uncertain system is stable and `1.25*mxeig` when it is unstable. `mxeig` corresponds to the value of the real, most positive pole of `usys`.

`usysout = ssbal(usys,wc,FSflag)` sets the scaling flag `FSflag` to handle repeated uncertain parameters. Setting `FSflag=1` uses full matrix scalings to balance the repeated uncertain parameter blocks. `FSflag=0`, the default, uses a single, positive scalar to balance the repeated uncertain parameter blocks.

`usysout = ssbal(usys,wc,FSflag,BLTflag)` sets the bilinear transformation flag, `BLTflag`. By default, `BLTflag=1` transforms the continuous-time system `usys` to a discrete-time system for balancing. `BLTflag=0` results in balancing the continuous-time

state-space data from `usys`. Note that if `usys` is a discrete-time system, no bilinear transformation is performed.

`ssbal` does not work on an array of uncertain systems. An error message is generated to alert you to this.

## Examples

Consider a two-input, two-output, two-state uncertain system with two real parameter uncertainties, `p1` and `p2`.

```
p2=ureal('p2',-17,'Range',[-19 -11]);
p1=ureal('p1',3.2,'Percentage',0.43);
A = [-12 p1;.001 p2];
B = [120 -809;503 24];
C = [.034 .0076;.00019 2];
usys = ss(A,B,C,zeros(2,2))
USS: 2 States, 2 Outputs, 2 Inputs, Continuous System
  p1: real, nominal = 3.2, variability = [-0.43 0.43]%, 1 occurrence
  p2: real, nominal = -17, range = [-19 -11], 1 occurrence
usys.NominalValue
a =
      x1      x2
x1   -12      3.2
x2   0.001    -17

b =
      u1      u2
x1   120   -809
x2   503     24

c =
      x1      x2
y1   0.034  0.0076
y2  0.00019      2

d =
      u1      u2
y1     0     0
y2     0     0
```

Continuous-time model.

`ssbal` is used to balance the uncertain system `usys`.

```
usysout = ssbal(usys)
USS: 2 States, 2 Outputs, 2 Inputs, Continuous System
  p1: real, nominal = 3.2, variability = [-0.43 0.43]%,
1 occurrence
  p2: real, nominal = -17, range = [-19 -11], 1 occurrence

usysout.NominalValue
```

```
a =
      x1      x2
x1      -12    0.3302
x2  0.009692    -17
```

```
b =
      u1      u2
x1  0.7802   -5.26
x2  31.7    1.512
```

```
c =
      x1      x2
y1  5.229    0.1206
y2  0.02922  31.74
```

```
d =
      u1  u2
y1     0   0
y2     0   0
```

Continuous-time model.

## See Also

[c2d](#) | [canon](#) | [d2c](#) | [mussv](#) | [mussvextract](#) | [ss2ss](#)

**Introduced before R2006a**

## stack

Construct array by stacking uncertain matrices, models, or arrays

### Syntax

```
umatout = stack(arraydim,umat1,umat2,...)
```

```
usysout = stack(arraydim,usys1,usys2,...)
```

### Description

`stack` constructs an uncertain array by stacking uncertain matrices, models, or arrays along array dimensions of an uncertain array.

`umatout = stack(arraydim,umat1,umat2,...)` produces an array of uncertain matrices, `umatout`, by stacking (concatenating) the `umat` matrices (or `umat` arrays) `umat1, umat2,...` along the array dimension `arraydim`. All models must have the same number of columns and rows. The column/row dimensions are not counted in the array dimensions.

`umatout = stack(arraydim,usys1,usys2,...)` produces an array of uncertain models, `ufrd` or `uss`, or `usysout`, by stacking (concatenating) the `ufrd` or `uss` matrices (or `ufrd` or `uss` arrays) `usys1, usys2,...` along the array dimension `arraydim`. All models must have the same number of columns and rows (the same input/output dimensions). Note that the input/output dimensions are not considered for arrays.

### Examples

Consider `usys1` and `usys2`, two single-input/single-output `uss` models:

```
zeta = ureal('zeta',1,'Range',[0.4 4]);  
wn = ureal('wn',0.5,'Range',[0.3 0.7]);  
P1 = tf(1,[1 2*zeta*wn wn^2]);  
P2 = tf(zeta,[1 10]);
```

You can stack along the first dimension to produce a 2-by-1 `uss` array.

```
stack(1,P1,P1)
USS: 2 States, 1 Output, 1 Input, Continuous System [array, 2 x 1]
    wn: real, nominal = 0.5, range = [0.3 0.7], 3 occurrences
    zeta: real, nominal = 1, range = [0.4 4], 1 occurrence
```

You can stack along the second dimension to produce a 1-by-2 uss array.

```
stack(2,P1,P2) % produces a 1-by-2 USS array.
USS: 2 States, 1 Output, 1 Input, Continuous System [array, 1 x 2]
    wn: real, nominal = 0.5, range = [0.3 0.7], 3 occurrences
    zeta: real, nominal = 1, range = [0.4 4], 1 occurrence
```

You can stack along the third dimension to produce a 1-by-1-by-2 uss array.

```
stack(3,P1,P2) % produces a 1-by-1-by-2 USS array.
USS: 2 States, 1 Output, 1 Input, Continuous System
[array, 1 x 1 x 2]
    wn: real, nominal = 0.5, range = [0.3 0.7], 3 occurrences
    zeta: real, nominal = 1, range = [0.4 4], 1 occurrence
```

## See Also

[append](#) | [blkdiag](#) | [horzcat](#) | [vertcat](#)

**Introduced before R2006a**

## symdec

Form symmetric matrix

### Syntax

`x = symdec(m,n)`

### Description

`symdec(m,n)` forms an  $m$ -by- $m$  symmetric matrix of the form

$$\begin{bmatrix} (n+1) & (n+2) & (n+4) & \dots \\ (n+2) & (n+3) & (n+5) & \dots \\ (n+4) & (n+5) & (n+6) & \dots \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix}$$

This function is useful to define symmetric matrix variables.  $n$  is the number of decision variables.

### Examples

#### Symmetric Matrix

Create a 4-by-4 symmetric matrix for an LMI problem in which  $n = 2$ . Display the matrix to verify its form.

`X = symdec(4,2)`

`X = 4×4`

```
    3    4    6    9
    4    5    7   10
```



6 7 8 11  
9 10 11 12

## See Also

[decinfo](#) | [skewdec](#)

**Introduced before R2006a**

## sysic

Build interconnections of certain and uncertain matrices and systems

### Syntax

```
sysout = sysic
```

### Description

`sysic` requires that 3 variables with fixed names be present in the calling workspace: `systemnames`, `inputvar` and `outputvar`.

`systemnames` is a char containing the names of the subsystems (`double`, `tf`, `zpk`, `ss`, `uss`, `frd`, `ufrd`, etc) that make up the interconnection. The names must be separated by spaces with no additional punctuation. Each named variable must exist in the calling workspace.

`inputvar` is a char, defining the names of the external inputs to the interconnection. The names are separated by semicolons, and the entire list is enclosed in square brackets [ ]. Inputs can be scalar or multivariate. For instance, a 3-component ( $x, y, z$ ) force input can be specified with 3 separate names, `Fx`, `Fy`, `Fz`. Alternatively, a single name with a defined integer dimension can be specified, as in `F{3}`. The order of names in `inputvar` determines the order of inputs in the interconnection.

`outputvar` is a char, describing the outputs of the interconnection. Outputs do not have names—they are simply linear combinations of individual subsystem's outputs and external inputs. Semicolons delineate separate components of the interconnections outputs. Between semicolons, signals can be added and subtracted, and multiplied by scalars. For multivariable subsystems, arguments within parentheses specify which subsystem outputs are to be used and in what order. For instance, `plant(2:4, 1, 9:11)` specifies outputs 2, 3, 4, 1, 9, 10, 11 from the subsystem `plant`. If a subsystem is listed in `outputvar` without arguments, then all outputs from that subsystem are used.

`sysic` also requires that for every subsystem name listed in `systemnames`, a corresponding variable, `input_to_ListedSubSystemName` must exist in the calling workspace. This variable is similar to `outputvar` - it defines the input signals to this

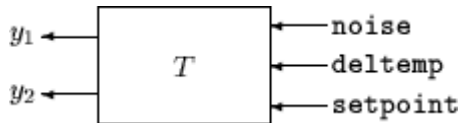
particular subsystem as linear combinations of individual subsystem's outputs and external inputs.

`sysout = sysic` will perform the interconnection described by the variables above, using the subsystem data in the names found in `systemnames`. The resulting interconnection is returned in the output argument, listed above as `sysout`.

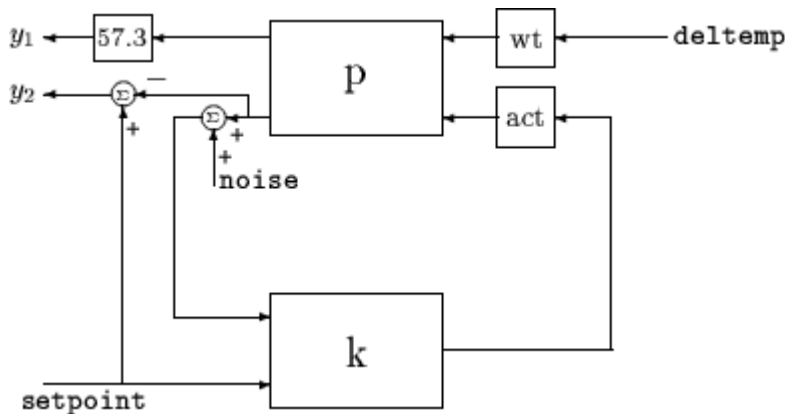
After running `sysic` the variables `systemnames`, `inputvar`, `outputvar` and all of the `input_to_ListedSubSystemName` will exist in the workspace. Setting the optional variable `cleanupsysic` to 'yes' will cause these variables to be removed from the workspace after `sysic` has formed the interconnection.

## Examples

A simple system interconnection, identical to the system illustrated in the `iconnect` description. Consider a three-input, two-output LTI matrix  $T$ ,



which has internal structure



```
P = rss(3,2,2);
K = rss(1,1,2);
```

```
A = rss(1,1,1);
W = rss(1,1,1);
systemnames = 'W A K P';
inputvar = '[noise;deltemp;setpoint]';
outputvar = '[57.3*P(1);setpoint-P(2)]';
input_to_W = '[deltemp]';
input_to_A = '[K]';
input_to_K = '[P(2)+noise;setpoint]';
input_to_P = '[W;A]';
cleanupysic = `yes`;
T = sysic;
exist(`inputvar`)
```

## Limitations

The syntax of `sysic` is limited, and for the most part is restricted to what is shown here. The `iconnect` interconnection object can also be used to define complex interconnections, and has a more flexible syntax.

Within `sysic`, error-checking routines monitor the consistency and availability of the subsystems and their inputs. These routines provide a basic level of error detection to aid the user in debugging.

## See Also

`iconnect`

**Introduced before R2006a**

# ucomplex

Create uncertain complex parameter

## Syntax

```
A = ucomplex('NAME',nominalvalue)
```

```
A = ucomplex('NAME',nominalvalue,'Property1',Value1,...  
            'Property2',Value2,...)
```

## Description

An uncertain complex parameter is used to represent a complex number whose value is uncertain. Uncertain complex parameters have a name (the `Name` property), and a nominal value (the `NominalValue` property).

The uncertainty (potential deviation from the nominal value) is described in two different manners:

- **Radius** (radius of disc centered at `NominalValue`)
- **Percentage** (disc size is percentage of magnitude of `NominalValue`)

The `Mode` property determines which description remains invariant if the `NominalValue` is changed (the other is derived). The default `Mode` is `'Radius'` and the default radius is 1.

Property/Value pairs can also be specified at creation. For instance,

```
B = ucomplex('B',6-j,'Percentage',25)
```

sets the nominal value to  $6 - j$ , the percentage uncertainty to 25 and, implicitly, the `Mode` to `'Percentage'`.

## Examples

### Sample Uncertain Complex Parameter

Compute 400 random samples of an uncertain complex parameter and visualize them in a plot.

Create an uncertain complex parameter with internal name A.

```
A = ucomplex('A',4+3*j)
```

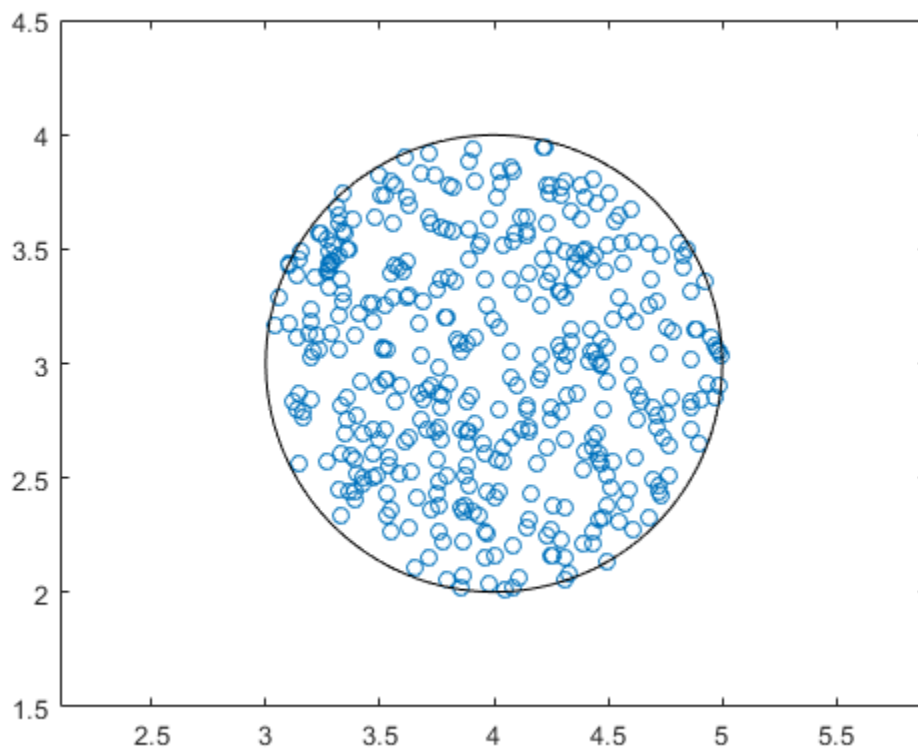
```
A =
```

```
Uncertain complex parameter "A" with nominal value 4+3i and radius 1.
```

The uncertain parameter's possible values are a complex disc of radius 1, centered at  $4 + 3j$ . The value of `A.percentage` is 20 (radius is 1/5 of the magnitude of the nominal value).

You can visualize the uncertain complex parameter by sampling and plotting the data.

```
sa = usample(A,400);  
w = linspace(0,2*pi,200);  
circ = sin(w) + j*cos(w);  
rc = real(A.NominalValue+circ);  
ic = imag(A.NominalValue+circ);  
plot(real(sa(:)),imag(sa(:)), 'o',rc,ic, 'k-')  
xlim([2.5 5.5])  
ylim([1.5 4.5])  
axis equal
```



## See Also

`get | ucomplexm | ultidyn | umat | ureal`

**Introduced before R2006a**

## ucomplexm

Create uncertain complex matrix

### Syntax

```
M = ucomplexm('Name',NominalValue)
```

```
M = ucomplexm('Name',NominalValue,'WL',WLvalue,'WR',WRvalue)
```

```
M = ucomplexm('Name',NominalValue,'Property',Value)
```

### Description

`M = ucomplexm('Name',NominalValue)` creates an uncertain complex matrix representing a ball of complex-valued matrices, centered at a `NominalValue` and named `Name`.

`M = ucomplexm('Name',NominalValue,'WL',WLvalue,'WR',WRvalue)` creates an uncertain complex matrix with weights `WL` and `WR`. Specifically, the values represented by `M` are all matrices `H` that satisfy  $\text{norm}(\text{inv}(M.WL)*(H - M.NominalValue)*\text{inv}(M.WR)) \leq 1$ . `WL` and `WR` are square, invertible, and weighting matrices that quantify the size and shape of the ball of matrices represented by this object. The default values for `WL` and `WR` are identity matrices of appropriate dimensions.

Trailing `Property/Value` pairs are allowed, as in

```
M = ucomplexm('NAME',nominalvalue,'P1',V1,'P2',V2,...)
```

The property `AutoSimplify` controls how expressions involving the uncertain matrix are simplified. Its default value is `'basic'`, which means elementary methods of simplification are applied as operations are completed. Other values for `AutoSimplify` are `'off'`, no simplification performed, and `'full'` which applies model-reduction-like techniques to the uncertain object.



## Examples

### Sample an Uncertain Complex Matrix

Create a `ucomplexm` with the name `F`, nominal value `[1 2 3; 4 5 6]`, and weighting matrices `WL = diag([.1 .3])`, `WR = diag([.4 .8 1.2])`.

```
F = ucomplexm('F',[1 2 3;4 5 6], 'WL',diag([.1 .3]),...
    'WR',diag([.4 .8 1.2]))
```

`F =`

```
Uncertain complex matrix "F" with 2 rows and 3 columns.
```

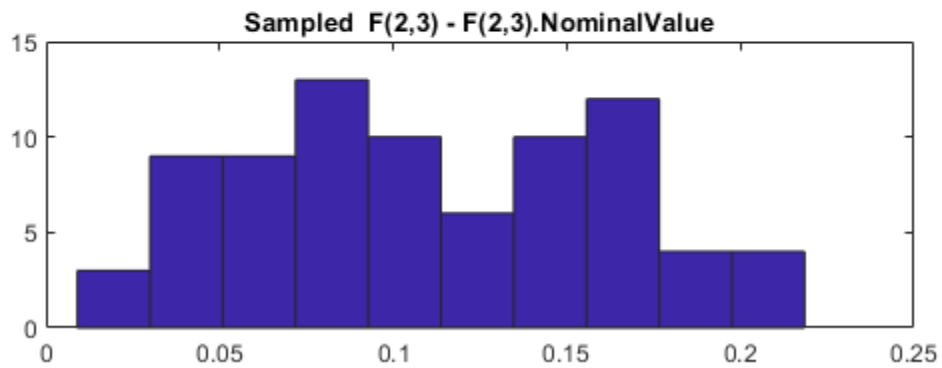
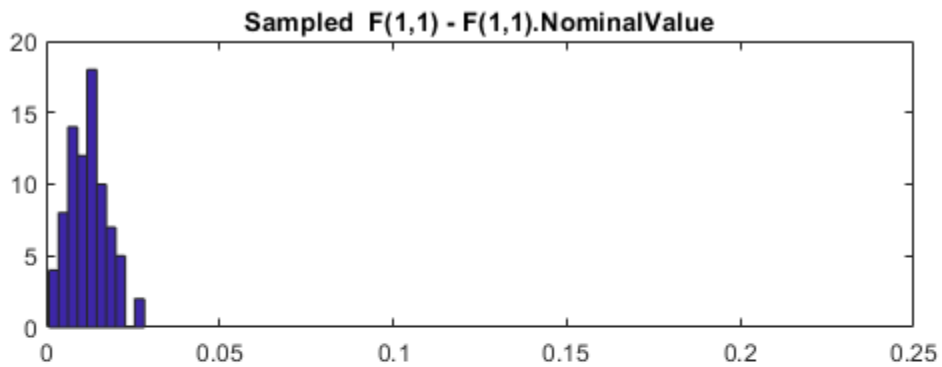
Sample the difference between the uncertain matrix and its nominal value at 80 points, yielding a 2-by-3-by-80 matrix `typicaldev`.

```
typicaldev = usample(F - F.NominalValue,80);
```

Plot histograms of the deviations in the (1,1) entry and the (2,3) entry of the complex matrix.

The absolute values of the (1,1) entry and the (2,3) entry are shown by histogram plots. Typical deviations in the (1,1) entry should be about 10 times smaller than the typical deviations in the (2,3) entry.

```
subplot(2,1,1);
td11 = squeeze(typicaldev(1,1,:));
hist(abs(td11));
xlim([0 .25])
title('Sampled F(1,1) - F(1,1).NominalValue')
subplot(2,1,2);
td23 = squeeze(typicaldev(2,3,:));
hist(abs(td23));
title('Sampled F(2,3) - F(2,3).NominalValue')
```



## See Also

`get | ucomplex | ultidyn | umat | ureal`

**Introduced before R2006a**

## ucover

Fit an uncertain model to set of LTI responses

### Syntax

```
usys = ucover(Parray,Pnom,ord)
usys = ucover(Parray,Pnom,ord1,ord2,utype)
[usys,info] = ucover(Parray,...)
[usys_new,info_new] = ucover(Pnom,info,ord1_new,ord2_new)
```

### Description

`usys = ucover(Parray,Pnom,ord)` returns an uncertain model `usys` with nominal value `Pnom` and whose range of behaviors includes all responses in the LTI array `Parray`.

The uncertain model structure is of the form  $usys = Pnom(1 + W(s)\Delta(s))$ , where

- $\Delta$  is an `ultidyn` object that represents uncertain dynamics with unit peak gain.
- $W$  is a stable, minimum-phase shaping filter that adjusts the amount of uncertainty at each frequency.

`ord` is the number of states (order) of  $W$ . `Pnom` and `Parray` can be `ss`, `tf`, `zpk`, or `zpk` models. `usys` is of class `ufrd` when `Pnom` is an `frd` model and is an `uss` model otherwise.

`usys = ucover(Parray,Pnom,ord1,ord2,utype)` specifies the order `ord1` and `ord2` of each diagonal entry of  $W1$  and  $W2$ , where  $W1$  and  $W2$  are diagonal, stable, minimum-phase shaping filters. `utype` specifies the uncertain model structure, as described in “Uncertain Model Structures” on page 1-458, and can be `'InputMult'` (default), `'OutputMult'` or `'Additive'`. `ord1` and `ord2` can be:

- `[]`, which implies that the corresponding filter is 1.
- Scalar, which implies that the corresponding filter is scalar-valued.
- Vectors with as many entries as diagonal entries in  $W1$  and  $W2$ .

`[usys,info] = ucover(Parray,...)` returns a structure `info` that contains optimization information. `info.W1opt` and `Info.W2opt` contain the values of  $W1$  and

W2 computed on a frequency grid and `info.W1` and `info.W2` contain the fitted shaping filters W1 and W2.

`[usys_new,info_new] = ucover(Pnom,info,ord1_new,ord2_new)` improves the fit using initial filter values in `info` and new orders `ord1_new` and `ord2_new` of W1 and W2. This syntax speeds up command execution by reusing previously computed information. Use this syntax when you are changing filter orders in an iterative workflow.

## Examples

### Fit Uncertain Model to Array of LTI Responses

Fit an uncertain model to an array of LTI responses. The responses might be, for example, the results of multiple runs of acquisition of frequency response data from a physical system.

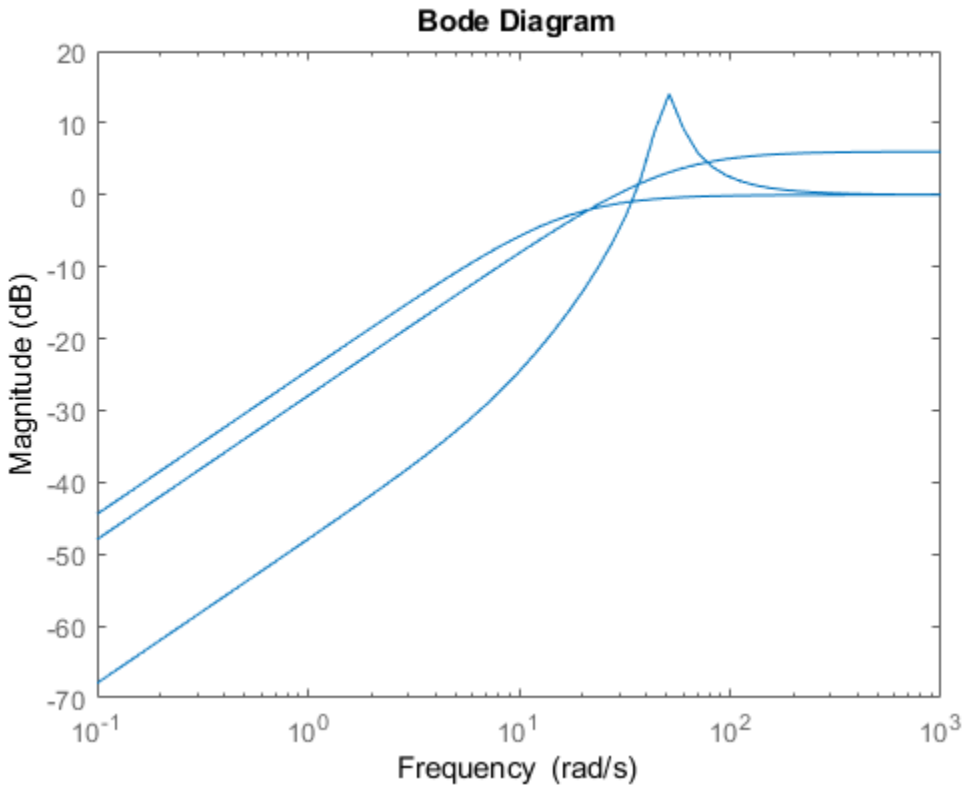
For the purpose of this example, generate the frequency response data by creating an array of LTI models and sampling the frequency response of those models.

```
Pnom = tf(2,[1 -2]);  
p1 = Pnom*tf(1,[.06 1]);  
p2 = Pnom*tf([- .02 1],[.02 1]);  
p3 = Pnom*tf(50^2,[1 2*.1*50 50^2]);  
array = stack(1,p1,p2,p3);  
Parray = frd(array,logspace(-1,3,60));
```

The frequency response data in `Parray` represents three separate data acquisition experiments on the system.

Plot relative errors between the nominal plant response and the three models in the LTI array.

```
relerr = (Pnom-Parray)/Pnom;  
bodemag(relerr)
```



If you use a multiplicative uncertainty model structure, the magnitude of the shaping filter should fit the maximum relative errors at each frequency.

Try a 1st-order shaping filter to fit the maximum relative errors.

```
[P,Info] = ucover(Parray,Pnom,1);
```

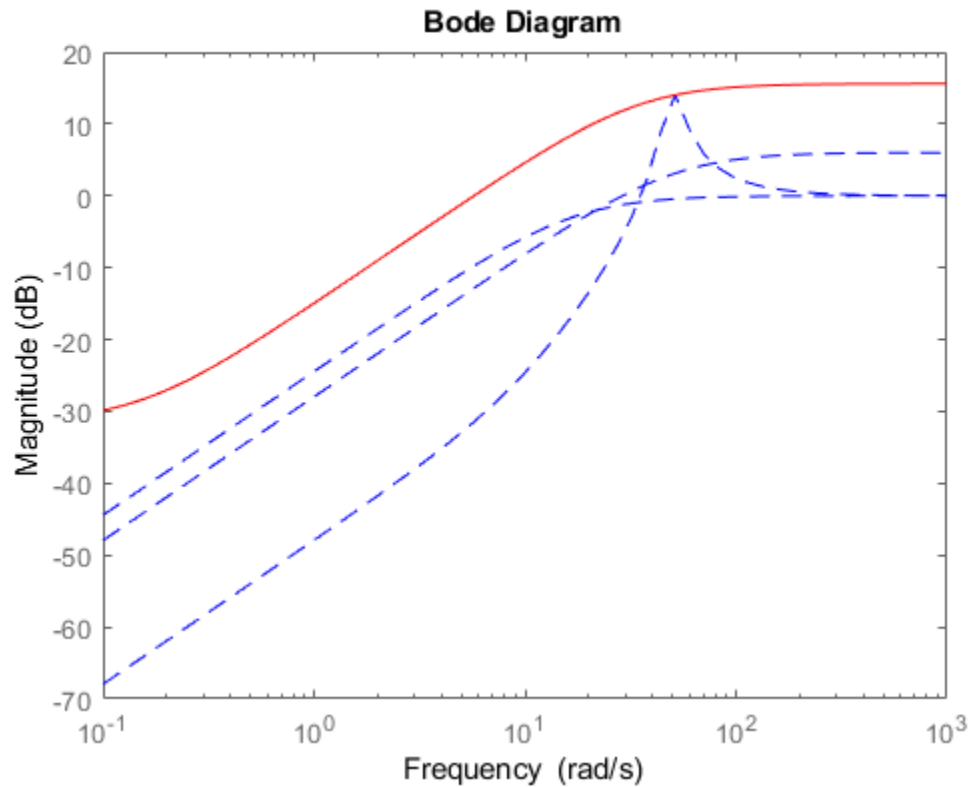
P is an uncertain state-space (uss) model that captures the uncertainty as a `ultidyn` uncertain dynamics block.

P.Uncertainty

```
ans = struct with fields:
  Parray_InputMultDelta: [1x1 ultidyn]
```

The `Info` structure contains other information about the fit, including the shaping filter. Plot the response to see how well the shaping filter fits the relative errors.

```
bodemag(releerr, 'b--', Info.W1, 'r', {0.1, 1000});
```

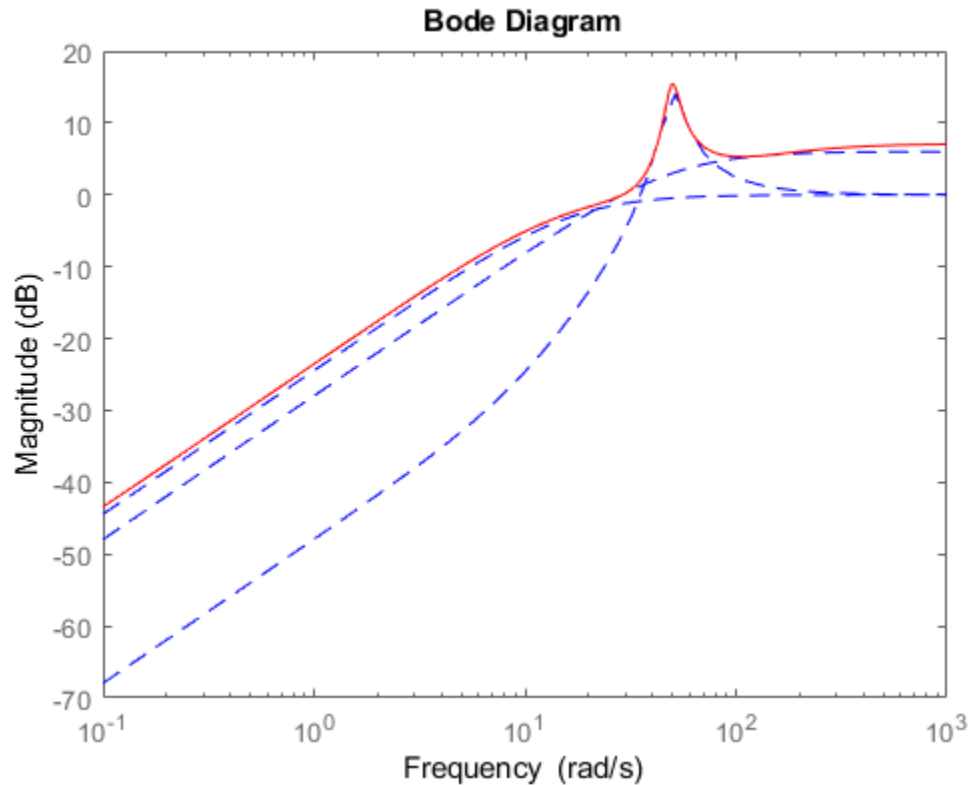


The plot shows that the filter `W1` is too conservative and exceeds the maximum relative error at most frequencies. To obtain a tighter fit, rerun the function using a 4th-order filter.

```
[P, Info] = ucover(Parray, Pnom, 4);
```

Evaluate the fit by plotting the Bode magnitude plot.

```
bodemag(releerr, 'b--', Info.W1, 'r', {0.1, 1000});
```



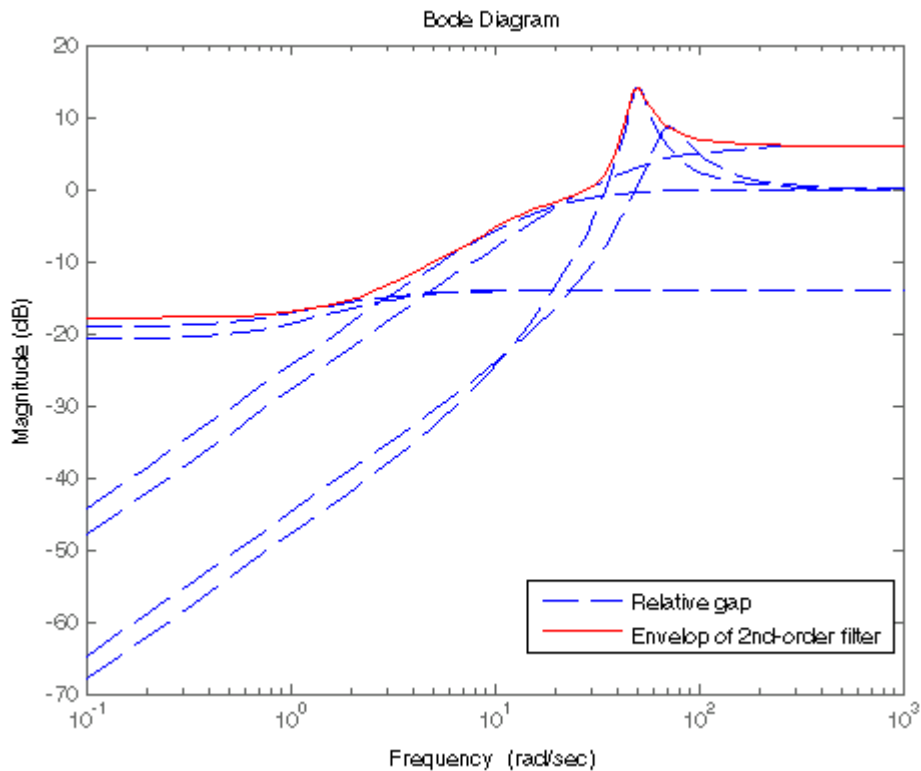
This plot shows that for the 4th-order filter, the magnitude of  $W1$  closely matches the minimum uncertainty amount.

- “Modeling a Family of Responses as an Uncertain System”
- “Simultaneous Stabilization Using Robust Control”
- “First-Cut Robust Design”

## Definitions

### Uncertain Model Structures

When fitting the responses of LTI models in `Parray`, the gaps between `Parray` and the nominal response `Pnom` of the uncertain model are modeled as uncertainty on the system dynamics. To model the frequency distribution of these unmodeled dynamics, `ucover` measures the gap between `Pnom` and `Parray` at each frequency and selects a shaping filter  $W$  whose magnitude approximates the maximum gap between `Pnom` and `Parray`. The following figure shows the relative gap between the nominal response and six LTI responses, enveloped using a second-order shaping filter.



The software then sets the uncertainty to  $W \cdot \Delta$ , where  $\Delta$  is an `ultidyn` object that represents unit-gain uncertain dynamics. This ensures that the amount of uncertainty at



each frequency is specified by the magnitude of  $W$  and therefore closely tracks the gap between  $P_{nom}$  and  $P_{array}$ .

There are three possible uncertainty model structures:

- Input Multiplicative of the form  $usys = P_{nom} \times (I + W_1 \times \Delta \times W_2)$ .
- Output Multiplicative of the form  $usys = (I + W_1 \times \Delta \times W_2) \times P_{nom}$ .
- Additive of the form  $usys = P_{nom} + W_1 \times \Delta \times W_2$ .

Use additive uncertainty to model the absolute gaps between  $P_{nom}$  and  $P_{array}$ , and multiplicative uncertainty to model relative gaps.

---

**Note** For SISO models, input and output multiplicative uncertainty are equivalent. For MIMO systems with more outputs than inputs, the input multiplicative structure may be too restrictive and not adequately cover the range of models.

---

The model structure  $usys = P_{nom} \times (I + W \times \Delta)$  that you obtain using `usys = ucover(Parray, Pnom, ord)`, corresponds to  $W_1 = W \times I$  and  $W_2 = 1$ .

## Algorithms

The `ucover` command designs the minimum-phase shaping filters  $W_1$  and  $W_2$  in two steps:

- 1 Computes the optimal values of  $W_1$  and  $W_2$  on a frequency grid.
- 2 Fits  $W_1$  and  $W_2$  values with the dynamic filters of the specified orders using the `fitmagfrd` command.

## See Also

`frd` | `ss` | `tf` | `usample` | `zpk`

## Topics

“Modeling a Family of Responses as an Uncertain System”  
 “Simultaneous Stabilization Using Robust Control”  
 “First-Cut Robust Design”

**Introduced in R2009b**

## udyn

Create unstructured uncertain dynamic system object

### Syntax

```
n = udyn('name',iosize);
```

### Description

`n = udyn('name',iosize)` creates an unstructured uncertain dynamic system class, with input/output dimension specified by `iosize`. This object represents the class of completely unknown multivariable, time-varying nonlinear systems.

For practical purposes, these uncertain elements represent noncommuting symbolic variables (placeholders). All algebraic operations, such as addition, subtraction, and multiplication (i.e., cascade) operate properly, and substitution (with `usubs`) is allowed.

The analysis tools (e.g., `robstab`) do not currently handle these types of uncertain elements. Therefore, these elements do not provide a significant amount of usability, and their role in the toolbox is small.

### Examples

You can create a 2-by-3 `udyn` element and check its size and properties.

```
N = udyn('N',[2 3])
Uncertain Dynamic System: Name N, size 2x3
size(N)
ans =
     2     3
get(N)
      Name: 'N'
      NominalValue: [2x3 double]
      AutoSimplify: 'basic'
```

## **See Also**

ucomplex | ucomplexm | ultidyn | ureal

**Introduced before R2006a**

## ufind

Find uncertain variables in Simulink model

### Syntax

```
uvars = ufind('mdl')  
[uvars,pathinfo] = ufind('mdl')  
uvars = ufind(usys_1,usys_2,...)
```

### Description

`uvars = ufind('mdl')` finds Uncertain State Space blocks in the Simulink model `mdl`. It returns a structure `uvars` that contains all uncertain variables associated with the Uncertain State Space blocks. Each uncertain variable is a `ureal` or `ultidyn` object and is listed by name in `uvars`.

`[uvars,pathinfo] = ufind('mdl')` returns a cell array `pathinfo` that contains paths to the Uncertain State Space blocks and the corresponding uncertain variables in the block. The first column of `pathinfo` lists the block paths through the model hierarchy and the second column lists the uncertain variables associated with the block. Use `pathinfo` to verify that all Uncertain State Space blocks in the model `mdl` have been identified.

`uvars = ufind(usys_1,usys_2,...)` collects all uncertain variables referenced by the uncertain model `usys_n`. `usys_n` can be `uss` or `ufrd` models. Use this syntax as an alternative to querying the model itself, when you know the uncertain models that the Uncertain State Space blocks use.

`ufind` can find Uncertain State Space blocks inside Masked Subsystems, Library Links, and Model References but not inside Accelerated Model References. `ufind` errors out if the same uncertain variable name has different definitions in the model. For example, if your model contains two Uncertain State Space blocks where the uncertain system variables define the same uncertain variable `'unc_par'` as `ultidyn('unc_par',[1 1])` and `ureal('unc_par',5)`, such an error occurs.

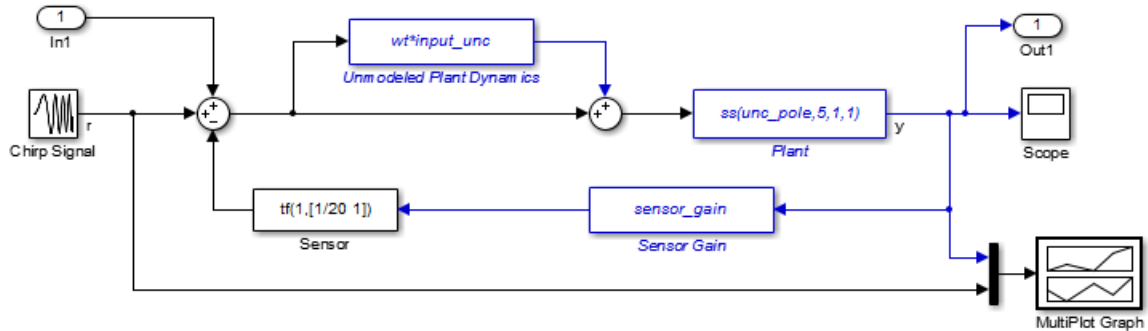
## Examples

Find all Uncertain State Space blocks and uncertain variables in a Simulink model:

- 1 Open the Simulink model.

```
open_system('usim_model')
```

The model, as shown in the following figure, contains three Uncertain State Space blocks named Unmodeled Plant Dynamics, Plant, and Sensor Gain. These blocks depend on three uncertain variables named `input_unc`, `unc_pole` and `sensor_gain`.



- 2 Use `ufind` to find all Uncertain State Space blocks and uncertain variables in the model.

```
[uvars,pathinfo] = ufind('usim_model')
```

- 3 Type `uvars` to view the structure `uvars`. MATLAB returns the following result:

```
uvars =
```

```
input_unc: [1x1 ultidyn]
sensor_gain: [1x1 ureal]
unc_pole: [1x1 ureal]
```

Each uncertain variable is a `ureal` or `ultidyn` object and is listed by name in `uvars`.

- 4 View the Uncertain State Space block paths and uncertain variables.
  - a Type `pathinfo(:,1)` to view paths of the Uncertain State Space blocks in the model. MATLAB returns the following result:

```
ans =  
  
    'usim_model/Plant'  
    'usim_model/Sensor Gain'  
    'usim_model/Unmodeled Plant Dynamics'
```

- b** Type `pathinfo(:,2)` to view the uncertain variables referenced by each Uncertain State Space block. MATLAB returns the following results:

```
ans =  
  
    'unc_pole'  
    'sensor_gain'  
    'input_unc'
```

## Tutorials

“Vary Uncertainty Values Using Individual Uncertain State Space Blocks”

“Vary Uncertainty Values Across Multiple Uncertain State Space Blocks”

Robustness Analysis in Simulink

## How To

“Simulate Uncertainty Effects”

## See Also

Uncertain State Space | `usample`

**Introduced in R2009b**

## ufrd

Uncertain frequency response data model

### Syntax

```
ufrd_sys = ufrd(M, freqs)
ufrd_sys = ufrd(M, freqs, frequits)
ufrd_sys = ufrd(M, freqs, frequits, timeunits)
```

### Description

Uncertain frequency response data models (`ufrd`) arise when combining numeric `frd` models with uncertain models such as `ureal`, `ultidyn`, or `uss`. A `ufrd` model keeps track of how the uncertain elements affect the frequency response. Use `ufrd` for robust stability and worst-case performance analysis.

There are three ways to construct a `ufrd` model:

- 1 Combine numeric `frd` models with uncertain models using model arithmetic. For example:

```
sys = frd(rand(100,1), logspace(-2,2,100));
k = ureal('k',1);
D = ultidyn('Delta',[1 1]);
ufrd_sys = k*sys*(1+0.1*D)
```

`ufrd_sys` is a `ufrd` model with uncertain elements `k` and `D`.

- 2 `ufrd_sys = ufrd(M, freqs)` converts the dynamic system model (Control System Toolbox) or static model (Control System Toolbox) `M` to `ufrd`. If `M` contains Control Design Blocks that do not represent uncertainty, these blocks are replaced by their current value. (To preserve both tunable and uncertain Control Design Blocks, use `genfrd` instead.)

Use `ufrd_sys = ufrd(M, freqs, frequits)` to specify the frequency units of the frequencies in `freqs`. The argument `frequits` can take the following values:



- 'rad/TimeUnit'
- 'cycles/TimeUnit'
- 'rad/s'
- 'Hz'
- 'kHz'
- 'MHz'
- 'GHz'
- 'rpm'

Use `ufrd_sys = ufrd(M, freqs, frequents, timeunits)` to specify the time unit of `ufrd_sys` when `M` is a static model. `timeunits` can take the following values:

- 'nanoseconds'
- 'microseconds'
- 'milliseconds'
- 'seconds'
- 'minutes'
- 'hours'
- 'days'
- 'weeks'
- 'months'
- 'years'

- 3 Use `frd` to construct a `ufrd` model from an uncertain matrix (`umat`) representing uncertain frequency response data. For example:

```
a = ureal('delta',1,'percent',50);
freq = logspace(-2,2,100);
RespData = rand(1,1,100) * a;
usys = frd(RespData,freq,0.1)
```

## Examples

Compute the uncertain frequency response of an uncertain system (`uss` model) with both parametric uncertainty (`ureal`) and unmodeled dynamics uncertainty (`ultidyn`).

```
p1 = ureal('p1',5,'Range',[2 6]);  
p2 = ureal('p2',3,'Plusminus',0.4);  
p3 = ultidyn('p3',[1 1]);  
Wt = makeweight(.15,30,10);  
A = [-p1 0;p2 -p1];  
B = [0;p2];  
C = [1 1];  
usys = uss(A,B,C,0)*(1+Wt*p3);
```

```
usysfrd = ufrd(usys,logspace(-2,2,60));
```

Plot 20 random samples and the nominal value of the uncertain frequency response.

```
bode(usysfrd,'r',usysfrd.NominalValue,'b+')
```

## See Also

frd | genfrd | ss | uss

## Topics

“Control Design Blocks” (Control System Toolbox)

**Introduced before R2006a**

# ulinearize

Linearize Simulink model with Uncertain State Space block

## Syntax

```
ulin = ulinearize('sys',io)
ulin = ulinearize('sys',op,io)
ulin = ulinearize('sys',op,io,options)
ulin = ulinearize('sys',op)
ulin_block = ulinearize('sys',op,'blockname')
[ulin,op] = ulinearize('sys',snapshottimes,...);
ulin = ulinearize('sys','StateOrder',stateorder)
```

## Description

`ulin = ulinearize('sys',io)` linearizes the Simulink model `sys` that contains Uncertain State Space blocks and returns a linear time-invariant uncertain system `ulin`. `ulin` is an `uss` object. `io` is an I/O object that specifies linearization I/O points in the model. Use `getlinio` or `linio` to create `io`. The linearization occurs at the operating point specified in the model.

`ulin=ulinearize('sys',io,op)` linearizes the model at the operating point specified in the operating point object `op`. Use `operpoint` or `findop` to create `op`. Both `op` and `io` are associated with the same model `sys`.

`ulin=ulinearize('sys',io,op,options)` takes a linearization options object `options` that contains several options for linearization and returns linear time-invariant uncertain system `ulin`. Use `linearizeOptions` to create `options`.

`ulin=ulinearize('sys',op)` linearizes the model `sys` at the operating point specified in the operating point object `op`. The software uses root-level inport and outport blocks in `sys` as I/O points for linearization.

`ulin_block=ulinearize('sys',op,'blockname',...)` takes the name of a block `blockname` in the model `sys` and returns a linear time-invariant uncertain system `ulin_block`. You can also specify a fourth argument `options` to provide options for the linearization.

`[ulin,op] = ulinearize('sys',snapshottimes,...)` creates operating points for linearization by simulating the model and taking snapshots of the system's states and inputs at times specified in the vector `snapshottimes`. `ulin` is a set of linear time-invariant uncertain systems and `op` is the set of operating point objects used in linearization. You can also specify I/O object for linearization, or a block name. If you do not specify an I/O object or block name, the linearization uses root-level inport and outport blocks in the model. You can also supply an additional argument, `options`, to provide options for linearization.

`ulin = ulinearize('sys','StateOrder',stateorder)` creates a linear-time-invariant uncertain system `ulin`, whose states are in a specified order. Specify the state order in the cell array `stateorder` by entering the names of the blocks containing states in the model. For all blocks, you can enter block names as the full block path. For continuous blocks, you can alternatively enter block names as the user-defined unique state name.

## Examples

Compute uncertain linearization of a Simulink model containing Uncertain State Space blocks:

```
% Define uncertain variables and uncertain system variables
% to use in Uncertain State Space blocks.
unc_pole = ureal('unc_pole',-5,'Range',[-10 -4]);
plant = ss(unc_pole,5,1,0);
wt = makeweight(0.25,130,2.5);
input_unc = ultidyn('input_unc',[1 1]);
sensor_pole = ureal('sensor_pole',-20,'Range',[-30 -10]);
sensor = tf(1,[1/(-sensor_pole) 1]);

% Open Simulink model. The model contains three Uncertain State
% Space blocks named Unmodeled Plant Dynamics, Uncertain Plant and
% Uncertain Sensor, and linearization I/O points.
open_system('rct_ulinearize_uss')

% Obtain linearization I/O points.
```

```
mdl = 'rct_ulinearize_uss';  
io = getlinio(mdl);  
  
% Compute the uncertain linearization of the model.  
ulin = ulinearize(mdl,io)  
% MATLAB returns an uss object with 5 states.
```

## Tutorials

“Linearize Block to Uncertain Model”

Linearization of Simulink Models with Uncertainty

## How To

“Obtain Uncertain State-Space Model from Simulink Model”

## See Also

udyn | ultidyn | ureal | uss

**Introduced in R2009b**

## ultidyn

Create uncertain linear time-invariant object

### Syntax

```
H = ultidyn('Name',iosize)
```

```
H = ultidyn('Name',iosize,'Property1',Value1,'Property2',Value2,...)
```

### Description

`H = ultidyn('Name',iosize)` creates an uncertain linear, time-invariant objects are used to represent unknown dynamic objects whose only known attributes are bounds on their frequency response. Uncertain linear, time-invariant objects have a name (the `Name` property), and an input/output size (`ioSize` property).

Trailing Property/Value pairs are allowed in the construction.

```
H = ultidyn('name',iosize,'Property1',Value1,'Property2',Value2,...)
```

The property `Type` is `'GainBounded'` (default) or `'PositiveReal'`, and describes in what form the knowledge about the object's frequency response is specified.

- If `Type` is `'GainBounded'`, then the knowledge is an upper bound on the magnitude (i.e., absolute value), namely  $\text{abs}(H) \leq \text{Bound}$  at all frequencies. The matrix generalization of this is  $\|H\| \leq \text{Bound}$ .
- If `Type` is `'PositiveReal'` then the knowledge is a lower bound on the real part, namely  $\text{Real}(H) \geq \text{Bound}$  at all frequencies. The matrix generalization of this is  $H + H' \geq 2 * \text{Bound}$

The property `Bound` is a real, scalar that quantifies the bound on the frequency response of the uncertain object as described above.

The property `SampleStateDimension` is a positive integer, defining the state dimension of random samples of the uncertain object when sampled with `usample`. The default value is 1.

The property `AutoSimplify` controls how expressions involving the uncertain matrix are simplified. Its default value is `'basic'`, which means elementary methods of simplification are applied as operations are completed. Other values for `AutoSimplify` are `'off'`, no simplification performed, and `'full'` which applies model-reduction-like techniques to the uncertain object.

Use the property `SampleMaxFrequency` to limit the natural frequency for sampling. Randomly sampled uncertain dynamics are no faster than the specified value. The default value is `Inf` (no limit).

To model frequency-dependent uncertainty levels, multiply the `ultidyn` object by a suitable shaping filter. For example, for a `ultidyn` object `dH`, the following commands specify an uncertainty bound that increases from 0.1 at low frequencies to 10 at high frequencies.

```
W = tf([1 .1],[.1 1]);
dH = W*dH;
```

## Examples

### MIMO Uncertain Dynamics

Create an `ultidyn` object with internal name `'H'`, dimensions 2-by-3, norm bounded by 7.

```
H = ultidyn('H',[2 3],'Bound',7)
```

```
Uncertain GainBounded LTI Dynamics: Name H, 2x3, Gain Bound = 7
```

### Nyquist Plot of Uncertain Dynamics

Create a scalar `ultidyn` object with an internal name `'B'`, whose frequency response has a real part greater than 2.5.

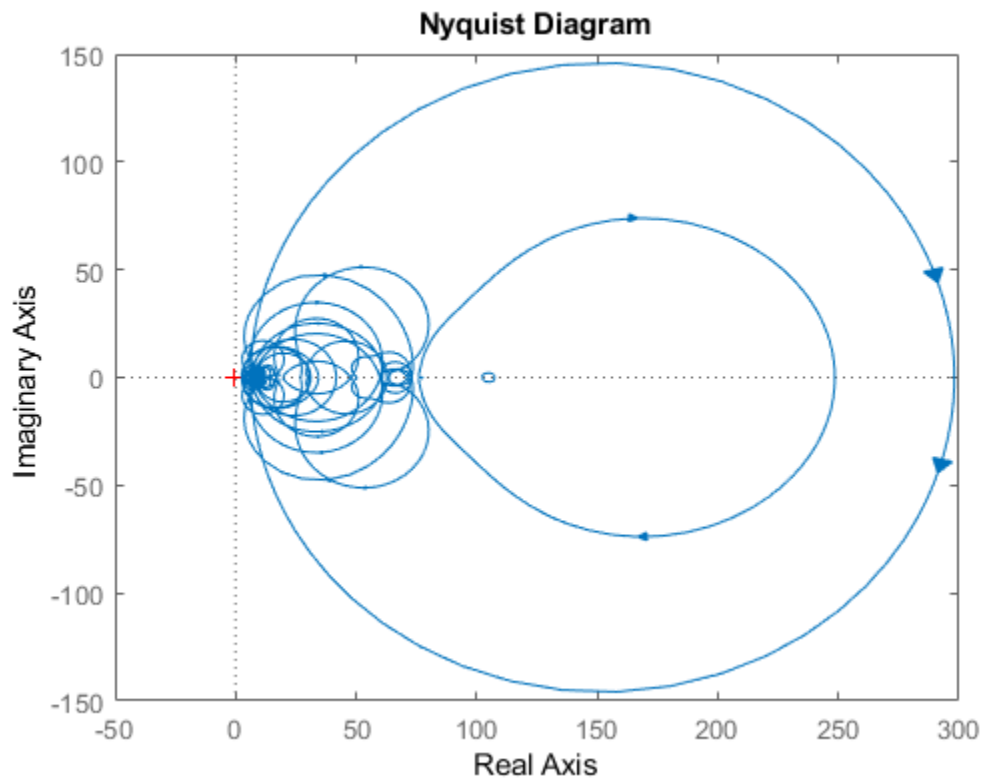
```
B = ultidyn('B',[1 1],'Type','PositiveReal','Bound',2.5)
```

```
B =
```

```
Uncertain LTI dynamics "B" with 1 outputs, 1 inputs, and positive real bound of 2.5.
```

Change the `SampleStateDimension` to 5, and plot the Nyquist plot of 30 random samples.

```
B.SampleStateDimension = 5;  
nyquist(usample(B,30))
```



## See Also

[get](#) | [ureal](#) | [uss](#)

**Introduced before R2006a**



# umat

Create uncertain matrix

## Syntax

```
M = umat(A)
```

## Description

Uncertain matrices are rational expressions involving uncertain elements of type `ureal`, `ucomplex`, or `ucomplexm`. Use uncertain matrices for worst-case gain analysis and for building uncertain state-space (uss) models.

Create uncertain matrices by creating uncertain elements and combining them using arithmetic and matrix operations. For example:

```
p = ureal('p',1);  
M = [0 p; 1 p^2]
```

creates a 2-by-2 uncertain matrix (a `umat` object) with the uncertain parameter `p`.

The syntax `M = umat(A)` converts the double array `A` to a `umat` object with no uncertainty.

Most standard matrix manipulations are valid on uncertain matrices, including addition, multiplication, inverse, horizontal and vertical concatenation. Specific rows/columns of an uncertain matrix can be referenced and assigned also.

If `M` is a `umat`, then `M.NominalValue` is the result obtained by replacing each uncertain element in `M` with its own nominal value.

If `M` is a `umat`, then `M.Uncertainty` is an object describing all the uncertain elements in `M`. All element can be referenced and their properties modified with this `Uncertainty` gateway. For instance, if `B` is an uncertain real parameter in `M`, then `M.Uncertainty.B` accesses the uncertain element `B` in `M`.

## Examples

Create 3 uncertain elements and then a 3-by-2 `umat`.

```
a = ureal('a',5,'Range',[2 6]);
b = ucomplex('b',1+j,'Radius',0.5);
c = ureal('c',3,'Plusminus',0.4);
M = [a b;b*a 7;c-a b^2]
```

M is an uncertain matrix (`umat` object) with the uncertain parameters a, b, and c.

View the properties of M with `get`

```
get(M)
```

The nominal value of M is the result when all atoms are replaced by their nominal values.

```
M.NominalValue
ans =
    5.0000          1.0000 + 1.0000i
    5.0000 + 5.0000i    7.0000
   -2.0000              0 + 2.0000i
```

Change the nominal value of a within M to 4. The nominal value of M reflects this change.

```
M.Uncertainty.a.NominalValue = 4;
M.NominalValue
ans =
    4.0000          1.0000 + 1.0000i
    4.0000 + 4.0000i    7.0000
   -1.0000              0 + 2.0000i
```

Get a random sample of M, obtained by taking random samples of the uncertain atoms within M.

```
usample(M)
ans =
    2.0072          0.8647 + 1.3854i
    1.7358 + 2.7808i    7.0000
    1.3829          -1.1715 + 2.3960i
```

Select the 1st and 3rd rows, and the 2nd column of M. The result is a 2-by-1 `umat`, whose dependence is only on b.

```
M([1 3],2)
```

## See Also

`ucomplex` | `ucomplexm` | `ultidyn` | `ureal` | `usample`

**Introduced before R2006a**

## **uplot**

Plot multiple frequency response objects and doubles on same graph

### **Syntax**

```
uplot(G1)
```

```
uplot(G1,G2)
```

```
uplot(G1,Xdata,Ydata)
```

```
uplot(G1,Xdata,Ydata,...)
```

```
uplot(G1,linetype)
```

```
uplot(G1,linetype,G2,...)
```

```
uplot(G1,linetype,Xdata,Ydata,linetype)
```

```
uplot(type,G1,linetype,Xdata,Ydata,linetype)
```

```
H = uplot(G1)
```

```
H = uplot(G1,G2)
```

```
H = uplot(G1,Xdata,Ydata)
```

```
H = uplot(G1,Xdata,Ydata,...)
```

```
H = uplot(G1,linetype)
```

```
H = uplot(G1,linetype,G2,...)
```

```
H = uplot(G1,linetype,Xdata,Ydata,linetype)
```

### **Description**

`uplot` plots double and `frd` objects. The syntax is the same as the MATLAB `plot` command except that all data is contained in `frd` objects, and the axes are specified by `type`.

The (optional) `type` argument must be one of

| Type     | Description   |
|----------|---|
| 'iv,d'   | Data versus independent variable (default)                  |
| 'iv,m'   | Magnitude versus independent variable                       |
| 'iv,lm'  | log(magnitude) versus independent variable                  |
| 'iv,p'   | Phase versus independent variable                           |
| 'liv,m'  | Magnitude versus log(independent variable)                  |
| 'liv,d'  | Data versus log(independent variable)                       |
| 'liv,m'  | Magnitude versus log(independent variable)                  |
| 'liv,lm' | log(magnitude) versus log(independent variable)             |
| 'liv,p'  | Phase versus log(independent variable)                      |
| 'r,i'    | Real versus imaginary (parametrize by independent variable) |
| 'nyq'    | Real versus imaginary (parametrize by independent variable) |
| 'nic'    | Nicholas plot   |
| 'bode'   | Bode magnitude and phase plot                               |

The remaining arguments of `uplot` take the same form as the MATLAB `plot` command. Line types (for example, '+', 'g-.', or '\*r') can be optionally specified after any frequency response argument.

There is a subtle distinction between constants and `frd` objects with only one independent variable. A constant is treated as such across all frequencies, and consequently shows up as a line on any graph with the independent variable as an axis. A `frd` object with only one frequency point always shows up as a point. You might need to specify one of the more obvious point types in order to see it (e.g., '+', 'x', etc.).

## Examples

### Plot Multiple Frequency Responses

Create two SISO second-order systems, and calculate their frequency responses over different frequency ranges.

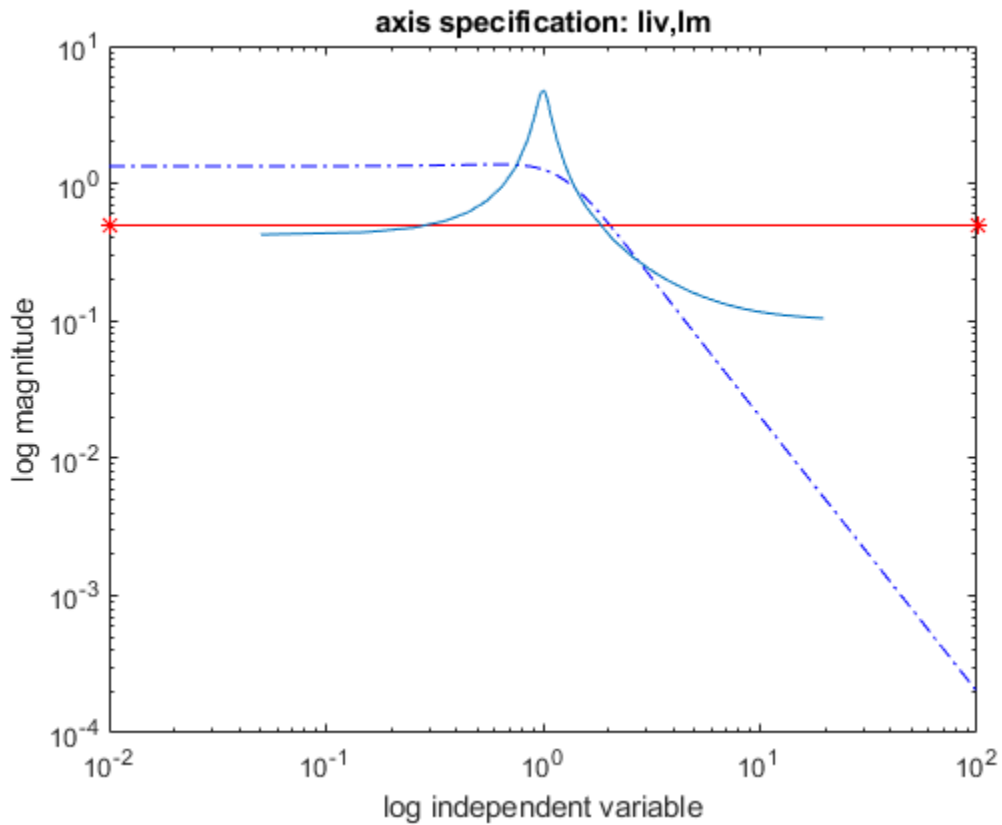
```
a1 = [-1,1;-1,-0.5];
b1 = [0;2]; c1 = [1,0]; d1 = 0;
sys1 = ss(a1,b1,c1,d1);
a2 = [-.1,1;-1,-0.05];
b2 = [1;1]; c2 = [-0.5,0]; d2 = 0.1;
sys2 = ss(a2,b2,c2,d2);
omega = logspace(-2,2,100);
sys1g = frd(sys1,omega);
omega2 = [ [0.05:0.1:1.5] [1.6:.5:20] [0.9:0.01:1.1] ];
omega2 = sort(omega2);
sys2g = frd(sys2,omega2);
```

Create an frd object with a single frequency.

```
sys3 = rss(1,1,1);
rspot = frd(sys3,2);
```

The following plot uses the plot\_type specification 'liv,lm'.

```
uplot('liv,lm',sys1g,'b-.',rspot,'r*-',sys2g);
xlabel('log independent variable')
ylabel('log magnitude')
title('axis specification: liv,lm')
```



## See Also

[bode](#) | [nichols](#) | [nyquist](#) | [plot](#) | [semilogx](#) | [semilogy](#) | [sigma](#)

Introduced before R2006a

## ureal

Create uncertain real parameter

### Syntax

```
p = ureal('name',nominalvalue)
```

```
p = ureal('name',nominalvalue,'Property1',Value1,...  
'Property2',Value2,...)
```

### Description

An uncertain real parameter is used to represent a real number whose value is uncertain. Uncertain real parameters have a name (the `Name` property), and a nominal value (`NominalValue` property).

The uncertainty (potential deviation from `NominalValue`) is described (equivalently) in 3 different properties:

- `PlusMinus`: the additive deviation from `NominalValue`
- `Range`: the interval containing `NominalValue`
- `Percentage`: the percentage deviation from `NominalValue`

The `Mode` property specifies which one of these three descriptions remains unchanged if the `NominalValue` is changed (the other two descriptions are derived). The possible values for the `Mode` property are `'Range'`, `'Percentage'` and `'PlusMinus'`.

The default `Mode` is `'PlusMinus'`, and `[-1 1]` is the default value for the `'PlusMinus'` property. The range of uncertainty need not be symmetric about `NominalValue`.

The property `AutoSimplify` controls how expressions involving the uncertain matrix are simplified. Its default value is `'basic'`, which means elementary methods of simplification are applied as operations are completed. Other values for `AutoSimplify` are `'off'`, no simplification performed, and `'full'`, which applies model-reduction-like techniques to the uncertain object.



## Examples

### Example 1

Create an uncertain real parameter and use `get` to display the properties and their values. Create uncertain real parameter object `a` with the internal name `'a'` and nominal value 5.

```
a = ureal('a',5)
Uncertain Real Parameter: Name a, NominalValue 5, variability = [-1 1]
get(a)
    Name: 'a'
NominalValue: 5
    Mode: 'PlusMinus'
    Range: [4 6]
    PlusMinus: [-1 1]
    Percentage: [-20 20]
AutoSimplify: 'basic'
```

Note that the `Mode` is `'PlusMinus'`, and that the value of `PlusMinus` is indeed `[-1 1]`. As expected, the range description of uncertainty is `[4 6]`, while the percentage description of uncertainty is `[-20 20]`.

Set the range to `[3 9]`. This leaves `Mode` and `NominalValue` unchanged, but all three descriptions of uncertainty have been modified.

```
a.Range = [3 9];
get(a)
    Name: 'a'
NominalValue: 5
    Mode: 'PlusMinus'
    Range: [3 9]
    PlusMinus: [-2 4]
    Percentage: [-40 80]
AutoSimplify: 'basic'
```

### Example 2

Property/Value pairs can also be specified at creation.

```
b = ureal('b',6,'Percentage',[-30 40],'AutoSimplify','full');
get(b)
    Name: 'b'
NominalValue: 6
```

```
Mode: 'Percentage'  
Range: [4.2000 8.4000]  
PlusMinus: [-1.8000 2.4000]  
Percentage: [-30.0000 40.0000]  
AutoSimplify: 'full'
```

Note that Mode is automatically set to 'Percentage'.

## Example 3

Specify the uncertainty in terms of percentage, but force Mode to 'Range'.

```
c = ureal('c',4,'Mode','Range','Percentage',25);  
get(c)  
Name: 'c'  
NominalValue: 4  
Mode: 'Range'  
Range: [3 5]  
PlusMinus: [-1 1]  
Percentage: [-25 25]  
AutoSimplify: 'basic'
```

## See Also

[getLimits](#) | [ucomplex](#) | [umat](#) | [uss](#)

**Introduced before R2006a**

## uss/usample

Generate random samples of uncertain or generalized model

### Syntax

```
B = usample(A);
```

```
B = usample(A,N)
```

```
[B,SampleValues] = usample(A,N)
```

```
[B,SampleValues] = usample(A,Names,N)
```

```
[B,SampleValues] = usample(A,Names1,N1,Names2,N2,...)
```

```
[B,SampleValues] = usample(A,N,Wmax)
```

```
[B,SampleValues] = usample(A,Names,N,Wmax)
```

### Description

`B = usample(A)` substitutes a random sample of the uncertain objects in `A`, returning a certain (i.e., not uncertain) array of size `[size(A)]`. The input `A` can be any uncertain element, matrix, or system, such as `ureal`, `umat`, `uss`, or `ufrd`. `A` can also be any generalized matrix or system, such as `genss` or `genmat`, that contains uncertain blocks and other types of Control Design Blocks (Control System Toolbox). If `A` contains non-uncertain control design blocks, these are unchanged in `B`. Thus, for example, `usample` applied to a `genss` with both tunable and uncertain blocks, the result is a `genss` array with only tunable blocks.

`B = usample(A,N)` substitutes `N` random samples of the uncertain objects in `A`, returning a certain (i.e., not uncertain) array of size `[size(A) N]`.

`[B,SampleValues] = usample(A,N)` additionally returns the specific sampled values (as a `Struct` whose field names are the names of `A`'s uncertain elements) of the uncertain elements. Hence, `B` is the same as `usubs(A,SampleValues)`.

`[B,SampleValues] = usample(A,Names,N)` samples only the uncertain elements listed in the `Names` variable (cell, or char array). If `Names` does not include all the

uncertain objects in *A*, then *B* will be an uncertain object. Any entries of *Names* that are not elements of *A* are simply ignored. Note that `usample(A, fieldnames(A.Uncertainty), N)` is the same as `usample(A, N)`.

`[B, SampleValues] = usample(A, Names1, N1, Names2, N2, ...)` takes *N1* samples of the uncertain elements listed in *Names1*, and *N2* samples of the uncertain elements listed in *Names2*, and so on. `size(B)` will equal `[size(A) N1 N2 ...]`.

The scalar parameter *Wmax* in

```
[B, SampleValues] = usample(A, N, Wmax)
[B, SampleValues] = usample(A, Names, N, Wmax)
[B, SampleValues] = usample(A, Names, N, Wmax)
```

affects how `ultidyn` elements within *A* are sampled, restricting the poles of the samples. If *A* is a continuous-time `uss` or `ufrd`, then the poles of sampled `GainBounded` `ultidyn` elements in *SampleValues* will each have magnitude  $\leq BW$ . If *A* is a discrete-time, then sampled `GainBounded` `ultidyn` elements are obtained by Tustin transformation, using  $BW/(2*TS)$  as the (continuous) pole magnitude bound. In this case, *BW* should be  $< 1$ . If the `ultidyn` type is `PositiveReal`, then the samples are obtained by bilinearly transforming (see “Normalizing Functions for Uncertain Elements”) the `GainBounded` elements described above.

## Examples

### Sample Real Parameter

Create a real uncertain parameter, sample it, and plot a histogram of the sampled values.

```
A = ureal('A', 5);
Asample = usample(A, 500);
```

Examine the size of the parameter and the sample array.

```
size(A)
```

```
Uncertain real scalar.
```

```
size(Asample)
```

```
ans = 1x3
```

```
1      1    500
```

A is a scalar parameter. The dimensions of `Asample` reflect that A is a 1-by-1 parameter. Examine the data type of `Asample`.

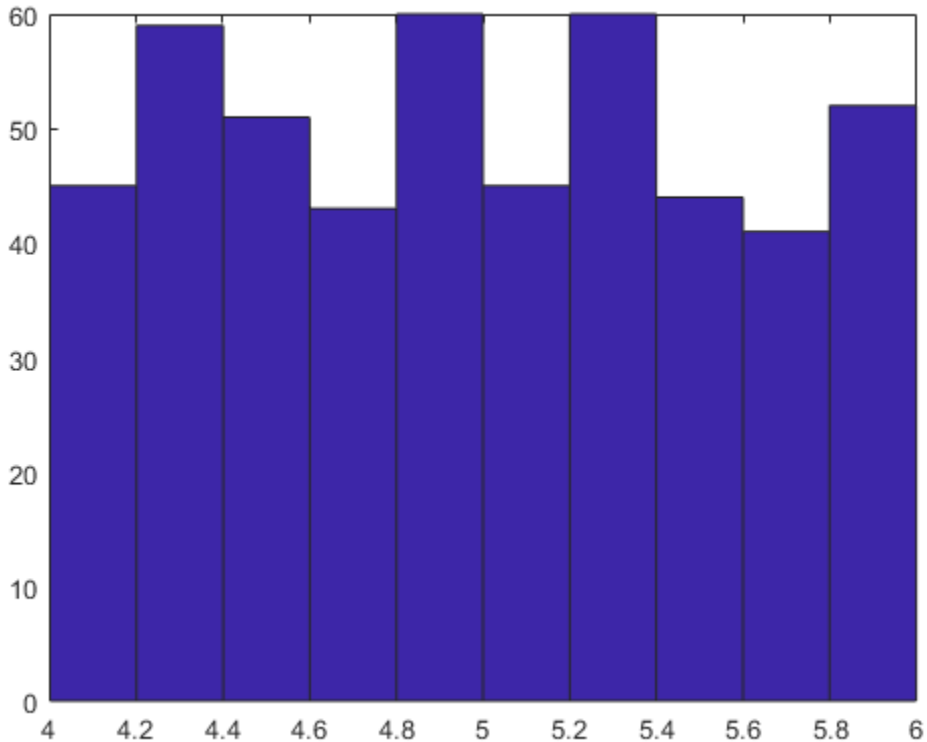
```
class(Asample)
```

```
ans =  
'double'
```

The samples of the scalar parameter are numerical values.

Plot the histogram of sampled values.

```
hist(Asample(:))
```



## Sample Responses of Uncertain Control System Model

This example illustrates how to sample the open and closed-loop response of an uncertain plant model for Monte Carlo analysis.

Create two uncertain real parameters and an uncertain plant.

```
gamma = ureal('gamma',4);  
tau = ureal('tau',.5,'Percentage',30);  
P = tf(gamma,[tau 1]);
```

Create an integral controller based on the nominal values of plant uncertainties.

```
KI = 1/(2*tau.Nominal*gamma.Nominal);  
C = tf(KI,[1 0]);
```

Now create an uncertain closed-loop system.

```
CLP = feedback(P*C,1);
```

Sample the plant at 20 values, distributed uniformly about the tau and gamma parameter cube.

```
[Psample1D,Values1D] = usample(P,20);  
size(Psample1D)
```

20x1 array of state-space models.  
Each model has 1 outputs, 1 inputs, and 1 states.

This sampling returns an array of 20 state-space models, each representing the closed-loop system within the uncertainty.

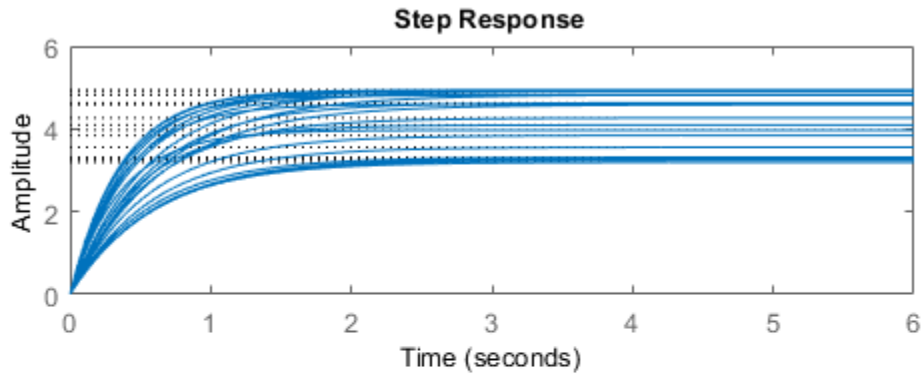
Now sample the plant at 10 values of tau and 15 values of gamma.

```
[Psample2D,Values2D] = usample(P,'tau',10,'gamma',15);  
size(Psample2D)
```

10x15 array of state-space models.  
Each model has 1 outputs, 1 inputs, and 1 states.

Plot the step responses of the 1-D sampled plant.

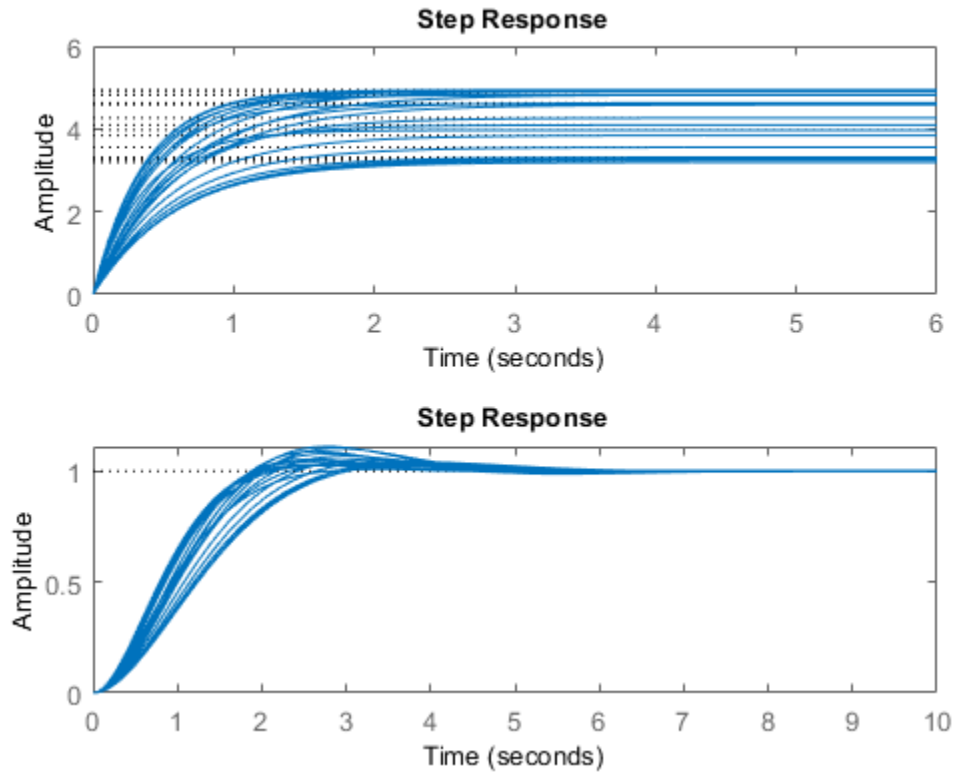
```
subplot(2,1,1);  
step(Psample1D)
```



Evaluate the uncertain closed-loop model at the same values using `usubs`, and plot the step response.

```
subplot(2,1,2);  
step(usubs(CLP,Values1D))
```





### Restrict Pole Locations in Sampled Uncertain Dynamics

To see the effect of limiting the bandwidth of sampled models with  $w_{\max}$ , create two `ultidyn` objects.

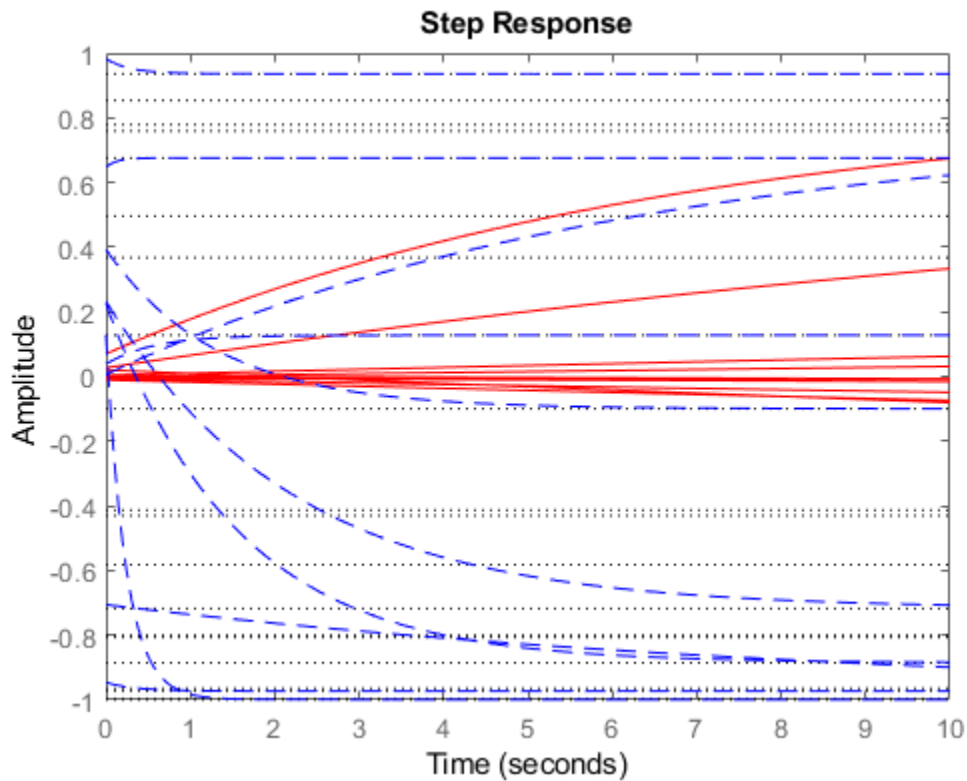
```
A = ultidyn('A',[1 1]);
B = ultidyn('B',[1 1]);
```

Sample 10 instances of each, using a bandwidth limit of 1 rad/sec on A, and 20 rad/sec on B.

```
Npts = 10;  
As = usample(A,Npts,1);  
Bs = usample(B,Npts,20);
```

Plot 10-second step responses, for the two sample sets.

```
step(As, 'r', Bs, 'b--', 10)
```



The lower bandwidth limit on the samples of A results in generally slower step responses for those samples.

## See Also

rsampleBlock | ucomplex | ufind | ufrd | ultidyn | umat | ureal | usample | uss |  
usubs

**Introduced in R2009b**

## usample

Generate random samples of uncertain variables in a Simulink model

This function is for generating random samples of uncertain variables stored in a data structure you obtain from a Simulink model, using `ufind`. To generate random samples from uncertain models or generalized state-space models, use `usample`.

### Syntax

```
samples = usample(uvars,N)
samples = usample(uvars)
samples = usample(uvars,N,Wmax)
```

### Description

`samples = usample(uvars,N)` generates  $N$  random samples of the uncertain variables in `uvars`. `uvars` is a structure that lists uncertain variables (`ureal`, `ucomplex` or `ultidyn`) by name. You can automatically obtain `uvars` for a Simulink model that contains Uncertain State Space blocks using `ufind`. `samples` is an  $N$ -by-1 structure array whose field names and values are the names and sample values of the uncertain variables. Use this syntax, together with `ufind`, to generate random samples for uncertain variables in Simulink models.

`samples = usample(uvars)` is equivalent to `usample(uvars,1)`.

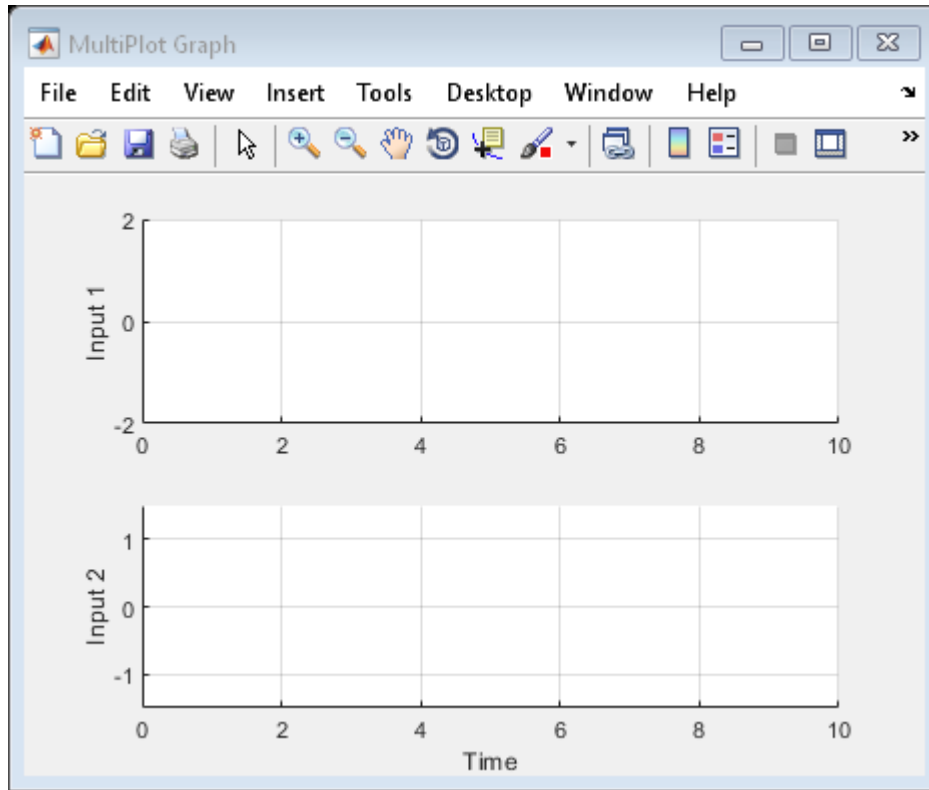
`samples = usample(uvars,N,Wmax)` specifies constraints, as described in `uss/usample`, for sampling uncertain variables of type `ultidyn` in `uvars`.

### Examples

#### Generate random samples of uncertain variables

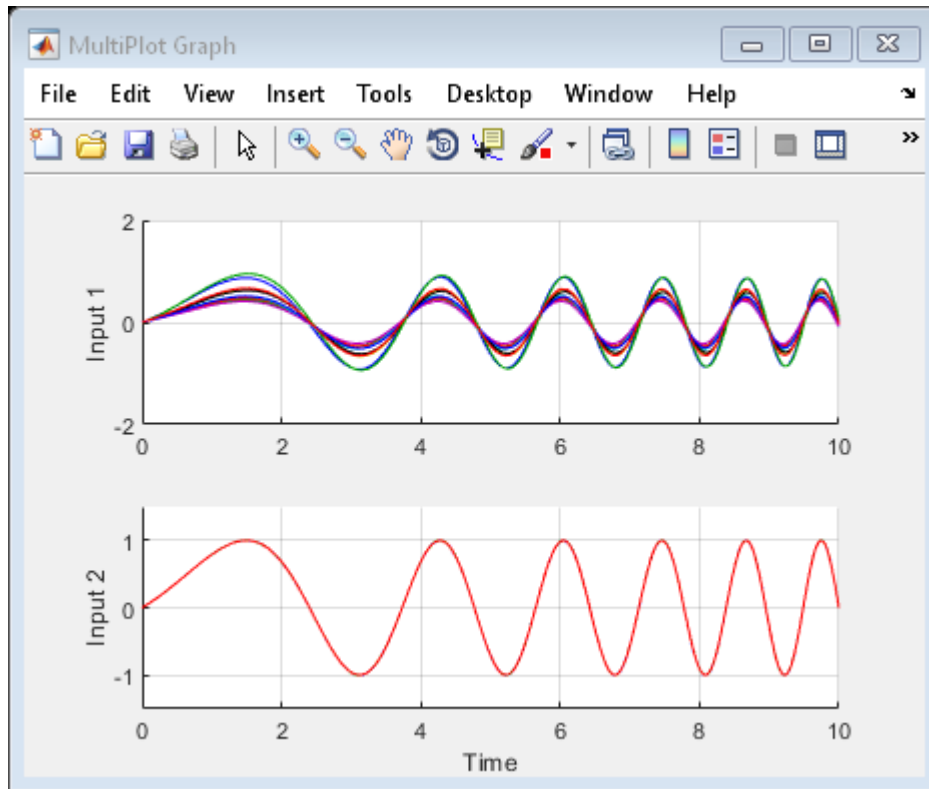
```
% Create a structure that contains uncertain variables a and % b.
uvars = struct('a',ureal('a',5),'b',ultidyn('b',[2 3],'Bound',7))
```





Use `usample` to generate random samples of `input_unc`, `unc_pole`, and `sensor_gain`. Simulate the closed-loop response for each of these random samples.

```
for i=1:10;
    uval = usample(uvars);
    sim('usim_model',10);
end
```



The MultiPlot Graph block displays the simulated responses.

## Tutorials

“Vary Uncertainty Values Using Individual Uncertain State Space Blocks”

“Vary Uncertainty Values Across Multiple Uncertain State Space Blocks”

Robustness Analysis in Simulink

## How To

“Simulate Uncertainty Effects”

## **See Also**

ucomplex | ufind | ufrd | ultidyn | umat | ureal | uss | usubs

**Introduced before R2006a**



## usimfill

Helper function for USS System blocks to set "User-defined Uncertainty" field or state of "Uncertainty value" menu

---

**Note** `usimfill` will be removed in a future release. Use `ufind` instead.

---

## Syntax

```
usimfill(ModelName, val)
```

```
usimfill(ModelName, 'Uncertainty value', 'Nominal')
```

```
usimfill(ModelName, 'Uncertainty value', 'User defined')
```

## Description

The command `usimfill` allows simple control of some parameters of all USS System blocks in a Simulink model.

`usimfill(ModelName, val)` pushes the character vector `val` into the Uncertainty value name field of all USS System blocks in the Simulink model specified by `ModelName`.

`usimfill(ModelName, 'Uncertainty value', 'Nominal')` sets the Uncertainty value pulldown menu to `Nominal` for all USS System blocks in the Simulink model specified by `ModelName`. Only a limited number of characters are needed to make this specification, so `usimfill(ModelName, 'U', 'N')` accomplishes the same effect.

`usimfill(ModelName, 'Uncertainty value', 'User defined')` sets the Uncertainty value pulldown menu to `User defined` for all USS System blocks in the Simulink model specified by `ModelName`. Only a limited number of characters are needed to make this specification, so `usimfill(ModelName, 'U', 'U')` accomplishes the same effect.

## Examples

Open the model file associated with the example.

```
open_system('usim_model');
unc_pole = ureal('unc_pole', -5, 'Range', [-10 -4]);
plant = ss(unc_pole, 5, 1, 1);
input_unc = ultidyn('input_unc', [1 1]);
wt = makeweight(0.25, 130, 2.5);
sensor_gain = ureal('sensor_gain', 1, 'Range', [0.1 2]);
```

This has three USS System blocks. They are plant with a `ureal` atom named `unc_pole`; `input_unc` which is a `ultidyn` object, and `sensor_gain` which is a `ureal` atom.

Run `usimfill` on the model, filling in the field with the label 'newData'.

```
usimfill('usim_model', 'newData');
```

View all of the dialog boxes, and see that 'newData' has been entered.

Run `usimfill` on the model, changing the Uncertainty Selection to Nominal.

```
usimfill('usim_model', 'Uncertainty value', 'Nominal');
```

Similarly run `usimfill` on the model, changing the Uncertainty Selection to User Specified Uncertainty.

```
usimfill('usim_model', 'Uncertainty value', 'User defined');
```

Now generate a random sample of the uncertain atoms, and run the simulation

```
newData = usimsamp('usim_model', 120);
sim('usim_model');
```

## See Also

[usample](#) | [usiminfo](#) | [usimsamp](#) | [usubs](#)

**Introduced in R2007a**

## usiminfo

Find USS System blocks within specified Simulink model and check for consistency

---

**Note** usiminfo will be removed in a future release. Use ufind instead.

---

### Syntax

```
[cflags,allupaths,allunames,upaths,unames,csumchar]
= usiminfo(sname, silent)
```

### Description

The command `usiminfo` returns information regarding the locations of all USS System blocks within a Simulink model and determines if these compatibility conditions are satisfied. It is possible to have uncertain objects of the same name through out a Simulink model. The helper functions `usimsamp` and `usimfill` assume that these are the same uncertainty. Hence uncertain objects of the same name should have the same object properties and `Uncertainty` value in the USS System pull-down menu. `usiminfo` provides information about the uncertainty in the Simulink diagram `sname`.

The following describes the input and outputs arguments of `usiminfo`:

| Input Arguments     | Description  |
|---------------------|--|
| <code>sname</code>  | Simulink diagram name  |
| <code>silent</code> | Display inconsistencies between uncertain atoms, when not empty. Default is empty. |

| <b>Output Arguments</b> | <b>Description</b>  |
|-------------------------|---|
| <code>cflag</code>      | Compatibility flag set to 1 if all uncertainties are consistent, set to 0 if an uncertainty definition(s) is consistent and set to -1 if common uncertainties in different blocks have different <b>Uncertainty value</b> . |
| <code>allpaths</code>   | Path names of USS System blocks in the model (cell).  |
| <code>allunames</code>  | Uncertainties names in Simulink model (cell).   |
| <code>upaths</code>     | Path names associated with each <code>allunames</code> entry (cell).  |
| <code>unames</code>     | Uncertainty names associated with each <code>allpaths</code> entry (cell).  |
| <code>csumchar</code>   | Character array with description of uncertainties and their associated block path names. Empty if there is a conflict with <code>unames</code> .  |

## See Also

`usample` | `usimfill` | `usimsamp` | `usubs`

**Introduced in R2007a**

## usimsamp

Generate random instance of all uncertain atoms present in all USS System blocks of Simulink model

---

**Note** usimsamp will be removed in a future release. Use `usample` instead.

---

### Syntax

```
sample = usimsamp(ModelName)
```

```
sample = usimsamp(ModelName, BW)
```

### Description

The command `usimsamp` samples a Simulink model. Note that if the model contains any USS System blocks, then the model can be interpreted as an uncertain Simulink model. The sample generated by `usimsamp` is a scalar structure, with `fieldnames` corresponding to the uncertain atoms within all of the USS System blocks, and the values are specific random samples of the atoms.

For `ultidyn` atoms, the magnitude of the sampled poles can be limited using an optional second bandwidth argument, `BW`. See `usample` for more information on this parameter.

### Examples

Open the model file associated with the example.

```
open_system('usim_model');
```

This has 3 USS System blocks. They are `plant` with a `ureal` atom named `unc_pole`; `input_unc` which is a `ultidyn` object, and `sensor_gain` which is a `ureal` atom.

Run `usimsamp` on the model, yielding a structure as described above.

```
unc_pole = ureal('unc_pole',-5,'Range',[-10 -4]);
plant = ss(unc_pole,5,1,1);
input_unc = ultidyn('input_unc',[1 1]);
wt = makeweight(0.25,130,2.5);
sensor_gain = ureal('sensor_gain',1,'Range',[0.1 2]);
data = usimsamp('usim_model')
data =
    input_unc: [1x1 ss]
    sensor_gain: 0.9935
    unc_pole: -4.1308
```

## See Also

[usample](#) | [usimfill](#) | [usiminfo](#) | [usubs](#)

**Introduced in R2007a**

## USS

Uncertain state-space model

### Description

Use `uss` model objects to represent uncertain dynamic systems.

The two dominant forms of model uncertainty are:

- Uncertainty in parameters of the underlying differential equation models (uncertain state-space matrices)
- Frequency-domain uncertainty, which often quantifies model uncertainty by describing absolute or relative uncertainty in the frequency response (uncertain or unmodeled linear dynamics)

`uss` model objects can represent dynamic systems with either or both forms of uncertainty. You can use `uss` to perform robust stability and performance analysis and to test the robustness of controller designs.

### Creation

There are several ways to create a `uss` model object, including:

- Use `tf` with one or more uncertain real parameters (`ureal`). For example:

```
p = ureal('p',1);  
usys = tf(p,[1 p]);
```

For another example, see “Transfer Function with Uncertain Coefficients” on page 1-517.

- Use `ss` with uncertain state-space matrices (`umat`). For example:

```
p = ureal('p',1);  
A = [0 3*p; -p p^2];  
B = [0; p];  
C = ones(2);
```

```
D = zeros(2,1);  
usys = ss(A,B,C,D);
```

For another example, see “Uncertain State-Space Model” on page 1-519.

- Combine numeric LTI models with uncertain elements using model interconnection commands such as `connect`, `series`, or `parallel`, or model arithmetic operators such as `*`, `+`, or `-`. For example:

```
sys = tf(1,[1 1]);  
p = ureal('p',1);  
D = ultidyn('Delta',[1 1]);  
usys = p*sys*(1 + 0.1*D);
```

For another example, see “System with Uncertain Dynamics” on page 1-520.

- Convert a double array or a numeric LTI model to `uss` form using `usys = uss(sys)`. In this case, the resulting `uss` model object has no uncertain elements. For example:

```
M = tf(1,[1 1 1]);  
usys = uss(M);
```

- Use `ucover` to create a `uss` model whose range of possible frequency responses includes all responses in an array of numeric LTI models. The resulting model expresses the range of behaviors as dynamic uncertainty (`ultidyn`).

## Properties

### **NominalValue** — Nominal value of uncertain model

`ss` model object

Nominal value of the uncertain model, specified as a state-space (`ss`) model object. The state-space model is obtained by setting all the uncertain control design blocks of the uncertain model to their nominal values.

### **Uncertainty** — Uncertain elements

structure

Uncertain elements of the model, specified as a structure whose fields are the names of the uncertain blocks, and whose values are the control design blocks themselves. Thus, the values stored in the structure can be `ureal`, `umat`, `ultidyn`, or other uncertain control design blocks. For instance, the following commands create an uncertain model `usys` with two uncertain parameters, `p1` and `p2`.



```

p1 = ureal('p1',1);
p2 = ureal('p2',3);
A = [0 3*p1; -p1 p1^2];
B = [0; p2];
C = ones(2);
D = zeros(2,1);
usys = ss(A,B,C,D);

```

The `Uncertainty` property of `usys` is a structure with two fields, `p1` and `p2`, whose values are the corresponding `ureal` uncertain parameters.

```
usys.Uncertainty
```

```
ans =
```

```

struct with fields:

    p1: [1x1 ureal]
    p2: [1x1 ureal]

```

You can access or examine each uncertain parameter individually. For example:

```
get(usys.Uncertainty.p1)
```

```

NominalValue: 1
           Mode: 'PlusMinus'
           Range: [0 2]
           PlusMinus: [-1 1]
           Percentage: [-100 100]
           AutoSimplify: 'basic'
           Name: 'p1'

```

## **A, B, C, D, E — State-space matrices**

numeric matrix | uncertain matrix

This property is read-only.

State-space matrices, specified as numeric matrices or uncertain matrices (`umat`). The state-space matrices are evaluated by fixing all dynamic uncertainty blocks (`udyn`, `ultidyn`) to their nominal values.

- **A** — State matrix *A*, specified as a square matrix or `umat` with as many rows and columns as there are system states.
- **B** — Input-to-state matrix *B*, specified as a matrix or `umat` with as many rows as there are system states and as many columns as there are system inputs.

- **C** — State-to-output matrix  $C$ , specified as a matrix or `umat` with as many rows as there are system outputs and as many columns as there are system states.
- **D** — Feedthrough matrix  $D$ , specified as a matrix or `umat` with as many rows as there are system outputs and as many columns as there are system inputs.
- **E** —  $E$  matrix for implicit (descriptor) state-space models, specified as a matrix or `umat` of the same dimensions as  $A$ . By default  $E = []$ , meaning that the state equation is explicit. To specify an implicit state equation  $E dx/dt = Ax + Bu$ , set this property to a square matrix of the same size as  $A$ . See `dss` for more information about descriptor state-space models.

### **StateName** — State names

`{ '' }` (default) | character vector | cell array of character vectors

State names, specified as one of these values:

- Character vector — For first-order models
- Cell array of character vectors — For models with two or more states
- `''` — For unnamed states

You can specify `StateName` using a string, such as "velocity", but the state name is stored as a character vector, 'velocity'.

Example: 'velocity'

Example: {'x1', 'x2'}

### **StateUnit** — State units

`{ '' }` (default) | character vector | cell array of character vectors

State units, specified as one of these values:

- Character vector — For first-order models
- Cell array of character vectors — For models with two or more states
- `''` — For states without specified units

Use `StateUnit` to keep track of the units each state is expressed in. `StateUnit` has no effect on system behavior.

You can specify `StateUnit` using a string, such as "mph", but the state units are stored as a character vector, 'mph'.

Example: 'mph'

Example: {'rpm', 'rad/s'}

### **InternalDelay — Internal delays**

scalar | vector

Internal delays, specified as a scalar or vector. For continuous-time models, internal delays are expressed in the time unit specified by the `TimeUnit` property of the model object. For discrete-time models, internal delays are expressed as integer multiples of the sample time  $T_s$ . For example, `InternalDelay = 3` means a delay of three sampling periods.

You can modify the values of internal delays. However, the number of entries in `InternalDelay` cannot change, because it is a structural property of the model.

Internal delays arise, for example, when closing feedback loops on systems with delays, or when connecting delayed systems in series or parallel. For more information about internal delays, see “Closing Feedback Loops with Time Delays” (Control System Toolbox).

### **InputDelay — Delay at inputs**

0 (default) | scalar | vector

Delay at each input, specified as a scalar or a vector. For a system with  $N_u$  inputs, set `InputDelay` to an  $N_u$ -by-1 vector. Each entry of this vector is a numerical value that represents the input delay for the corresponding input channel. For continuous-time models, specify input delays in the time unit stored in the `TimeUnit` property of the model object. For discrete-time models, specify input delays in integer multiples of the sample time  $T_s$ . For example, `InputDelay = 3` means a delay of three sample times.

Set `InputDelay` to a scalar value to apply the same delay to all channels.

### **OutputDelay — Delay at outputs**

0 (default) | scalar | vector

Delay at each output, specified as a scalar or a vector. For a system with  $N_y$  outputs, set `OutputDelay` to an  $N_y$ -by-1 vector. Each entry of this vector is a numerical value that represents the output delay for the corresponding output channel. For continuous-time models, specify output delays in the time unit stored in the `TimeUnit` property of the model object. For discrete-time models, specify output delays in integer multiples of the sample time  $T_s$ . For example, `OutputDelay = 3` means a delay of three sample times.

Set `OutputDelay` to a scalar value to apply the same delay to all channels.

## **Ts — Sample time**

0 (default) | -1 | positive scalar

Sample time, specified as:

- 0 — For continuous-time models.
- Positive scalar value — For discrete-time models. Specify the sample time in the units given in the `TimeUnit` property of the model.
- -1 — For discrete-time models with unspecified sample time.

Changing this property does not discretize or resample the model. Use `chgTs` to convert between continuous-time and discrete-time representations. Use `chgTs` to change the sample time of a discrete-time system.

## **TimeUnit — Model time units**

'seconds' (default) | 'minutes' | 'milliseconds' | ...

Model time units, specified as one of these values:

- 'nanoseconds'
- 'microseconds'
- 'milliseconds'
- 'seconds'
- 'minutes'
- 'hours'
- 'days'
- 'weeks'
- 'months'
- 'years'

You can specify `TimeUnit` using a string, such as "hours", but the time units are stored as a character vector, 'hours'.

Model properties such as sample time `Ts`, `InputDelay`, `OutputDelay`, and other time delays are expressed in the units specified by `TimeUnit`. Changing this property has no effect on other properties, and therefore changes the overall system behavior. Use `chgTimeUnit` to convert between time units without modifying system behavior.

**InputName — Names of input channels**

{' ' } (default) | character vector | cell array of character vectors

Names of input channels, specified as one of these values:

- Character vector — For single-input models
- Cell array of character vectors — For models with two or more inputs
- ' ' — For inputs without specified names

You can use automatic vector expansion to assign input names for multi-input models. For example, if `sys` is a two-input model, enter:

```
sys.InputName = 'controls';
```

The input names automatically expand to {'controls(1)'; 'controls(2)'}

You can use the shorthand notation `u` to refer to the `InputName` property. For example, `sys.u` is equivalent to `sys.InputName`.

Input channel names have several uses, including:

- Identifying channels on model display and plots
- Extracting subsystems of MIMO systems
- Specifying connection points when interconnecting models

You can specify `InputName` using a string, such as "voltage", but the input name is stored as a character vector, 'voltage'.

**InputUnit — Units of input signals**

{' ' } (default) | character vector | cell array of character vectors

Units of input signals, specified as one of these values:

- Character vector — For single-input models
- Cell array of character vectors — For models with two or more inputs
- ' ' — For inputs without specified units

Use `InputUnit` to keep track of the units each input signal is expressed in. `InputUnit` has no effect on system behavior.

You can specify `InputUnit` using a string, such as "voltage", but the input units are stored as a character vector, 'voltage'.

Example: 'voltage'

Example: {'voltage', 'rpm'}

## **InputGroup — Input channel groups**

structure with no fields (default) | structure

Input channel groups, specified as a structure where the fields are the group names and the values are the indices of the input channels belonging to the corresponding group. When you use `InputGroup` to assign the input channels of MIMO systems to groups, you can refer to each group by name when you need to access it. For example, suppose you have a five-input model `sys`, where the first three inputs are control inputs and the remaining two inputs represent noise. Assign the control and noise inputs of `sys` to separate groups.

```
sys.InputGroup.controls = [1:3];  
sys.InputGroup.noise = [4 5];
```

Use the group name to extract the subsystem from the control inputs to all outputs.

```
sys(:, 'controls')
```

Example: `struct('controls', [1:3], 'noise', [4 5])`

## **OutputName — Names of output channels**

{' ' } (default) | character vector | cell array of character vectors

Names of output channels, specified as one of these values:

- Character vector — For single-output models
- Cell array of character vectors — For models with two or more outputs
- '' — For outputs without specified names

You can use automatic vector expansion to assign output names for multi-output models. For example, if `sys` is a two-output model, enter:

```
sys.OutputName = 'measurements';
```

The output names automatically expand to {'measurements(1)'; 'measurements(2)'}.

You can use the shorthand notation `y` to refer to the `OutputName` property. For example, `sys.y` is equivalent to `sys.OutputName`.

Output channel names have several uses, including:

- Identifying channels on model display and plots
- Extracting subsystems of MIMO systems
- Specifying connection points when interconnecting models

You can specify `OutputName` using a string, such as `"rpm"`, but the output name is stored as a character vector, `'rpm'`.

### **OutputUnit — Units of output signals**

`{ '' }` (default) | character vector | cell array of character vectors

Units of output signals, specified as one of these values:

- Character vector — For single-output models
- Cell array of character vectors — For models with two or more outputs
- `''` — For outputs without specified units

Use `OutputUnit` to keep track of the units each output signal is expressed in. `OutputUnit` has no effect on system behavior.

You can specify `OutputUnit` using a string, such as `"voltage"`, but the output units are stored as a character vector, `'voltage'`.

Example: `'voltage'`

Example: `{'voltage','rpm'}`

### **OutputGroup — Output channel groups**

structure with no fields (default) | structure

Output channel groups, specified as a structure where the fields are the group names and the values are the indices of the output channels belonging to the corresponding group. When you use `OutputGroup` to assign the output channels of MIMO systems to groups, you can refer to each group by name when you need to access it. For example, suppose you have a four-output model `sys`, where the second output is a temperature, and the rest are state measurements. Assign these outputs to separate groups.

```
sys.OutputGroup.temperature = [2];
sys.InputGroup.measurements = [1 3 4];
```

Use the group name to extract the subsystem from all inputs to the measurement outputs.

```
sys('measurements',:)
```

```
Example: struct('temperature',[2],'measurement',[1 3 4])
```

### Notes — Text notes about model

[0×1 string] (default) | string | cell array of character vector

Text notes about the model, stored as a string or a cell array of character vectors. The property stores whichever of these two data types you provide. For instance, suppose that `sys1` and `sys2` are dynamic system models, and set their `Notes` properties to a string and a character vector, respectively.

```
sys1.Notes = "sys1 has a string.";
sys2.Notes = 'sys2 has a character vector.';
sys1.Notes
sys2.Notes
```

```
ans =
```

```
    "sys1 has a string."
```

```
ans =
```

```
    'sys2 has a character vector.'
```

### UserData — Data associated with model

[] (default) | any data type

Data of any kind that you want to associate and store with the model, specified as any MATLAB data type.

### Name — Model name

' ' (default) | character vector

Model name, stored as a character vector. You can specify `Name` using a string, such as "DCmotor", but the output units are stored as a character vector, 'DCmotor'.

```
Example: 'system_1'
```

### SamplingGrid — Sampling grid for model arrays

structure with no fields (default) | structure



Sampling grid for model arrays, specified as a structure. For model arrays that are derived by sampling one or more independent variables, this property tracks the variable values associated with each model in the array. This information appears when you display or plot the model array. Use this information to trace results back to the independent variables.

Set the field names of the data structure to the names of the sampling variables. Set the field values to the sampled variable values associated with each model in the array. All sampling variables should be numeric and scalar valued, and all arrays of sampled values should match the dimensions of the model array.

For example, suppose you create a 11-by-1 array of linear models, `sysarr`, by taking snapshots of a linear time-varying system at times  $t = 0:10$ . The following code stores the time samples with the linear models.

```
sysarr.SamplingGrid = struct('time',0:10)
```

Similarly, suppose you create a 6-by-9 model array, `M`, by independently sampling two variables, `zeta` and `w`. The following code attaches the (`zeta`,`w`) values to `M`.

```
[zeta,w] = ndgrid(<6 values of zeta>,<9 values of w>)  
M.SamplingGrid = struct('zeta',zeta,'w',w)
```

When you display `M`, each entry in the array includes the corresponding `zeta` and `w` values.

`M`

```
M(:,:,1,1) [zeta=0.3, w=5] =
```

```
          25  
-----  
s^2 + 3 s + 25
```

```
M(:,:,2,1) [zeta=0.35, w=5] =
```

```
          25  
-----  
s^2 + 3.5 s + 25
```

...

For model arrays generated by linearizing a Simulink model at multiple parameter values or operating points, the software populates `SamplingGrid` automatically with the

variable values that correspond to each entry in the array. For example, the Simulink Control Design commands `linearize` and `slLinearizer` populate `SamplingGrid` in this way.

## Object Functions

Most functions that work on numeric LTI models also work on `uss` models. These include model interconnection functions such as `connect` and `feedback`, and linear analysis functions such as `bode` and `stepinfo`. Some functions that generate plots, such as `bode` and `step`, plot random samples of the uncertain model to give you a sense of the distribution of uncertain dynamics. When you use these commands to return data, however, they operate on the nominal value of the system only.

In addition, you can use functions such as `robstab` and `wcgain` to perform robustness and worst-case analysis of uncertain systems represented by `uss` models. You can also use tuning functions such as `systemtune` for robust controller tuning.

The following lists contain a representative subset of the functions you can use with `uss` models.

### Model Interconnection

|                       |   |
|-----------------------|---|
| <code>feedback</code> | Feedback connection of two models                 |
| <code>connect</code>  | Block diagram interconnections of dynamic systems |
| <code>series</code>   | Series connection of two models                   |
| <code>parallel</code> | Parallel connection of two models                 |

### Linear Analysis

|                         |  |
|-------------------------|--|
| <code>step</code>       | Step response plot of dynamic system; step response data     |
| <code>bode</code>       | Bode plot of frequency response, or magnitude and phase data |
| <code>sigma</code>      | Singular values plot of dynamic system                       |
| <code>margin</code>     | Gain margin, phase margin, and crossover frequencies         |
| <code>loopmargin</code> | Stability margin analysis of LTI and Simulink feedback loops |

### Robustness and Worst-Case Analysis

|                          |   |
|--------------------------|---|
| <code>uss/usample</code> | Generate random samples of uncertain or generalized model |
| <code>robstab</code>     | Robust stability of uncertain system                      |
| <code>robgain</code>     | Robust performance of uncertain system                    |

wcgain Worst-case gain of uncertain system  
wcsigma Plot worst-case gain of uncertain system

## Control System Design and Tuning

dksyn Robust controller design using  $\mu$ -synthesis  
systeme Tune fixed-structure control systems modeled in MATLAB

## Examples

### Transfer Function with Uncertain Coefficients

Create a second-order transfer function with uncertain natural frequency and damping coefficient.

```
w0 = ureal('w0',10);
zeta = ureal('zeta',0.7,'Range',[0.6,0.8]);
```

```
usys = tf(w0^2,[1 2*zeta*w0 w0^2])
```

```
usys =
```

```
Uncertain continuous-time state-space model with 1 outputs, 1 inputs, 2 states.
The model uncertainty consists of the following blocks:
  w0: Uncertain real, nominal = 10, variability = [-1,1], 5 occurrences
  zeta: Uncertain real, nominal = 0.7, range = [0.6,0.8], 1 occurrences
```

Type "usys.NominalValue" to see the nominal value, "get(usys)" to see all properties, a

usys is an uncertain state-space (uss) model with two Control Design Blocks. The uncertain real parameter w0 occurs five times in the transfer function, twice in the numerator and three times in the denominator. To reduce the number of occurrences, you can rewrite the transfer function by dividing numerator and denominator by w0^2.

```
usys = tf(1,[1/w0^2 2*zeta/w0 1])
```

```
usys =
```

```
Uncertain continuous-time state-space model with 1 outputs, 1 inputs, 2 states.
The model uncertainty consists of the following blocks:
  w0: Uncertain real, nominal = 10, variability = [-1,1], 3 occurrences
```

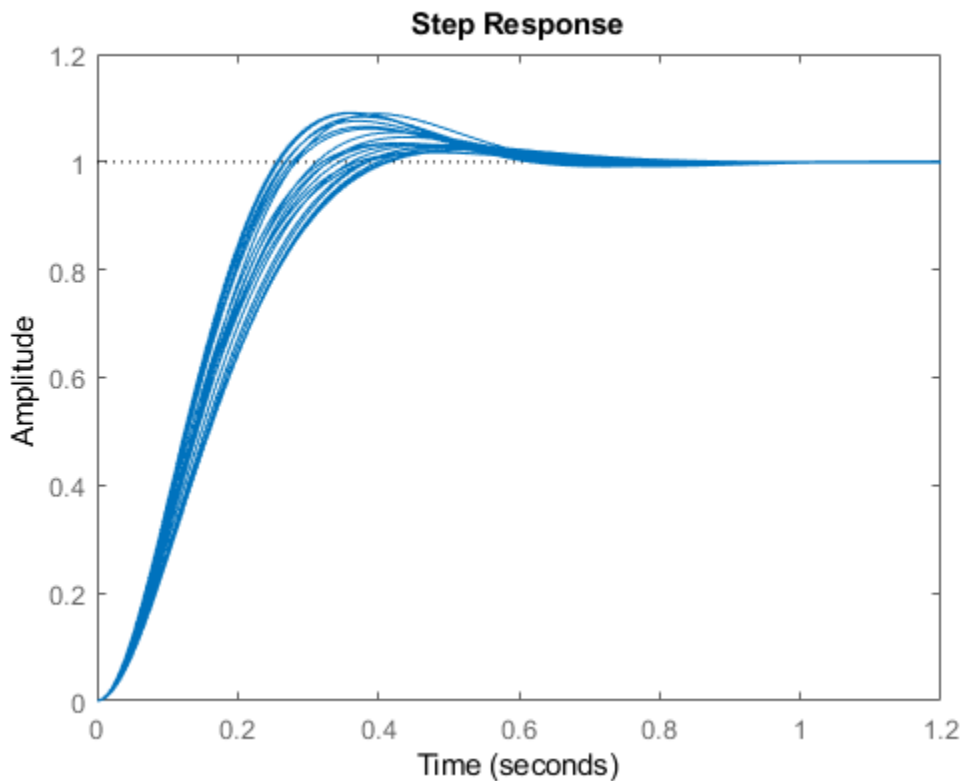
zeta: Uncertain real, nominal = 0.7, range = [0.6,0.8], 1 occurrences

Type `usys.NominalValue` to see the nominal value, `get(usys)` to see all properties, and

In the new formulation, there are only three occurrences of the uncertain parameter  $w_0$ . Reducing the number of occurrences of a Control Design Block in a model can improve the performance of calculations involving the model.

Examine the step response of the system to get a sense of the range of responses that the uncertainty represents.

```
step(usys)
```



When you use linear analysis commands like `step` and `bode` to create response plots of uncertain systems, they automatically plot random samples of the system. While these samples give you a sense of the range of responses that fall within the uncertainty, they do not necessarily include the worst-case response. To analyze worst-case responses of uncertain systems, use `wcgain` or `wcsigma`.

### Uncertain State-Space Model

To create an uncertain state-space model, you first use Control Design Blocks to create uncertain elements. Then, use the elements to specify the state-space matrices of the system.

For instance, create three uncertain real parameters and build state-spaces matrices from them.

```
p1 = ureal('p1',10,'Percentage',50);
p2 = ureal('p2',3,'PlusMinus',[-.5 1.2]);
p3 = ureal('p3',0);

A = [-p1 p2; 0 -p1];
B = [-p2; p2+p3];
C = [1 0; 1 1-p3];
D = [0; 0];
```

The matrices constructed with uncertain parameters, A, B, and C, are uncertain matrix (`umat`) objects. Using them as inputs to `ss` results in a 2-output, 1-input, 2-state uncertain system.

```
sys = ss(A,B,C,D)
```

```
sys =
```

```
Uncertain continuous-time state-space model with 2 outputs, 1 inputs, 2 states.
The model uncertainty consists of the following blocks:
  p1: Uncertain real, nominal = 10, variability = [-50,50]%, 2 occurrences
  p2: Uncertain real, nominal = 3, variability = [-0.5,1.2], 2 occurrences
  p3: Uncertain real, nominal = 0, variability = [-1,1], 2 occurrences
```

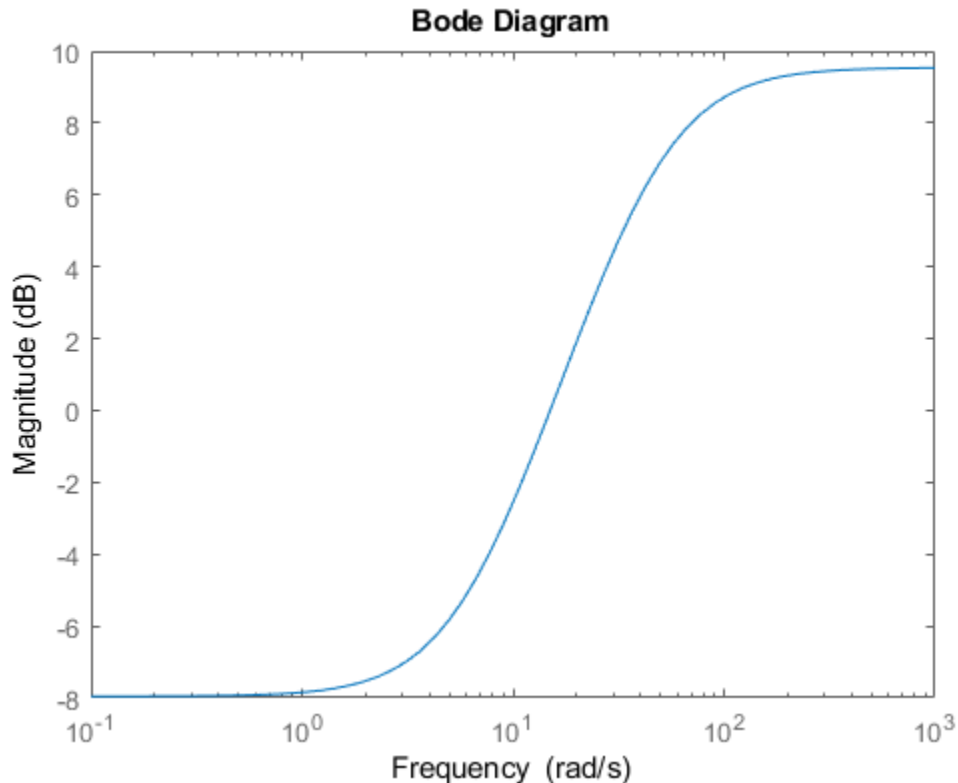
Type `"sys.NominalValue"` to see the nominal value, `"get(sys)"` to see all properties, and

The display shows that the system includes the three uncertain parameters.

## System with Uncertain Dynamics

Create an uncertain system comprising a nominal model with a frequency-dependent amount of uncertainty. You can model such uncertainty using `ultidyn` and a weighting function that represents the frequency profile of the uncertainty. Suppose that at low frequency, below 3 rad/s, the model can vary up to 40% from its nominal value. Around 3 rad/s, the percentage variation starts to increase. The uncertainty crosses 100% at 15 rad/s and reaches 2000% at approximately 1000 rad/s. Create a transfer function with an appropriate frequency profile, `Wunc`, to use as a weighting function that modulates the amount of uncertainty with frequency.

```
Wunc = makeweight(0.40,15,3);  
bodemag(Wunc)
```



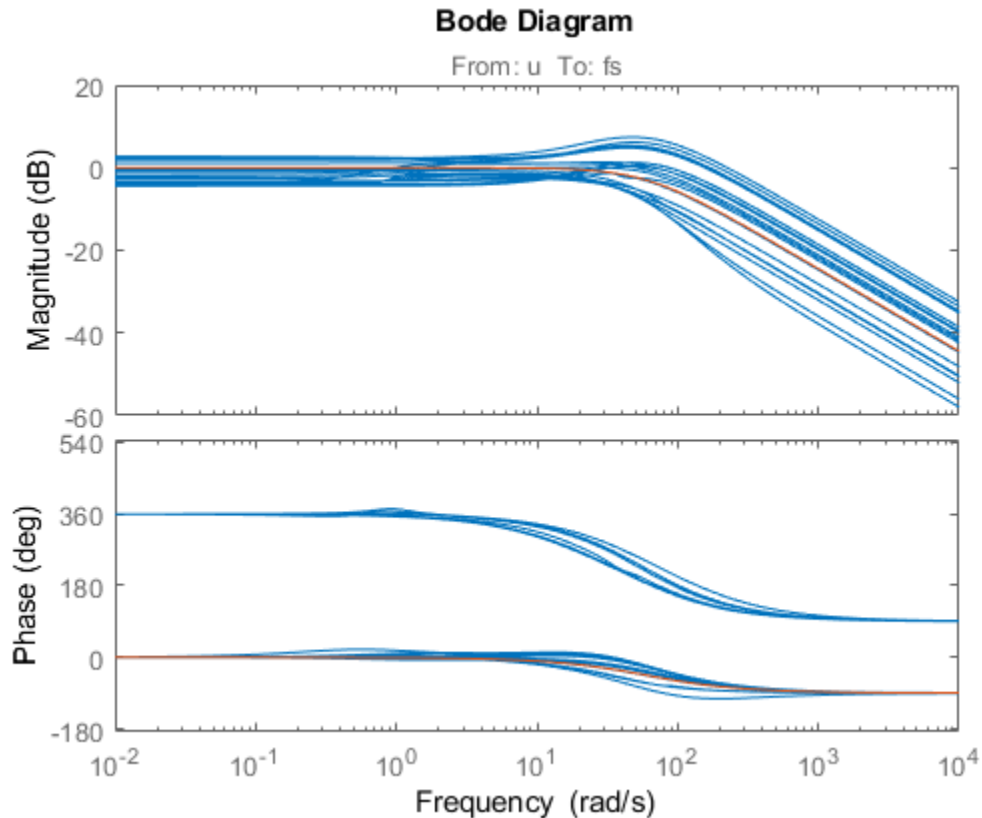
Next, create a transfer function representing the nominal value of the system. For this example, use a transfer function with a single pole at  $s = -60$  rad/s. Then, create a `ultidyn` model to represent 1-input, 1-output uncertain dynamics, and add the weighted uncertainty to the nominal transfer function.

```
sysNom = tf(1,[1/60 1]);
unc = ultidyn('unc',[1 1],'SampleStateDim',3); % samples of uncertain dynamics have the
usys = sysNom*(1 + Wunc*unc);

% Set properties of usys
usys.InputName = 'u';
usys.OutputName = 'fs';
```

Examine random samples of usys to see the effect of the uncertain dynamics.

```
bode(usys,usys.Nominal)
```



## Properties of uss Objects

uss models, like all model objects, include properties that store dynamics and model metadata. View the properties of an uncertain state-space model.

```
p1 = ureal('p1',10,'Percentage',50);  
p2 = ureal('p2',3,'PlusMinus',[-.5 1.2]);  
p3 = ureal('p3',0);
```



```

A = [-p1 p2; 0 -p1];
B = [-p2; p2+p3];
C = [1 0; 1 1-p3];
D = [0; 0];
sys = ss(A,B,C,D);      % create uss model

```

```
get(sys)
```

```

NominalValue: [2x1 ss]
Uncertainty: [1x1 struct]
      A: [2x2 umat]
      B: [2x1 umat]
      C: [2x2 umat]
      D: [2x1 double]
      E: []
      StateName: {2x1 cell}
      StateUnit: {2x1 cell}
InternalDelay: [0x1 double]
InputDelay: 0
OutputDelay: [2x1 double]
      Ts: 0
      TimeUnit: 'seconds'
InputName: {''}
InputUnit: {''}
InputGroup: [1x1 struct]
OutputName: {2x1 cell}
OutputUnit: {2x1 cell}
OutputGroup: [1x1 struct]
      Notes: [0x1 string]
      UserData: []
      Name: ''
SamplingGrid: [1x1 struct]

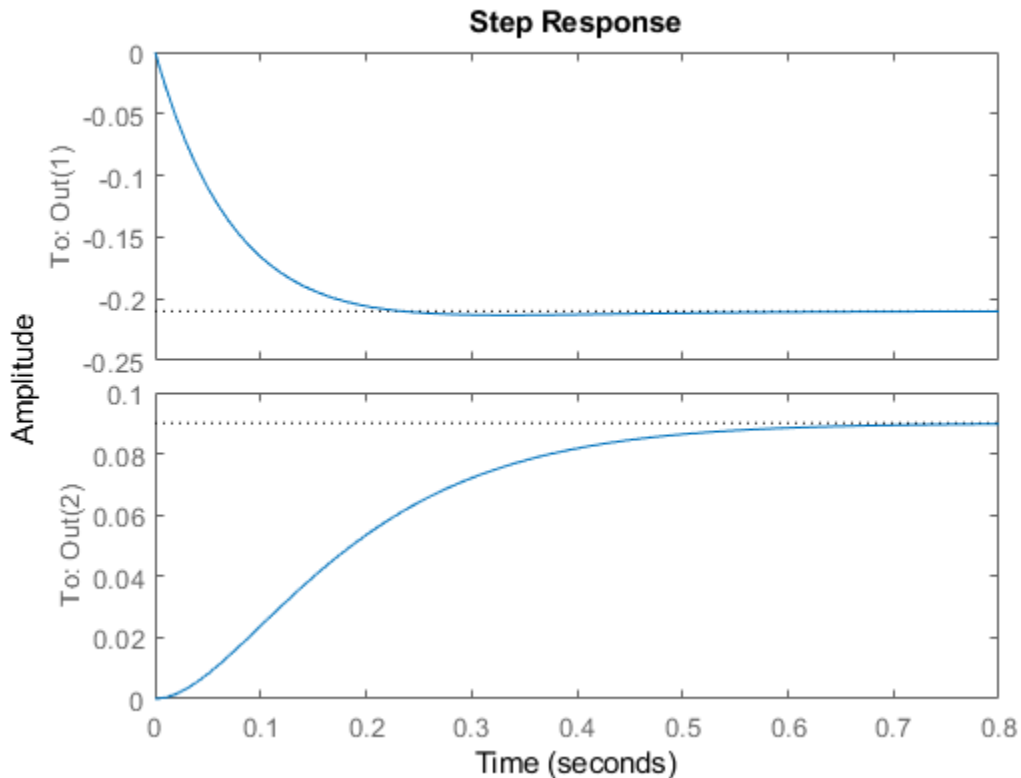
```

Most of the properties behave similarly to how they behave for `ss` model objects. The `NominalValue` property is itself an `ss` model object. You can therefore analyze the nominal value as you would any state-space model. For instance, compute the poles and step response of the nominal system.

```
pole(sys.NominalValue)
```

```
ans = 2x1
    -10
    -10
```

```
step(sys.NominalValue)
```



As with the uncertain matrices (`umat`), the `Uncertainty` property is a structure containing the uncertain elements. You can use this property for direct access to the uncertain elements. For instance, check the `Range` of the uncertain element named `p2` within `sys`.

```
sys.Uncertainty.p2.Range
```

```
ans = 1x2
```

```
2.5000 4.2000
```

Change the uncertainty range of p2 within sys.

```
sys.Uncertainty.p2.Range = [2 4];
```

This command changes only the range of the parameter called p2 in sys. It does not change the variable p2 in the MATLAB workspace.

```
p2.Range
```

```
ans = 1×2
```

```
    2.5000    4.2000
```

## See Also

[ucomplex](#) | [ultidyn](#) | [umat](#) | [ureal](#) | [uss/usample](#)

## Topics

“Uncertain State-Space Models”

**Introduced before R2006a**

## usubs

Substitute given values for uncertain elements of uncertain objects

## Syntax

```
B = usubs(M,ElementName1,value1,ElementName2,value2,...)
```

```
B = usubs(M,S)
```

```
B = usubs(M,...,'-once')
```

```
B = usubs(M,...,'-batch')
```

## Description

Use `usubs` to substitute a specific value for an uncertain element of an uncertain model object. The value can itself be uncertain. It needs to be the correct size, but otherwise can be of any class, and can be an array. Hence, the result can be of any class. In this manner, uncertain elements act as symbolic placeholders, for which specific values (which can also contain other placeholders too) can be substituted.

`B = usubs(M,ElementName1,value1,ElementName2,value2,...)` sets the elements in `M`, identified by `ElementName1`, `ElementName2`, etc., to the values in `value1`, `value2`, etc. respectively.

You can also use the character vectors `'NominalValue'` or `'Random'` as any `value` argument. If you do so, the nominal value or a random instance of the uncertain element is used. You can partially specify these character vectors, instead of typing the full expression. For example, you can use `'Nom'` or `'Rand'`.

`B = usubs(M,S)` instantiates the uncertain elements of `M` to the values specified in the structure `S`. The field names of `S` are the names of the uncertain elements to replace. The values are the corresponding replacement values. To provide several replacement values, make `S` a struct array, where each struct contains one set of replacement values. A structure such as `S` typically comes from robustness analysis commands such as `robstab`, `usample`, or `wcgain`.

`B = usubs(M,...,'-once')` performs vectorized substitution in the uncertain model array `M`. Each uncertain element is replaced by a single value, but this value may change

across the model array. To specify different substitute values for each model in the array `M`, use:

- A cell array for each `valueN` that causes the uncertain element `ElementNameN` in `M(:, :, k)` to be replaced by `valueN(k)`. For example, if `M` is a 2-by-3 array, then a 2-by-3 cell array `value1` replaces `ElementName1` of the model `M(:, :, k)` with the corresponding `value1(k)`.
- A struct array `S` that specifies one set of substitute values `S(k)` for each model `M(:, :, k)`.

Numeric array formats are also accepted for `value1, value2, ...`. For example, `value1` can be a 2-by-3 array of LTI models, a numeric array of size `[size(name1) 2 3]`, or a 2-by-3 matrix when the uncertain element `name1` is scalar-valued. The array sizes of `M, S, value1, value2, ...` must agree along non-singleton dimensions. Scalar expansion takes place along singleton dimensions.

Vectorized substitution ('-once') is the default for model arrays when no substitution method is specified.

`B = usubs(M, ..., '-batch')` performs batch substitution in the uncertain model array `M`. Each uncertain element is replaced by an array of values, and the same values are used for all models in `M`. In batch substitution, `B` is a model array of size `[size(M) VS]`, where `VS` is the size of the array of substitute values.

## Examples

### Evaluate Uncertain Matrix for Multiple Values of Uncertain Parameters

Evaluate an uncertain matrix at several different values of the uncertain parameters of the matrix.

Create an uncertain matrix with two uncertain parameters.

```
a = ureal('a',5);
b = ureal('b',-3);
M = [a b];
```

Evaluate the matrix at four different combinations of values for the uncertain parameters `a` and `b`.

```
B = usubs(M, 'a', [1;2;3;4], 'b', [10;11;12;13]);
```

This command evaluates  $M$  for the four different  $(a, b)$  combinations  $(1,10)$ ,  $(2,11)$ , and so on. Therefore,  $B$  is a 1-by-2-by-4 array of numeric values containing the four evaluated values of  $M$ .

## Evaluate Uncertain Matrix over Grid of Uncertain Parameters

Evaluate an uncertain matrix over a 3-by-4 grid of values of the uncertain parameters of the matrix.

Create a 2-by-2 uncertain matrix with two uncertain parameters.

```
a = ureal('a',5);  
b = ureal('b',-3);  
M = [a b;0 a*b];
```

Build arrays of values for the uncertain parameters.

```
aval = [1;2;3;4];  
bval = [10;20;30];  
[as,bs] = ndgrid(aval,bval);
```

This command builds two 4-by-3 grids of values.

Evaluate  $M$  over the parameter grids  $A$  and  $B$ .

```
B = usubs(M, 'a', as, 'b', bs);
```

This command evaluates  $M$  for each four different combination of values  $(A(k), B(k))$ .  $B$  is a 2-by-2-by-4-by-3 array of numeric values, which is a 4-by-3 array of values of  $M$ , i.e.,  $M$  evaluated over the parameter grids.

## Instantiate Uncertain Parameter by Batch Substitution of Parameter for Array of Values

Evaluate an array of uncertain models, substituting an array of values for an uncertain parameter.

Create a 1-by-2 uncertain matrix with two uncertain parameters.

```
a = ureal('a',5);
b = ureal('b',-3);
M = [a b];
```

Replace **a** by each of the values 1, 2, 3, and 4.

```
Ma = usubs(M, 'a', [1;2;3;4]);
```

This command returns a 4-by-1 array of 1-by-2 uncertain matrices that contain one uncertain parameter **b**.

For each model in the array **Ma**, evaluate **b** at 10, 20, and 30.

```
B = usubs(Ma, 'b', [10;20;30], '-batch');
```

The `'-batch'` flag causes `usubs` to evaluate each model in the array at all three values of **b**. Thus **B** is a 4-by-3 array of **M** values.

The `'-batch'` syntax here yields the same result as the parameter grid approach used in the previous example:

```
aval = [1;2;3;4];
bval = [10;20;30];
[as,bs] = ndgrid(aval,bval);
B = usubs(M, 'a', as, 'b', bs);
```

### Instantiate Uncertain Parameter Using Different Value for Each Entry in Array

Evaluate an array of uncertain models, substituting a different value for the uncertain parameter in each entry in the array.

Create a 1-by-2 uncertain matrix with two uncertain parameters.

```
a = ureal('a',5);
b = ureal('b',-3);
M = [a b];
```

Replace **a** by each of the values 1, 2, 3, and 4.

```
Ma = usubs(M, 'a', [1;2;3;4]);
```

This command returns a 4-by-1 array of 1-by-2 uncertain matrices that contain one uncertain parameter `b`.

For each model in the array `Ma`, evaluate `b`. Use `b = 10` for the first entry in the array, `b = 20` for the second entry, and so on.

```
B = usubs(Ma, 'b', {10;20;30;40}, '-once');
```

The `'-once'` flag causes `usubs` to evaluate the first model in the array using the first specified value for `b`, the second model for the second specified value, etc.

### Replace Uncertain Parameters with Values Returned by `usample`

Replace the uncertain parameters in an uncertain models by values specified in struct array form, as returned by `usample`.

This is useful, for example, when you have multiple uncertain models that use the same set of parameters, and you want to evaluate all models at the same parameter values.

Create two uncertain matrices that have the same uncertain parameters, `a` and `b`.

```
a = ureal('a',5);  
b = ureal('b',-3);  
M1 = [a b];  
M2 = [a b;0 a*b];
```

Generate some random samples of `M1`.

```
[M1rand,samples] = usample(M1,5);
```

`M1rand` is an array of five values of `M1`, evaluated at randomly generated values of `a` and `b`. These `a` and `b` values are returned in the struct array `samples`.

Examine the struct array `samples`.

```
samples
```

```
samples = 5x1 struct array with fields:  
    a  
    b
```



The field names of `samples` correspond to the uncertain parameters of `M1`. The values are the parameter values used to generate `M1rand`. Because `M2` has the same parameters, you can use this structure to evaluate `M2` at the same set of values.

```
M2rand = usubs(M2,samples);
```

This command returns a 1-by-5 array of instantiations of `M2`.

## See Also

`gridureal` | `simplify` | `usample`

**Introduced before R2006a**

## wcgain

Worst-case gain of uncertain system

### Syntax

```
[wcg,wcu] = wcgain(usys)
[wcg,wcu] = wcgain(usys,w)
[wcg,wcu] = wcgain( ____,opts)
[wcg,wcu,info] = wcgain( ____,info)
```

### Description

`[wcg,wcu] = wcgain(usys)` calculates the worst-case peak gain of the uncertain system `usys`. Peak gain refers to the maximum gain over frequency ( $H_\infty$  norm). For multi-input, multi-output (MIMO) systems, gain refers to the largest singular value of the frequency response matrix. (See `sigma` for more information about singular values.) The structure `wcg` contains upper and lower bounds on the worst-case gain and the critical frequency at which the lower bound peaks. (See “Worst-Case Gain” on page 1-546.) The structure `wcu` contains the values of the uncertain elements of `usys` that cause the worst-case peak gain.

`[wcg,wcu] = wcgain(usys,w)` restricts worst-case computation to the frequencies specified by `w`.

- If `w` is a cell array of the form `{wmin,wmax}`, then `wcgain` returns the worst-case gain in the interval between `wmin` and `wmax`.
- If `w` is a vector of frequencies, then `wcgain` calculates the worst-case gain at the specified frequencies only, and returns the worst of those gains.

`[wcg,wcu] = wcgain( ____,opts)` specifies additional options for the computation. Use `wcOptions` to create `opts`. You can use this syntax with any of the previous input-argument combinations.

`[wcg,wcu,info] = wcgain( ____,info)` returns a structure with additional information about the worst-case gains and the perturbations that generate them. See `info` for

details about this structure. You can use this syntax with any of the previous input-argument combinations.

## Examples

### Worst-Case Peak Gain of Closed-Loop System

Consider a control system whose plant is nominally an integrator with some additive dynamic uncertainty. Create a model of the plant.

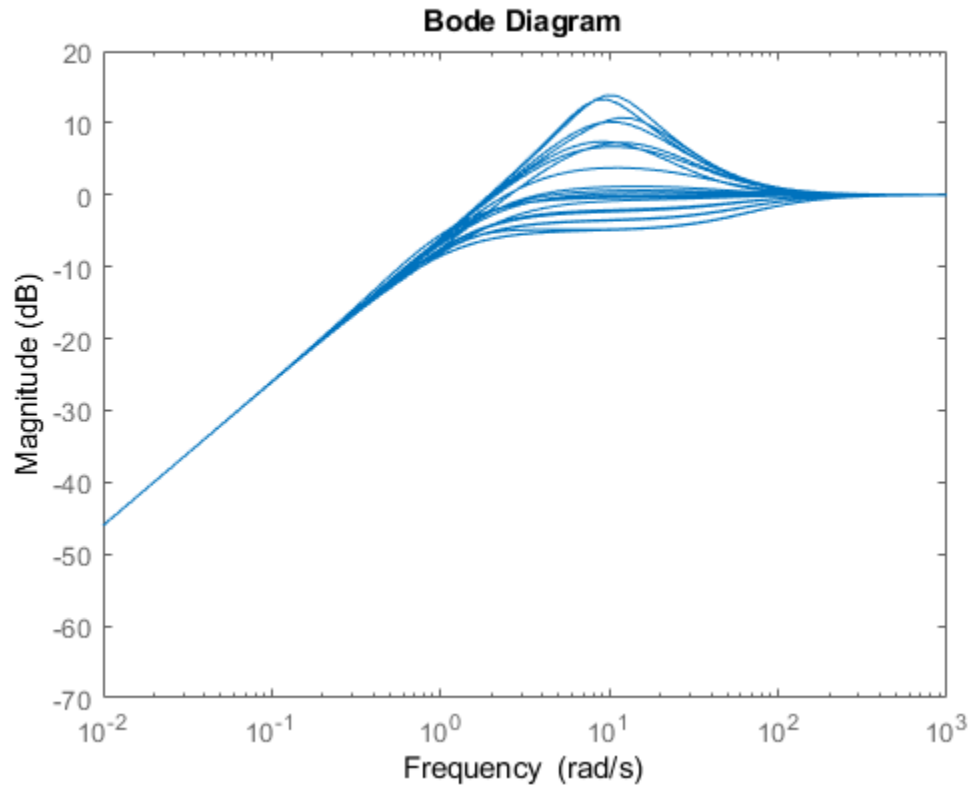
```
delta = ultidyn('delta',[1 1],'bound',0.4);  
G = tf(1,[1 0]) + delta;
```

Create a PD controller for the model. Suppose you want to examine the worst-case disturbance rejection performance. Build the closed-loop sensitivity function to examine the worst-case gain of a disturbance at the plant input.

```
C = pid(2,0,-0.04,0.02);  
S = feedback(1,G*C);
```

Because of the uncertainty, the frequency response of this transfer function falls within some envelope. The frequency-response magnitude of a few samples of the system gives a sense of that envelope.

```
bodemag(S)
```



Each sample has a different peak gain. Find the highest peak-gain value within the envelope and the corresponding values for the uncertain elements.

```
[wcg,wcu] = wcgain(S);  
wcg
```

```
wcg = struct with fields:  
    LowerBound: 5.1042  
    UpperBound: 5.1145  
    CriticalFrequency: 10.5819
```

The `LowerBound` and `UpperBound` fields of `wcg` show that the worst-case peak gain is around 5.1. This gain occurs at the critical frequency around 10.6 rad/s.

The output `wcu` is a structure that contains the perturbation to `delta` that causes the worst-case gain. Confirm the result by substituting this value into the sensitivity function.

```
Swc = usubs(S,wcu);
getPeakGain(Swc)
```

```
ans = 5.1042
```

Because the system has dynamic uncertainty `delta` with gain not exceeding 0.4, the worst-case value of `delta` should be a system with peak gain of 0.4. Confirm this result.

```
getPeakGain(wcu.delta)
```

```
ans = 0.4000
```

### Worst-Case Gain at Frequencies in a Range

Consider a model of a control system containing uncertain elements.

```
k = ureal('k',10,'Percent',40);
delta = ultidyn('delta',[1 1]);
G = tf(18,[1 1.8 k]) * (1 + 0.5*delta);
C = pid(2.3,3,0.38,0.001);
CL = feedback(G*C,1);
```

By default, `wcgain` returns only the worst-case peak gain over all frequencies. To obtain worst-case gain values at multiple frequencies, use the `'VaryFrequency'` option of `wcOptions`. For example, compute the highest possible gain of the system at frequency points between 0.1 and 10 rad/s.

```
opts = wcOptions('VaryFrequency','on');
[wcg1,wcu1,info1] = wcgain(CL,{0.1,10},opts);
info1
```

```
info1 =
```

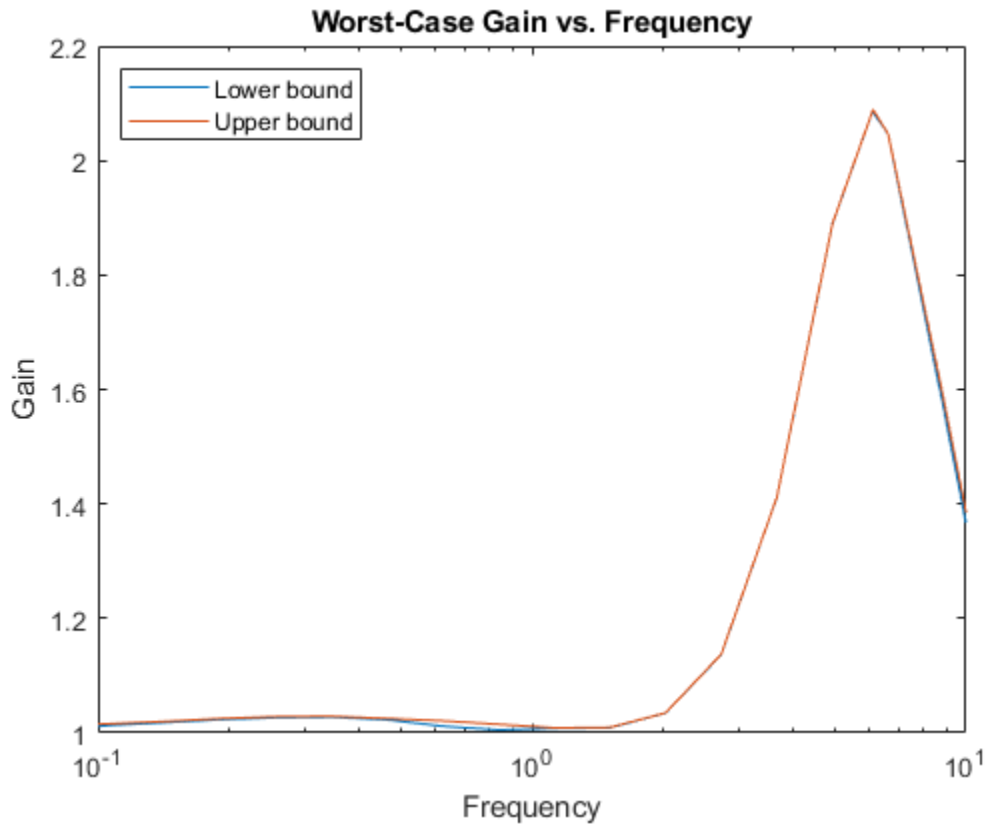
```
struct with fields:
```

```
Model: 1
Frequency: [20x1 double]
Bounds: [20x2 double]
```

```
WorstPerturbation: [20x1 struct]
  Sensitivity: [1x1 struct]
BadUncertainValues: [20x1 struct]
  ArrayIndex: 1
```

wcgain returns the vector of frequencies in the `info` output, in the `Frequencies` field. `info.Bounds` contains the upper and lower bounds on the worst-case gain at each frequency. Use these values to plot the frequency dependence of the worst-case gain.

```
semilogx(info1.Frequency,info1.Bounds)
title('Worst-Case Gain vs. Frequency')
ylabel('Gain')
xlabel('Frequency')
legend('Lower bound','Upper bound','Location','northwest')
```



The curve shows the high-gain envelope for all systems within the uncertainty ranges of CL. You can also use `wcsigma` to plot this envelope along with samples of the system.

When you use the 'VaryFrequency' option, `wcgain` chooses frequency points automatically. The frequencies it selects are guaranteed to include the frequency at which the worst-case gain is highest (within the specified range). Display the returned frequency values to confirm that they include the critical frequency.

```
info1.Frequency
wcg1.CriticalFrequency
```

```
ans =
```

```
0.1000
0.1061
0.1425
0.1914
0.2572
0.3455
0.4642
0.6236
0.8377
1.1253
1.5118
2.0309
2.7283
3.6652
4.9239
6.0828
6.1003
6.6147
8.8862
10.0000
```

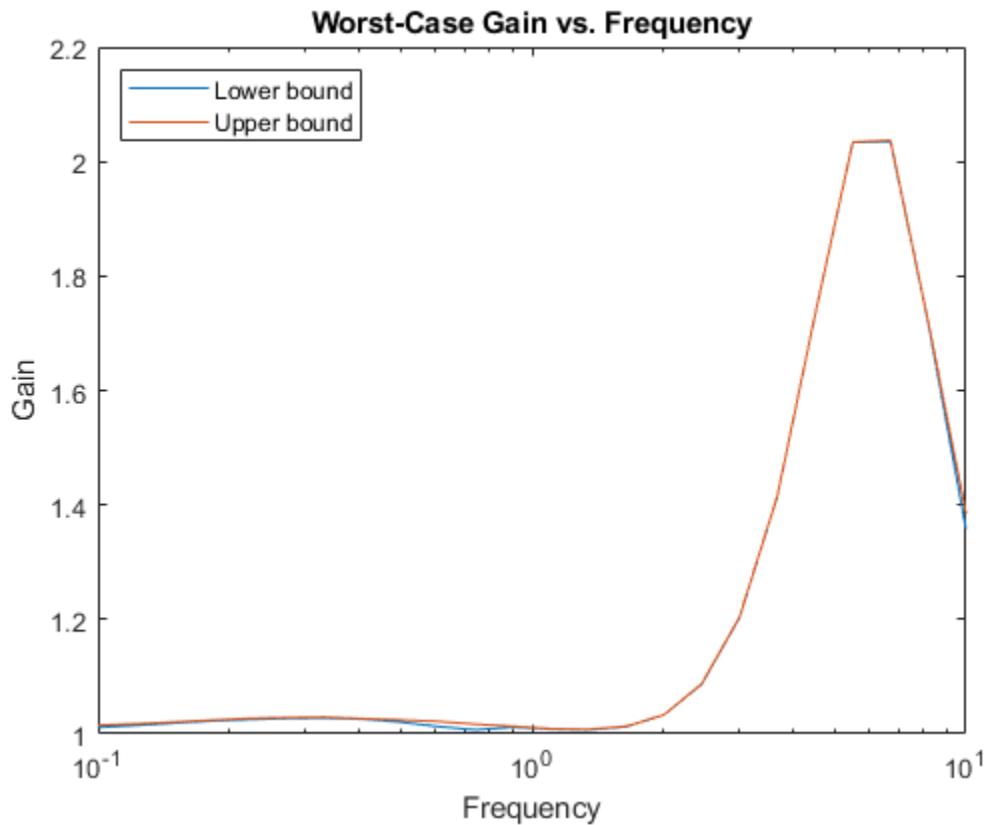
```
ans =
```

```
6.0828
```

Alternatively, instead of using 'VaryFrequency', you can specify particular frequencies at which to compute the worst-case gains. `info.Bounds` contains the worst-case gains at all specified frequencies.

```
w = logspace(-1,1,24);
[wcg2,wcu2,info2] = wcgain(CL,w);
semilogx(w,info2.Bounds)
title('Worst-Case Gain vs. Frequency')
ylabel('Gain')
xlabel('Frequency')
legend('Lower bound', 'Upper bound', 'Location', 'northwest')
```





When you provide the frequency grid in this way, the results are not guaranteed to include the overall worst-case gain, which might fall between specified frequency points. To see this, examine `wcg1` and `wcg2`, which contain the bounds for the two approaches.

```
wcg1
```

```
wcg1 =
```

```
struct with fields:
```

```
LowerBound: 2.0847
```

```
UpperBound: 2.0892
```

```
CriticalFrequency: 6.0828
```

```
wcg2
```

```
wcg2 =
```

```
struct with fields:
```

```
    LowerBound: 2.0349  
    UpperBound: 2.0371  
    CriticalFrequency: 6.7002
```

wcg1, computed using VaryFrequency, finds a higher peak gain than the specified frequency grid.

## Improve Worst-Case Perturbation

Specifying certain options for the structured-singular-value computation that underlies the worst-gain computation can yield better results in some cases. For example, consider a sample plant and controller.

```
load(fullfile(matlabroot, 'examples', 'robust', 'wcgExampleData.mat'))
```

This command loads gPlant, a MIMO plant with 10 outputs, 8 inputs, and 11 uncertain elements. It also loads Kmu, a state-space controller model. Form a closed-loop system with these models, and examine the worst-case gain.

```
CL = lft(gPlant, Kmu);  
[wcg, wcu] = wcgain(CL);  
wcg
```

```
wcg = struct with fields:  
    LowerBound: 10.7780  
    UpperBound: 11.2130  
    CriticalFrequency: 6.5759
```

There is a large difference between the lower and upper bounds on the worst-case gain. To get a better estimate of the actual worst-case gain, increase the number of restarts that wcgain uses for computing of the worst-case perturbation and associated lower

bound. Doing so can result in a tighter lower bound. This option does not affect the upper-bound calculation.

```
opt = wcOptions('MussvOptions','m3');
[wcg,wcg] = wcgain(CL,opt);
wcg
```

```
wcg = struct with fields:
    LowerBound: 10.8142
    UpperBound: 11.2130
    CriticalFrequency: 6.6564
```

The difference between the lower bound and upper bound on the worst-case gain is much smaller. The critical frequency is different as well.

- “Robust Stability and Worst-Case Gain of Uncertain System”

## Input Arguments

### **usys** — Dynamic system with uncertainty

uss | ufrd | genss | genfrd

Dynamic system with uncertainty, specified as a `uss`, `ufrd`, `genss`, or `genfrd` model that contains uncertain elements. For `genss` or `genfrd` models, `wcgain` uses the current value of any tunable blocks and folds them into the known (not uncertain) part of the model.

`usys` can also be an array of uncertain models. In that case, `wcgain` returns the worst-case gain across all models in the array, and the `info` output contains the index of the corresponding model.

### **w** — Frequencies

{wmin,wmax} | vector

Frequencies at which to compute worst-case gains, specified as the cell array `{wmin,wmax}` or as a vector of frequency values.

- If `w` is a cell array of the form `{wmin,wmax}`, then the function returns the worst-case gain in the interval between `wmin` and `wmax`.

- If `w` is a vector of frequencies, then the function computes the worst-case gain at each specified frequency.

Specify frequencies in units of `rad/TimeUnit`, where `TimeUnit` is the `TimeUnit` property of the model.

## **opts — Options for margin computation**

`wcOptions` object

Options for worst-case computation, specified as an object you create with `wcOptions`. The available options include settings that let you:

- Extract frequency-dependent worst-case gains (see “Worst-Case Gain” on page 1-546).
- Examine the sensitivity of the worst-case gain to each uncertain element.
- Improve the results of the worst-case gain calculation by setting certain options for the underlying `mussv` calculation. For an example, see “Improve Worst-Case Perturbation” on page 1-540.

For more information about all available options, see `wcOptions`.

Example: `wcOptions('Sensitivity','on','MussvOptions','m3')`

## **Output Arguments**

### **wcg — Worst-case peak gain and critical frequency**

structure

Worst-case peak gain and critical frequency, returned as a structure containing the following fields:

| Field      | Description   |
|------------|---|
| LowerBound | Lower bound on the actual worst-case peak gain of the model, returned as a scalar value. This value is the peak gain corresponding to the worst-case perturbation <code>wcu</code> . The exact worst-case peak gain is guaranteed to be no smaller than <code>LowerBound</code> . |

| Field             | Description  |
|-------------------|--|
| UpperBound        | Upper bound on the actual worst-case peak gain of the model, returned as a scalar value. The exact worst-case peak gain is guaranteed to be no larger than UpperBound. When you specify a frequency grid as a vector <code>w</code> , the guarantee only applies at the specified frequencies. |
| CriticalFrequency | Frequency at which the worst-case peak gain occurs, in rad/TimeUnit, where TimeUnit is the TimeUnit property of <code>usys</code> .  |

### wcu — Worst-case perturbation

structure

Worst case perturbation of uncertain elements, returned as a structure whose fields are the names of the uncertain elements of `usys`. Each field contains the actual value of the corresponding uncertain element of `usys` when the worst-case peak gain occurs. For example, if `usys` includes an uncertain matrix `M` and SISO uncertain dynamics `delta`, then `wcu.M` is a numeric matrix and `wcu.delta` is a SISO state-space model.

Use `usubs(usys,wcu)` to substitute these values for the uncertain elements in `usys`, to obtain the dynamic system that has the worst-case peak gain.

### info — Additional information about worst-case values

structure

Additional information about the worst-case values, returned as a structure with the following fields:

| Field | Description   |
|-------|---|
| Model | Index of the model that has the largest worst-case peak gain, when <code>usys</code> is an array of models. |

| Field     | Description   |
|-----------|---|
| Frequency | <p>Frequency points at which <code>wcgain</code> returns the worst-case gain, returned as a vector.</p> <ul style="list-style-type: none"> <li>• If the 'VaryFrequency' option of <code>wcOptions</code> is 'off', then <code>info.Frequency</code> is the critical frequency, the frequency at which the worst-case peak gain occurs. If the largest lower bound and the largest upper bound on the worst-case gain occur at different frequencies, then <code>info.Frequency</code> is a vector containing these two frequencies.</li> <li>• If the 'VaryFrequency' option of <code>wcOptions</code> is 'on', then <code>info.Frequency</code> contains the frequencies selected by <code>wcgain</code>. These frequencies are guaranteed to include the frequency at which the worst-case peak gain occurs.</li> <li>• If you specify a vector of frequencies <code>w</code> at which to compute the worst-case gains, then <code>info.Frequency = w</code>. When you specify a frequency vector, these frequencies are not guaranteed to include the frequency at which the worst-case peak gain occurs.</li> </ul> <p>The 'VaryFrequency' option is meaningful only for <code>uss</code> and <code>genss</code> models. <code>wcgain</code> ignores the option for <code>ufrd</code> and <code>genfrd</code> models.</p> |
| Bounds    | <p>Lower and upper bounds on the actual worst-case gain of the model, returned as an array. <code>info.Bounds(:,1)</code> contains the lower bound at each corresponding frequency in <code>info.Frequency</code>, and <code>info.Bounds(:,2)</code> contains the corresponding upper bounds.</p>   |

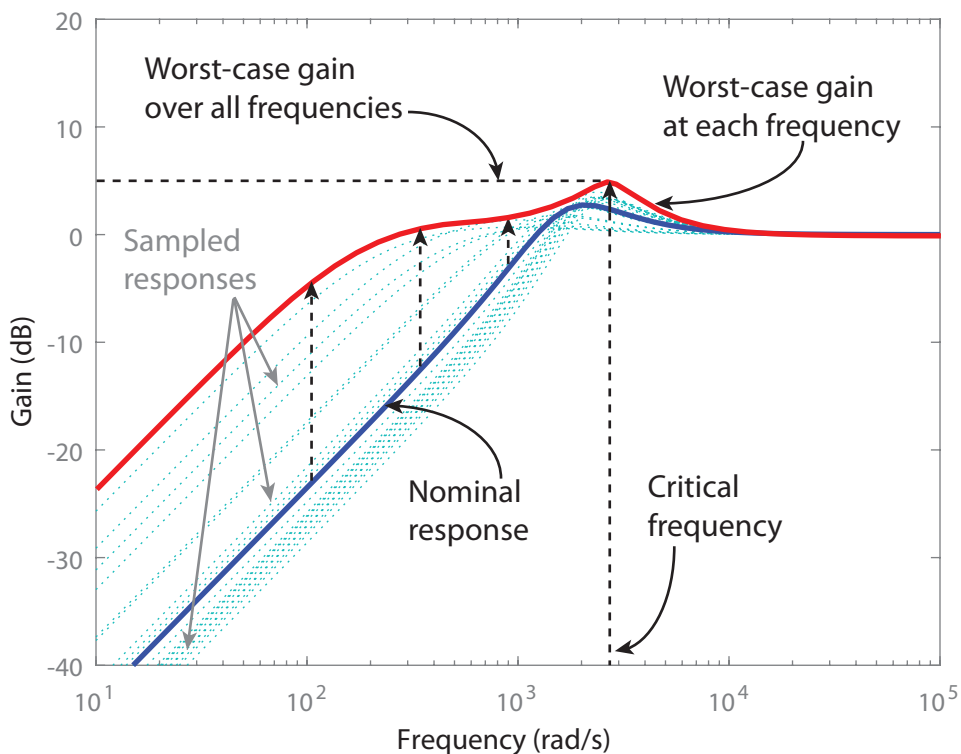
| Field              | Description  |
|--------------------|--|
| WorstPerturbation  | <p>Perturbations that cause the worst-case gain at each frequency point in <code>info.Frequency</code>, returned as a structure array. The fields of <code>info.WorstPerturbation</code> are the names of the uncertain elements in <code>usys</code>, and each field contains the worst-case value of the corresponding element at each frequency. For example, if <code>usys</code> includes an uncertain parameter <code>p</code> and SISO uncertain dynamics <code>delta</code>, then <code>info.WorstPerturbation.p</code> is a collection of numeric values and <code>info.WorstPerturbation.delta</code> is a collection of SISO state-space models.</p>  |
| Sensitivity        | <p>Sensitivity of the worst-case gain to each uncertain element, returned as a structure when the 'Sensitivity' option of <code>wcOptions</code> is 'on'. The fields of <code>info.Sensitivity</code> are the names of the uncertain elements in <code>usys</code>. Each field contains a percentage that measures how much the uncertainty in the corresponding element affects the worst-case gain. For example, if <code>info.Sensitivity.p</code> is 50, then a given fractional change in the uncertainty range of <code>p</code> causes half as much fractional change in the worst-case gain.</p> <p>If the 'Sensitivity' option of <code>wcOptions</code> is off (the default setting), then <code>info.Sensitivity</code> is NaN.</p> |
| BadUncertainValues | <p>Same as <code>WorstPerturbation</code>. Included for compatibility with R2016a and earlier.</p>   |
| ArrayIndex         | <p>Same as <code>Model</code>. Included for compatibility with R2016a and earlier.</p>   |

## Definitions

### Worst-Case Gain

By default, `wcgain` returns the peak gain (or peak singular value, for MIMO systems) achievable within the uncertainty range, over all frequencies (or the frequencies specified by `w`). You can obtain the peak gain as a function of frequency using the `VaryFrequency` option of `wcOptions`.

To understand the difference, consider the following illustration, representing the magnitude of the frequency response of an uncertain system.





The dark blue curve is the nominal response of the system. The light blue curves show various sampled responses of the system. The `wcg` output of `wcgain` contains the bounds on the worst-case gain over all frequencies, about 5 dB in the illustration. The frequency at which this gain occurs is the critical frequency, also returned in `wcg`.

If you set the `VaryFrequency` option of `wcOptions` to 'on', then `wcgain` also calculates the maximum gain at each frequency point, shown by the red curve. `wcgain` returns these values in `info.Bounds`. See “Worst-Case Gain at Frequencies in a Range” on page 1-535 for an example. You can also use `wcsigma` to visualize the worst-case gain as a function of frequency.

## Algorithms

Computing the worst-case gain at a particular frequency is equivalent to computing the structured singular value,  $\mu$ , for some appropriate block structure ( $\mu$ -analysis).

For `uss` and `genss` models, `wcgain(usys)` and `wcgain(usys, {wmin, wmax})` use an algorithm that finds the worst-case gain across frequency. This algorithm does not rely on frequency gridding and is not adversely affected by sharp peaks of the  $\mu$  structured singular value. See “Getting Reliable Estimates of Robustness Margins” for more information.

For `ufrd` and `genfrd` models, `wcgain` computes the  $\mu$  lower and upper bounds at each frequency point. This computation offers no guarantee between frequency points and can be inaccurate if the uncertainty gives rise to sharp resonances. The syntax `wcgain(uss, w)`, where `w` is a vector of frequency points, is the same as `wcgain(ufrd(uss, w))` and also relies on frequency gridding to compute the worst-case gain.

In general, the algorithm for state-space models is faster and safer than the frequency-gridding approach. In some cases, however, the state-space algorithm requires many  $\mu$  calculations. In those cases, specifying a frequency grid as a vector `w` can be faster, provided that the worst-case gain varies smoothly with frequency. Such smooth variation is typical for systems with dynamic uncertainty.

## See Also

`mussv` | `robstab` | `wcOptions` | `wcmargin` | `wcsens` | `wcsigma`

**Topics**

“Robust Stability and Worst-Case Gain of Uncertain System”

“Robustness and Worst-Case Analysis”

**Introduced before R2006a**

# wcgainOptions

(Not recommended) Option set for `wcgain`, `wcgainplot`, `wcnorm`, or `wcsens`

---

**Note** `wcgainOptions` is not recommended. Use `wcOptions` instead.

---

## Syntax

```
opt = wcgainOptions  
opt = wcgainOptions(Name,Value,...)
```

## Description

`opt = wcgainOptions` returns the default option set for a `wcgain` calculation. The commands `wcgainplot`, `wcnorm`, and `wcsens` also use `wcgain` to compute their results. Use a `wcgainOptions` options set to control options for those calculations.

`opt = wcgainOptions(Name,Value,...)` creates an option set with the options specified by one or more `Name,Value` pair arguments.

## Input Arguments

### Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

### Sensitivity

Determines whether to compute the sensitivity of worst-case gain with respect to each individual uncertain element.

Sensitivity takes the following values:

- 'on' — `wcgain` computes the sensitivity of the worst-case gain with respect to each individual uncertain element. This provides an indication of which elements are most problematic.
- 'off' — `wcgain` does not compute the sensitivity of the worst-case gain with respect to each individual uncertain element.

**Default:** 'on'

## **VaryUncertainty**

Percentage variation of uncertainty for sensitivity calculations. The sensitivity estimate uses a finite difference calculation.

**Default:** 25

## **LowerBoundOnly**

Determines whether only the lower bound is computed.

`LowerBoundOnly` takes the following values:

- 'on' — `wcgain` only computes a lower bound on the worst-case gain
- 'off' — `wcgain` computes lower and upper bounds on the worst-case gain

**Default:** 'off'

## **MaxOverFrequency**

`MaxOverFrequency` takes the following values:

- 'on' — `wcgain` computes the worst-case  $H_\infty$  norm (peak gain over frequency)
- 'off' — `wcgain` computes the worst-case gain at each frequency point

**Default:** 'on'

## **MaxOverArray**

For uncertain model arrays, determines if worst-case gain is calculated over entire array or individually for all models in array.

`MaxOverArray` takes the following values:

- 'on' — `wcgain` computes the worst-case gain over all models
- 'off' — `wcgain` computes the worst-case gain for each model individually

**Default:** 'on'

### **AbsTol**

Absolute tolerance on computed bound.

The algorithm terminates if  $\text{UpperBound} - \text{LowerBound} \leq \max(\text{AbsTol}, \text{RelTol} * \text{UpperBound})$ .

Relaxing tolerance speeds up the computation.

**Default:** 0.02

### **RelTol**

Relative tolerance on computed bound.

The algorithm terminates if  $\text{UpperBound} - \text{LowerBound} \leq \max(\text{AbsTol}, \text{RelTol} * \text{UpperBound})$ .

**Default:** 0.05

### **AbsMax**

Absolute threshold for lower bound.

The algorithm terminates if  $\text{LowerBound} \geq \text{AbsMax} + \text{RelMax} * \text{NominalGain}$ .

Specify `AbsMax` and `RelMax` to terminate when the lower bound is large enough compared to the nominal gain.

**Default:** 5

### **RelMax**

Relative threshold for lower bound.

The algorithm terminates if  $\text{LowerBound} \geq \text{AbsMax} + \text{RelMax} * \text{NominalGain}$ .

Specify `AbsMax` and `RelMax` to terminate when the lower bound is large enough compared to the nominal gain.

**Default:** 20

### **NSearch**

Number of lower bound searches at each frequency

**Default:** 2

## **Output Arguments**

### **opt**

Option set containing the specified options for `wcgain`.

## **Examples**

Create an options set for `wcgain` with only the lower bound being calculated and 5 lower bound searches at each frequency.

```
opt = wcgainOptions('LowerBoundOnly','on','Nsearch',5)
```

Alternatively, use dot notation to set the values of `opt`.

```
opt = wcgainOptions;  
opt.LowerBound = 'on';  
opt.NSearch = 5;
```

## **See Also**

`wcOptions` | `wcgain` | `wcgainplot` | `wcnorm` | `wcsens`

**Introduced in R2011b**

# wcgainplot

(Not recommended) Graphical worst-case gain analysis

---

**Note** `wcgainplot` is not recommended. Use `wcsigma` instead.

---

## Syntax

```
wcgainplot(sys)
wcgainplot(sys,w)
wcgainplot(sys,...,options)
```

## Description

`wcgainplot(sys)` plots the nominal and worst-case gains of the uncertain system `sys` as a function of frequency. For multi-input, multi-output (MIMO) systems, gain refers to the largest singular value of the frequency response matrix. (See `sigma` for more information about singular values.) The plot includes:

- Nominal — nominal gain of `sys`
- Worst — the response falling within the uncertainty of `sys` that has the highest peak gain
- Worst-case gain (lower bound) — the lowest worst-case gain at each frequency
- Worst-case gain (upper bound) — the highest gain within the uncertainty at each frequency
- Sampled Uncertainty — 20 responses randomly sampled from `sys`

`wcgainplot(sys,w)` focuses the plot on the frequencies specified by `w`.

- If `w` is a cell array `{wmin,wmax}`, `wcgainplot` plots the worst-case gains in the range `{wmin,wmax}`.
- If `w` is an array of frequencies, `wcgainplot` plots the worst-case gains at each frequency in the array.

`wcgainplot(sys, ..., options)` uses the options set `options` to specify additional options for the computation of the worst-case gains. Use `wcgainOptions` to create the options set.

## Input Arguments

### **sys**

Uncertain dynamic system (Control System Toolbox).

### **w**

Frequencies of worst-case gain plots. Specify frequencies in radians/TimeUnit, where TimeUnit is the time unit of `sys`.

- If `w` is a cell array `{wmin, wmax}`, `wcgainplot` plots the worst-case gains in the range `{wmin, wmax}`.
- If `w` is an array of frequencies, `wcgainplot` plots the worst-case gains at each frequency in the array.

### **options**

Options set specifying additional options for the computation of the worst-case gains. Use `wcgainOptions` to create the options set.

## Algorithms

`wcgainplot` uses `wcgain` to compute the worst-case gains. Use the `options` argument to `wcgainplot` to set options for the `wcgain` algorithm.

`wcgainplot` uses `usample` to compute the Sampled Uncertainty curves.

## See Also

`sigma` | `usample` | `wcgain` | `wcsigma`

**Introduced in R2011b**



## wcmargin

Worst-case disk stability margins of uncertain feedback loops

### Syntax

```
wcmarg = wcmargin(L)
wcmargi = wcmargin(p,c)
[wcmargi,wcmargo] = wcmargin(p,c)
wcmargi = wcmargin(p,c,opt)
[wcmargi,wcmargo] = wcmargin(p,c,opt)
```

### Description

Classical gain and phase margins define the allowable loop-at-a-time variations in the nominal system gain and phase for which the feedback loop retains stability.

An alternative to classical gain and phase margins is the disk margin. The disk margin is the largest region for each channel such that for all gain and phase variations inside the region the nominal closed-loop system is stable. See the `dmargin` and `loopmargin` reference pages to learn more about the algorithm.

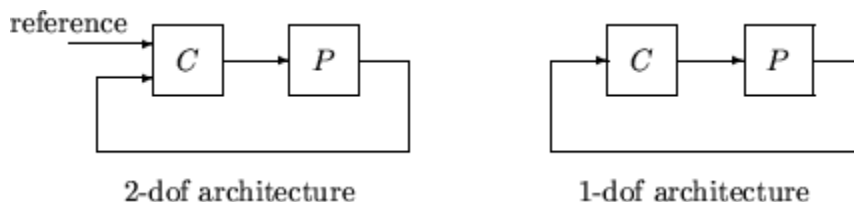
Consider a system with uncertain elements. It is of interest to determine the margin of each individual channel in the presence of uncertainty. These margins are called worst-case margins. Worst-case margin, `wcmargin` calculates the largest disk margin such that for values of the uncertainty and all gain and phase variations inside the disk, the closed-loop system is stable. The worst-case gain and phase margin bounds are defined based on the balanced sensitivity function. Hence, results from the worst-case margin calculation imply that the closed-loop system is stable for a given uncertainty set and would remain stable in the presence of an additional gain and phase margin variation in the specified input/output channel.

`wcmargL = wcmargin(L)` calculates the combined worst-case input and output loop-at-a-time gain/phase margins of the feedback loop consisting of the loop transfer matrix `L` in

negative feedback with an identity matrix.  $L$  must be an uncertain system, `uss` or `ufrd` object. If  $L$  is a `uss` object, the frequency range and number of points used to calculate `wcmargL` are chosen automatically. Note that in this case, the worst-case margins at the input and output are equal because an identity matrix is used in feedback. `wcmarg` is a  $NU$ -by-1 structure with the following fields:

| Field       | Description  |
|-------------|--|
| GainMargin  | Guaranteed bound on worst-case, single-loop gain margin at plant inputs. Loop-at-a-time analysis.  |
| PhaseMargin | Loop-at-a-time worst-case phase margin at plant inputs. Units are degrees.   |
| Frequency   | Frequency associated with the worst-case margin (rad/s).   |
| WCUnc       | Structure of the worst-case uncertainty values associated with the worst-case disk gain and phase margins for the $i$ -th loop $L(i, i)$ .   |
| Sensitivity | Struct with $M$ fields. Field names are names of uncertain elements of $P$ and $C$ . Values of fields are positive numbers, which each entry indicating the local sensitivity of the worst-case margins to all the individual uncertain element's uncertainty ranges. For instance, a value of 50 indicates that if the uncertainty range is enlarged by 8%, then the worst-case gain should increase by about 4%. If the <code>Sensitivity</code> property of the <code>wcOptions</code> object is 'off', the values are NaN. |

`[wcmargi,wcmargo] = wcmargin(P,C)` calculates the combined worst-case input and output loop-at-a-time gain/phase margins of the feedback loop consisting of  $C$  in negative feedback with  $P$ .  $C$  should only be the compensator in the feedback path, without reference channels, if it is a 2-Dof architecture. That is, if the closed-loop system has a 2-Dof architecture the reference channel of the controller should be eliminated resulting in a 1-Dof architecture as shown in the following figure. Either  $P$  or  $C$  must be an uncertain system, `uss` or `ufrd`, or an uncertain matrix, `umat`. If  $P$  and  $C$  are `ss/tf/zpk` or `uss` objects, the frequency range and number of points used to calculate `wcmargi` and `wcmargo` are chosen automatically.



## Basic Syntax

```
[wcmargi,wcmargo] = wcmargin(L)
[wcmargi,wcmargo] = wcmargin(P,C)
```

wcmargi and wcmargo are structures corresponding to the loop-at-a-time worst-case, single-loop gain and phase margin of the channel. For the single-loop transfer matrix  $L$  of size  $N$ -by- $N$ , wcmargi is a  $N$ -by-1 structure. For the case with two input arguments, the plant model  $P$  will have  $N_Y$  outputs and  $N_U$  inputs and hence the controller  $C$  must have  $N_U$  outputs and  $N_Y$  inputs. wcmargi is a  $N_U$ -by-1 structure with the following fields:

| Field       | Description  |
|-------------|--|
| GainMargin  | Guaranteed bound on worst-case, single-loop gain margin at plant inputs. Loop-at-a-time analysis.  |
| PhaseMargin | Loop-at-a-time worst-case phase margin at plant inputs. Units are degrees.   |
| Frequency   | Frequency associated with the worst-case margin (rad/s).   |
| WCUnc       | Structure of the worst-case uncertainty values associated with the worst-case disk gain and phase margins for the $i$ -th loop $L(i, i)$ .   |
| Sensitivity | Struct with $M$ fields. Field names are names of uncertain elements of $P$ and $C$ . Values of fields are positive numbers, which each entry indicating the local sensitivity of the worst-case margins to all the individual uncertain element's uncertainty ranges. For instance, a value of 50 indicates that if the uncertainty range is enlarged by 8%, then the worst-case gain should increase by about 4%. If the Sensitivity property of the wcOptions object is 'off', the values are NaN. |

wcmargo is an  $N$ -by-1 structure for the single loop transfer matrix input and wcmargo is an  $N_Y$ -by-1 structure when the plant and controller are input. In both these cases,

wcmargo has the same fields as wcmargi. The worst-case bound on the gain and phase margins are calculated based on a balanced sensitivity function.

[wcmargi,wcmargo] = wcmargin(L,opt) and

[wcmargi,wcmargo] = wcmargin(p,c,opt) specify options described in opt. (See wcOptions for more details on the options for wcmargin.)

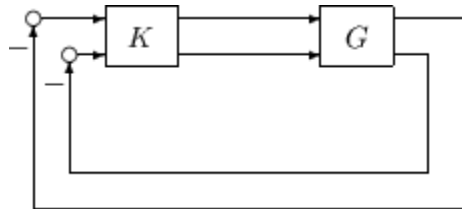
The sensitivity of the worst-case margin calculations to the individual uncertain elements is selected using the options object opt. To compute sensitivities, create a wcOptions options object, and set the Sensitivity property to 'on'.

## Examples

### MIMO Loop-at-a-Time Margins

This example is designed to illustrate that loop-at-a-time margins (gain, phase, and/or distance to -1) can be inaccurate measures of multivariable robustness margins. Margins of the individual loops can be very sensitive to small perturbations within other loops.

The nominal closed-loop system considered here is shown as follows.



$G$  and  $K$  are 2-by-2 multi-input/multi-output (MIMO) systems, defined as

$$G := \frac{1}{s^2 + \alpha^2} \begin{bmatrix} s - \alpha^2 & \alpha(s+1) \\ -\alpha(s+1) & s - \alpha^2 \end{bmatrix}, K = I_2$$

Set  $\alpha := 10$ , construct the nominal model  $G$  in state-space form, and compute its frequency response.

```
a = [0 10; -10 0];
b = eye(2);
```

```

c = [1 8;-10 1];
d = zeros(2,2);
G = ss(a,b,c,d);
K = [1 -2;0 1];

```

The nominal plant was analyzed previously using the command. Based on experimental data, the gain of the first input channel,  $b(1,1)$ , is found to vary between 0.97 and 1.06. The following statement generates the updated uncertain model.

```

ingain1 = ureal('ingain1',1,'Range',[0.97 1.06]);
b = [ingain1 0;0 1];
Gunc = ss(a,b,c,d);

```

Because of differences between measured data and the plant model an 8% unmodeled dynamic uncertainty is added to the plant outputs.

```

unmod = ultidyn('unmod',[2 2],'Bound',0.08);
Gmod = (eye(2)+unmod)*Gunc;
Gmodg =ufrd(Gmod,logspace(-1,3,60));

```

You can use the command `wcmargin` to determine the worst-case gain and phase margins in the presences of the uncertainty.

```

[wcmi,wcmo] = wcmargin(Gmodg,K);

```

The worst-case analysis corresponds to maximum allowable disk margin for all possible defined uncertainty ranges. The worst-case single-loop margin analysis performed using `wcmargin` results in a maximum allowable gain margin variation of 1.31 and phase margin variations of  $\pm 15.6$  degs in the second input channel in the presence of the uncertainties 'unmod' and 'ingain1'. `wcmi(1)`

```

ans =
  GainMargin: [0.3613 2.7681]
  PhaseMargin: [-50.2745 50.2745]
  Frequency: 0.1000
  Sensitivity: [1x1 struct]
wcmi(2)
ans =
  GainMargin: [0.7585 1.3185]
  PhaseMargin: [-15.6426 15.6426]
  Frequency: 0.1000
  Sensitivity: [1x1 struct]

```

Hence even though the second channel had infinite gain margin and 90 degrees of phase margin, allowing variation in both uncertainties, 'unmod' and 'ingain1' leads to a dramatic reduction in the gain and phase margin.

You can display the sensitivity of the worst-case margin in the second input channel to 'unmod' and 'ingain1' as follows:

```
wcni(2).Sensitivity
ans =
    ingain1: 12.1865
    unmod: 290.4557
```

The results indicate that the worst-case margins are not very sensitive to the gain variation in the first input channel, 'ingain1', but very sensitive to the LTI dynamic uncertainty at the output of the plant.

The worst-case single-loop margin at the output results in a maximum allowable gain margin variation of 1.46 and phase margin variation of  $\pm 21.3$  degs in the second output channel in the presence of the uncertainties 'unmod' and 'ingain1'.

```
wcmo(1)
ans =
    GainMargin: [0.2521 3.9664]
    PhaseMargin: [-61.6995 61.6995]
    Frequency: 0.1000
    Sensitivity: [1x1 struct]
wcmo(2)
ans =
    GainMargin: [0.6835 1.4632]
    PhaseMargin: [-21.2984 21.2984]
    Frequency: 0.1000
    Sensitivity: [1x1 struct]
```

You can display the sensitivity of the worst-case margin in the second output channel to 'unmod' and 'ingain1' as follows:

```
wcmo(2).Sensitivity
ans =
    ingain1: 16.3435
    unmod: 392.1320
```

The results are similar to the worst-case margins at the input. However, the worst-case margins at the second output channel are even more sensitive to the LTI dynamic uncertainty than the input channel margins.

## **See Also**

`dmplo`t | `loopsens` | `robstab` | `usubs` | `wcOptions` | `wcgain` | `wcsens`

**Introduced before R2006a**

## wcmarginOptions

(Not recommended) Option set for wcmargin

---

**Note** wcmarginOptions is not recommended. Use wcOptions instead.

---

### Syntax

```
opt = wcmarginOptions  
opt = wcmarginOptions(Name,Value,...)
```

### Description

opt = wcmarginOptions returns the default option set for wcmargin.

opt = wcmarginOptions(Name,Value,...) creates an option set with the options specified by one or more Name,Value pair arguments.

### Input Arguments

#### Name-Value Pair Arguments

Specify optional comma-separated pairs of Name,Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes ( ' '). You can specify several name and value pair arguments in any order as Name1,Value1,...,NameN,ValueN.

#### Sensitivity

Determines whether to compute the sensitivity of worst-case gain with respect to each individual uncertain element.

Sensitivity takes the following values:



- 'on' — Sensitivity of the worst-case gain is computed with respect to each individual uncertain element. This provides an indication of which elements are most problematic.
- 'off' — wcmargin does not compute the sensitivity of the worst-case gain with respect to each individual uncertain element.

**Default:** 'off'

### **AbsTol**

Absolute tolerance on computed worst-case margin bounds.

The algorithm terminates if  $\text{UpperBound} - \text{LowerBound} \leq \max(\text{AbsTol}, \text{RelTol} * \text{UpperBound})$

**Default:** 0.02

### **RelTol**

Relative tolerance on computed worst-case margin bounds.

The algorithm terminates if  $\text{UpperBound} - \text{LowerBound} \leq \max(\text{AbsTol}, \text{RelTol} * \text{UpperBound})$

**Default:** 0.05

## **Output Arguments**

### **opt**

Option set containing the specified options for wcmargin.

## **Examples**

Create an options set for wcmargin with an 0.01 and 0.03 as the absolute and relative tolerances on the worst-case margin bounds, respectively.

```
opt = wcmarginOptions('AbsTol',0.01,'RelTol',0.03);
```

Alternatively, use dot notation to set the values of opt.

```
opt = wcmarginOptions;  
opt.AbsTol = 0.01;  
opt.RelTol = 0.03;
```

## **See Also**

wcOptions | wcmargin

**Introduced in R2011b**

## wcnorm

Worst-case norm of uncertain matrix

### Syntax

```
maxnorm = wcnorm(m)
```

```
[maxnorm,wcu] = wcnorm(m)
```

```
[maxnorm,wcu] = wcnorm(m,opts)
```

```
[maxnorm,wcu,info] = wcnorm(m)
```

```
[maxnorm,wcu,info] = wcnorm(m,opts)
```

### Description

The norm of an uncertain matrix generally depends on the values of its uncertain elements. Determining the maximum norm over all allowable values of the uncertain elements is referred to as a *worst-case norm* analysis. The maximum norm is called the *worst-case norm*.

As with other *uncertain-system* analysis tools, only bounds on the worst-case norm are computed. The exact value of the worst-case norm is guaranteed to lie between these upper and lower bounds.

### Basic syntax

Suppose `mat` is a `umat` or a `uss` with  $M$  uncertain elements. The results of

```
[maxnorm,maxnormunc] = wcnorm(mat)
```

`maxnorm` is a structure with the following fields.

| Field      | Description                                      |
|------------|--|
| LowerBound | Lower bound on worst-case norm, positive scalar. |

| Field      | Description                                      |
|------------|--|
| UpperBound | Upper bound on worst-case norm, positive scalar. |

`maxnormunc` is a structure that includes values of uncertain elements and maximizes the matrix norm. There are  $M$  field names, which are the names of uncertain elements of `mat`. The value of each field is the corresponding value of the uncertain element, such that when jointly combined, lead to the norm value in `maxnorm.LowerBound`. The following command shows the norm:

```
norm(usubs(mat,maxnormunc))
```

### Basic syntax with third output argument

A third output argument provides information about sensitivities of the worst-case norm to the uncertain elements ranges.

```
[maxnorm,maxnormunc,info] = wcnorm(mat)
```

The third output argument `info` is a structure with the following fields:

| Field              | Description  |
|--------------------|--|
| Model              | Index of model with largest gain (when <code>mat</code> is an array of uncertain matrices)   |
| WorstPerturbation  | Structure of worst-case uncertainty values. The fields of <code>info.WorstPerturbation</code> are the names of the uncertain elements in <code>mat</code> , and each field contains the worst-case value of the corresponding element.   |
| Sensitivity        | A struct with $M$ fields. Fieldnames are names of uncertain elements of <code>sys</code> . Field values are positive numbers, each entry indicating the local sensitivity of the worst-case norm in <code>maxnorm.LowerBound</code> to all of the individual uncertain elements' uncertainty ranges. For instance, a value of 25 indicates that if the uncertainty range is increased by 8%, then the worst-case norm should increase by about 2%. If the <code>Sensitivity</code> property of the <code>wcOptions</code> object is 'off', the values are <code>NaN</code> . |
| BadUncertainValues | Same as <code>WorstPerturbation</code> . Included for compatibility with R2016a and earlier.   |
| ArrayIndex         | Same as <code>Model</code> . Included for compatibility with R2016a and earlier.   |

## Examples

### Worst-Case Norm and Condition Number of an Uncertain Matrix

Construct an uncertain matrix and compute the worst-case norm of the matrix and of its inverse. These computations let you accurately estimate the worst-case, or the largest, value of the condition number of the matrix.

```
a = ureal('a',5,'Range',[4 6]);
b = ureal('b',3,'Range',[2 10]);
c = ureal('c',9,'Range',[8 11]);
d = ureal('d',1,'Range',[0 2]);
```

```
M = [a b;c d];
Mi = inv(M);
```

```
maxnormM = wcnorm(M)
```

```
maxnormM = struct with fields:
    LowerBound: 14.7199
    UpperBound: 14.7227
```

```
maxnormMi = wcnorm(Mi)
```

```
maxnormMi = struct with fields:
    LowerBound: 2.5963
    UpperBound: 2.5975
```

The condition number of  $M$  must be less than the product of the two upper bounds for all values of the uncertain elements of  $M$ . Conversely, the condition number of the largest value of  $M$  must be at least equal to the condition number of the nominal value of  $M$ . Compute these bounds on the worst-case value of the condition number.

```
condUpperBound = maxnormM.UpperBound*maxnormMi.UpperBound;
condLowerBound = cond(M.NominalValue);
[condLowerBound condUpperBound]
```

```
ans = 1×2
```

```
5.0757    38.2414
```

The range between these lower and upper bounds is fairly large. You can get a more accurate estimate. Recall that the condition number of an  $n$ -by- $m$  matrix  $M$  can be expressed as an optimization, where a free norm-bounded matrix  $\Delta$  tries to align the gains of  $M$  and  $\text{inv}(M)$ :

$$\kappa(M) = \max_{\Delta \in \mathbb{C}^{m \times m}} (\sigma_{\max}(M\Delta M^{-1}))$$
$$\sigma_{\max}(\Delta) \leq 1$$

If  $M$  is uncertain, then the worst-case condition number involves further maximization over the possible values of  $M$ . Therefore, you can compute the worst-case condition number of an uncertain matrix by using a `ucomplexm` uncertain element and using `wcnorm` to carry out the maximization.

Create a 2-by-2 `ucomplexm` element with nominal value 0.

```
Delta = ucomplexm('Delta', zeros(2,2));
```

The range of values represented by `Delta` includes 2-by-2 matrices with the maximum singular value less than or equal to 1.

Create the expression involving `M`, `Delta`, and `inv(M)`.

```
H = M*Delta*Mi;
```

```
opt = wcOptions('MussvOptions','m5');  
[maxKappa,wcu,info] = wcnorm(H,opt);  
maxKappa
```

```
maxKappa = struct with fields:  
  LowerBound: 26.8406  
  UpperBound: 38.2579
```

Verify that the values in `wcu` make the condition number as large as `maxKappa`. `LowerBound`.

```
cond(usubs(M,wcu))
```

```
ans = 26.9629
```

## Algorithms

See `wcgain`.

## See Also

`norm` | `wcOptions` | `wcgain`

**Introduced before R2006a**

## wcOptions

Option set for worst-case analysis

### Syntax

```
opts = wcOptions  
opts = wcOptions(Name,Value,...)
```

### Description

`opts = wcOptions` returns the default option set for worst-case analysis commands `wcgain`, `wcsens`, `wcmargin`, or `wcsigma`.

`opts = wcOptions(Name,Value,...)` creates an option set with the options specified by one or more `Name,Value` pair arguments.

### Examples

#### Options for Worst-Case Gain Calculation

Create an options set to calculate the worst-case gain while allowing the uncertain parameters to vary by 20% more than the range specified in the model. Also, configure the options to include the element-by-element sensitivity in the calculation.

```
opts = wcOptions('ULevel',1.2,'Sensitivity','on');
```

Alternatively, create a default option set, and use dot notation to set the values of particular options.

```
opts = wcOptions;  
opts.ULevel = 1.2;  
opts.Sensitivity = 'on';
```

Use `opts` as an input argument to a worst-case analysis command such as `wcgain`.



## Improve Worst-Case Perturbation

Specifying certain options for the structured-singular-value computation that underlies the worst-gain computation can yield better results in some cases. For example, consider a sample plant and controller.

```
load(fullfile(matlabroot, 'examples', 'robust', 'wcgExampleData.mat'))
```

This command loads `gPlant`, a MIMO plant with 10 outputs, 8 inputs, and 11 uncertain elements. It also loads `Kmu`, a state-space controller model. Form a closed-loop system with these models, and examine the worst-case gain.

```
CL = lft(gPlant, Kmu);
[wcg, wcu] = wcgain(CL);
wcg

wcg = struct with fields:
    LowerBound: 10.7780
    UpperBound: 11.2130
    CriticalFrequency: 6.5759
```

There is a large difference between the lower and upper bounds on the worst-case gain. To get a better estimate of the actual worst-case gain, increase the number of restarts that `wcgain` uses for computing of the worst-case perturbation and associated lower bound. Doing so can result in a tighter lower bound. This option does not affect the upper-bound calculation.

```
opt = wcOptions('MussvOptions', 'm3');
[wcg, wcu] = wcgain(CL, opt);
wcg

wcg = struct with fields:
    LowerBound: 10.8142
    UpperBound: 11.2130
    CriticalFrequency: 6.6564
```

The difference between the lower bound and upper bound on the worst-case gain is much smaller. The critical frequency is different as well.

## Input Arguments

### Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes ( `' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

Example: `'ULevel','1.5','Sensitivity','on'`

#### **ULevel — Uncertainty level**

1 (default) | positive scalar

Uncertainty level to use for the worst-case computation, specified as the comma-separated pair consisting of `'ULevel'` and a positive scalar value. This option scales the normalized uncertainty by the factor you specify. Such scaling lets you examine the effect of smaller or larger uncertainty range without changing the uncertainty levels in your model. For instance, to see the effect of doubling the ranges of all uncertain parameters, set `'ULevel'` to 2. To see the effect of shrinking the ranges, set `'ULevel'` to 0.5. The default value, 1, corresponds to the amount of uncertainty specified in the model.

#### **Display — Display progress of computation and summary report**

'off' (default) | 'on'

Display progress and summary report of the worst-case gain computation, specified as the comma-separated pair consisting of `'Display'` and one of these values:

- `'off'` — Do not display progress and report.
- `'on'` — Display progress and report. When you use this option, a progress indicator and summary of results is displayed in the command window, similar to the following.

```
The worst-case gain is at most 11.2.  
-- There is a bad perturbation amounting to 100% of the modeled uncertainty.  
-- This perturbation causes a gain of 9.03 at the frequency 5.5 rad/seconds.
```

This setting overrides the silent (`'s'`) option in the `MussvOptions` option.

#### **VaryFrequency — Compute worst-case gain as function of frequency**

'off' (default) | 'on'

Return worst-case gain as a function of frequency, specified as the comma-separated pair consisting of 'VaryFrequency' and one of these values:

- 'off' — Only return worst-case gains at frequencies where the worst values occur.
- 'on' — Compute worst-case gains over a frequency grid suitable for plotting. The frequency grid is chosen automatically based on system dynamics. This calculation is done in addition to identifying the critical frequency where the gain peaks. Access the frequency values and corresponding gains in the `info` output of `wcgain` or other worst-case analysis command.

This option is ignored for `ufrd` and `genfrd` models.

### **Sensitivity — Calculate sensitivity of worst-case gains**

'off' (default) | 'on'

Calculate the sensitivity of the worst-case gain to each uncertain element in the model, specified as the comma-separated pair consisting of 'Sensitivity' and either 'off' or 'on'.

Each uncertain element contributes to the overall worst case in a coupled manner. Set this option to 'on' to estimate the sensitivity of the margin to each element. This element-by-element sensitivity provides an indication of which elements are most problematic. Access the sensitivity estimates in the `info` output of the worst-case computation command.

### **SensitivityPercent — Percentage variation of uncertainty for computing sensitivity**

25 (default) | positive scalar value

Percentage variation of uncertainty level for computing sensitivity, specified as the comma-separated pair consisting of 'SensitivityPercent' and a positive scalar value. The sensitivity to a particular uncertain element is estimated using a finite difference calculation. This calculation increases the (normalized) amount of uncertainty on this element by some percentage, computes the resulting worst-case gain, and computes the ratio of percent variations. This option specifies the percentage increase in uncertainty level applied to each element. The default value is 25%.

### **MussvOptions — Options for mussv calculation**

' ' (default) | character vector

Options for the `mussv` calculation that underlies the worst-case calculations, specified as the comma-separated pair consisting of `'MussvOptions'` and a character vector such as `'sm3'` or `'ad'`.

Some `MussvOptions` values that are especially useful for improving worst-case calculations include:

- `'a'` — Force the use of LMI optimization to compute the  $\mu$  upper bound, which yields better results in general but can be expensive when some `ureal` elements are repeated multiple times.
- `'mN'` — Use multiple restarts when computing the  $\mu$  lower bound, which corresponds to the lower bound for the worst-case gain. This option can reduce the gap between the lower bound and upper bound on the worst-case gains. `N` is the number of restarts. For example, setting `'MussvOptions'` to `'m3'` causes three restarts. See “Improve Worst-Case Perturbation” on page 1-571 for an example.

See `mussv` for the remaining available options and corresponding characters. The default, `' '`, uses the default options for `mussv`.

## Output Arguments

### **opts** — Options for worst-case calculations

`wcOptions` object

Options for worst-case calculations, returned as a `wcOptions` object. Use the options as an input argument to any of the worst-case analysis functions, such as `wcgain` and `wcsigma`. For example:

```
[wcgain,wcu,info] = wcgain(usys,opts)
```

## See Also

`wcgain` | `wcmargin` | `wcsens` | `wcsigma`

## Topics

“Robustness and Worst-Case Analysis”

**Introduced in R2016b**

## wcsens

Calculate worst-case sensitivity and complementary sensitivity functions of plant-controller feedback loop

### Syntax

```
wcst = wcsens(L)
```

```
wcst = wcsens(L,type)
```

```
wcst = wcsens(L,opt)
```

```
wcst = wcsens(L,type,scaling)
```

```
wcst = wcsens(L,type,scaling,opt)
```

```
wcst = wcsens(P,C)
```

```
wcst = wcsens(P,C,type)
```

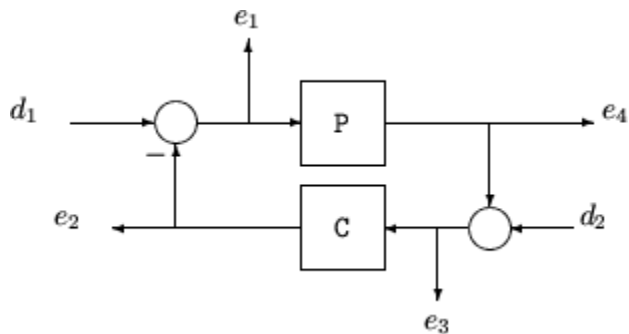
```
wcst = wcsens(P,C,opt)
```

```
wcst = wcsens(P,C,type,scaling)
```

```
wcst = wcsens(P,C,type,scaling,opt)
```

### Description

The sensitivity function,  $S = (I + L)^{-1}$ , and the complementary sensitivity function,  $T = L(I + L)^{-1}$ , where  $L$  is the loop gain matrix associated with the input,  $CP$ , or output,  $PC$ , are two transfer functions related to the robustness and performance of the closed-loop system. The multivariable closed-loop interconnection structure, shown below, defines the input/output sensitivity, complementary sensitivity and loop transfer functions.



| Description  | Equation          |
|--|-------------------|
| Input sensitivity ( $TF_{e_1-d_1}$ )                 | $(I + CP)^{-1}$   |
| Input complementary sensitivity ( $TF_{e_2-d_1}$ )   | $CP(I + CP)^{-1}$ |
| Output sensitivity ( $TF_{e_3-d_2}$ )                | $(I + PC)^{-1}$   |
| Output complementary sensitivity ( $-TF_{e_4-d_2}$ ) | $PC(I + PC)^{-1}$ |
| Input loop transfer function                         | $CP$              |
| Output loop transfer function                        | $PC$              |

`wcst = wcsens(L)` calculates the worst-case sensitivity and complementary sensitivity functions for the loop transfer matrix `L` in feedback in negative feedback with an identity matrix. If `L` is a `uss` object, the frequency range and number of points are chosen automatically.

`wcst = wcsens(P,C)` calculates the worst-case sensitivity and complementary sensitivity functions for the feedback loop `C` in negative feedback with `P`. `C` should only be the compensator in the feedback path, not any reference channels, if it is a 2-dof architecture (see `loopsens`). If `P` and `C` are `ss`/`tf`/`zpk` or `uss` objects, the frequency range and number of points are chosen automatically. `wcst` is a structure with the following substructures:

**Fields of wcst**

| <b>Field</b> | <b>Description</b>   |
|--------------|--|
| Si           | Worst-case input-to-plant sensitivity function                             |
| Ti           | Worst-case input-to-plant complementary sensitivity function               |
| So           | Worst-case output-to-plant sensitivity function                            |
| To           | Worst-case output-to-plant complementary sensitivity function              |
| PSi          | Worst-case plant times input-to-plant sensitivity function                 |
| CSo          | Worst-case compensator times output-to-plant sensitivity function          |
| Stable       | 1 if nominal closed loop is stable, 0 otherwise. NaN for frd/ufrd objects. |

Each sensitivity substructure is a structures with five fields **MaximumGain**, **BadUncertainValues**, **System**, **BadSystem**, **Sensitivity** derived from the outputs of **wcgain**.

**Fields of Si, So, Ti, To, PSi, CSo**

| <b>Field</b>       | <b>Description</b>   |
|--------------------|--|
| MaximumGain        | struct with fields LowerBound, UpperBound and CriticalFrequency. LowerBound and UpperBound are bounds on the unweighted maximum gain of the uncertain sensitivity function. CriticalFrequency is the frequency at which the maximum gain occurs.   |
| BadUncertainValues | Struct, containing values of uncertain elements which maximize the sensitivity gain. There are $M$ fluidness, which are the names of uncertain elements of sensitivity function. The value of each field is the corresponding value of the uncertain element, such that when jointly combined, lead to the gain value in MaximumGain.LowerBound.   |
| System             | Uncertain sensitivity function (ufrd or uss).  |
| BadSystem          | Worst-case system based on the uncertain object values in BadUncertainValues. BadSystem is defined as $\text{BadSystem} = \text{usubs}(\text{System}, \text{BadUncertainValues})$ .  |
| Sensitivity        | Struct with $M$ fields, fieldnames are names of uncertain elements of system. Values of fields are positive numbers, each entry indicating the local sensitivity of the maximum gain to all of the individual uncertain elements uncertainty ranges. For instance, a value of 50 indicates that if the uncertainty range is enlarged by 8%, then the maximum gain should increase by about 4%. If the 'Sensitivity' property of the wcOptions object is 'off', the values are NaN. |

$\text{wcst} = \text{wcsens}(L, \text{type})$  and  $\text{wcst} = \text{wcsens}(P, C, \text{type})$  allow selection of individual Sensitivity and Complementary Sensitivity functions,  $\text{type}$ , as 'Si', 'Ti', 'So', 'To', 'PSi', 'CSo' corresponding to the sensitivity and complementary sensitivity functions. Setting  $\text{type}$  to 'S' or 'T' selects all sensitivity functions ('Si', 'So', 'PSi', 'CSo') or all complementary sensitivity functions ('Ti', 'To'). Similarly, setting  $\text{type}$  to 'Input' or 'Output' selects all input Sensitivity functions ('Si', 'Ti', 'PSi') or all output sensitivity functions ('So', 'To', 'CSo'). 'All' selects all six Sensitivity functions for analysis (default).  $\text{type}$  may also be a cell array containing multiple function types, such as {'Si', 'To'}.

$\text{wcst} = \text{wcsens}(L, \text{type}, \text{scaling})$  and  $\text{wcst} = \text{wcsens}(P, C, \text{type}, \text{scaling})$  add a scaling to the worst-case sensitivity analysis.  $\text{scaling}$  is one of the following:



- 'Absolute' (default) — Calculates bounds on the maximum gain of the uncertain sensitivity function.
- 'Relative' — Finds bounds on the maximum relative gain of the uncertain sensitivity function. That is, the maximum relative gain is the largest ratio of the worst-case gain and the nominal gain evaluated at each frequency point in the analysis.
- LTI model (ss, tf, zpk, or frd) — Calculates bounds on the maximum scaled gain of the uncertain sensitivity function. The input/output dimensions of the LTI model must be either 1-by-1, or compatible with P and C.

You can also combine `type` and `scaling` in a cell array, e.g.

```
wcst = wcsens(P,C,{'Ti','So'},'Abs','Si','Rel','PSi',wt)
```

`wcst = wcsens(P,C,opt)` or `wcst = wcsens(P,C,type,scaling,opt)` specifies options for the worst-case gain calculation as defined by `opt`. (See `wcOptions` for more details on the options for `wcsens`.)

The sensitivity of the worst-case sensitivity calculations to the individual uncertain components can be determined using the options object `opt`. To compute the sensitivities to the individual uncertain components, create a `wcOptions` options object, and set the `Sensitivity` property to 'on'.

```
opt = wcOptions('Sensitivity','on');
wcst = wcsens(P,C,opt)
```

## Examples

The following constructs a feedback loop with a first order plant and a proportional-integral controller. The time constant is uncertain and the model also includes an multiplicative uncertainty. The nominal (input) sensitivity function has a peak of 1.09 at  $\omega = 1.55$  rad/sec. Since the plant and controller are single-input / single-output, the input/output sensitivity functions are the same.

```
delta = ultidyn('delta',[1 1]);
tau = ureal('tau',5,'range',[4 6]);
P = tf(1,[tau 1])*(1+0.25*delta);
C=tf([4 4],[1 0]);
looptransfer = loopsens(P,C);
Snom = looptransfer.Si.NominalValue;
```

```
norm(Snom,inf)
ans =
    1.0864
```

wcsens is then used to compute the worst-case sensitivity function as the uncertainty ranges over its possible values. More information about the fields in `wcst.Si` can be found in the `wcgain` help. The `badsystem` field of `wcst.Si` contains the worst case sensitivity function. This worst case sensitivity has a peak of 1.52 at  $\omega = 1.02$  rad/sec. The `maxgainunc` field of `wcst.Si` contains the perturbation that corresponds to this worst case sensitivity function.

```
wcst = wcsens(P,C)
wcst =
    Si: [1x1 struct]
    Ti: [1x1 struct]
    So: [1x1 struct]
    To: [1x1 struct]
    PSi: [1x1 struct]
    CSo: [1x1 struct]
    Stable: 1
Swc = wcst.Si.BadSystem;
omega = logspace(-1,1,50);
bodemag(Snom, '-', Swc, '-.', omega);
legend('Nominal Sensitivity', 'Worst-Case Sensitivity', ...
    'Location', 'SouthEast')
norm(Swc,inf)
ans =
    1.5075
```

For multi-input/multi-output systems the various input/output sensitivity functions will, in general, be different.

## References

J. Shin, G.J. Balas, and A.K. Packard, "Worst case analysis of the X-38 crew return vehicle flight control system," *AIAA Journal of Guidance, Dynamics and Control*, vol. 24, no. 2, March-April 2001, pp. 261-269.

## See Also

`loopsens` | `robstab` | `usubs` | `wcOptions` | `wcgain` | `wcmargin`

**Introduced before R2006a**

## wcsigma

Plot worst-case gain of uncertain system

### Syntax

```
wcsigma(usys)  
wcsigma(usys,w)  
wcsigma( ____,opts)
```

### Description

`wcsigma(usys)` plots the nominal and worst-case gains of the uncertain system `usys` as a function of frequency. For multi-input, multi-output (MIMO) systems, gain refers to the largest singular value of the frequency response matrix. (See `sigma` for more information about singular values.) The plot includes:

- Nominal — Nominal gain of `usys`.
- Worst — The response falling within the uncertainty of `usys` that has the highest peak gain.
- Worst-case gain (lower bound) — The lowest possible worst-case gain at each frequency.
- Worst-case gain (upper bound) — The highest possible gain within the uncertainty at each frequency. This curve represents the envelope produced by finding the highest possible gain at each frequency.
- Sampled Uncertainty — 20 responses randomly sampled from `usys`.

`wcsigma(usys,w)` focuses the plot on the frequencies specified by `w`.

- If `w` is a cell array of the form `{wmin,wmax}`, then `wcsigma` plots the worst-case gains in the range `{wmin,wmax}`.
- If `w` is an array of frequencies, then `wcsigma` plots the worst-case gains at each frequency in the array.

`wcsigma( ____,opts)` specifies additional options for the computation. Use `wcOptions` to create `opts`.

## Examples

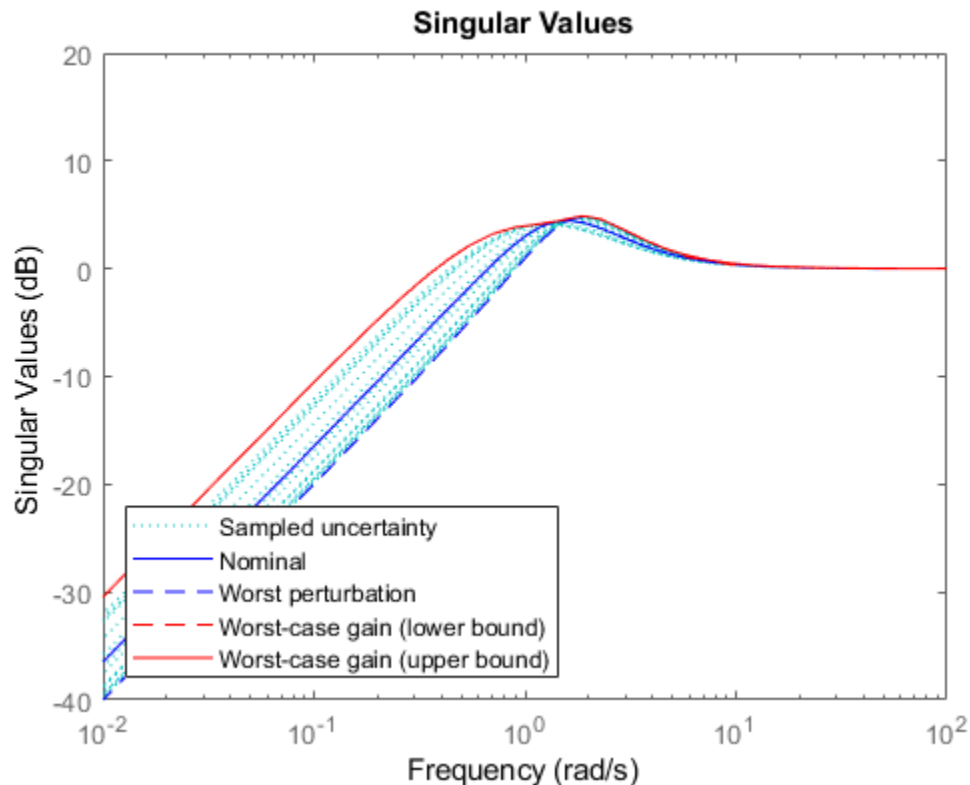
### Plot Worst-Case Gain of Uncertain System

Plot the worst-case gain of the following system:

$$sys = \frac{s^2 + 3s}{s^2 + 2s + a}$$

The uncertain parameter  $a = 2 \pm 1$ .

```
a = ureal('a',2);  
usys = tf([1 3 0],[1 2 a]);  
wcsigma(usys)
```

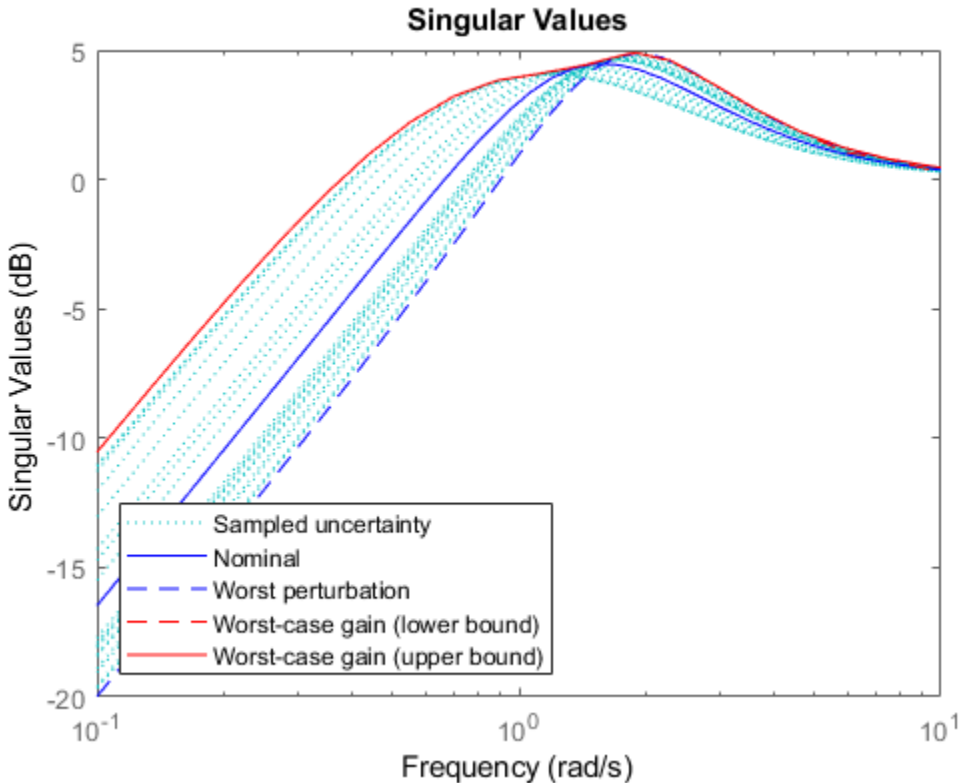


The **Worst** curve identifies the single response within the uncertainty that yields the highest gain at any frequency. The **Worst-case gain (upper bound)** curve is the envelope produced by finding the highest gain within the uncertainty at each frequency.

The **Worst perturbation** curve identifies the combination of uncertain elements within the specified range that yields the highest overall gain. This perturbation corresponds to the wcu output of wcgain. The **Worst-case gain** curves show the lower and upper bounds on the worst-case gain at each frequency. For any perturbation within the specified uncertainty range, the principal gains (singular values) of the perturbed system lie below the **Worst-case gain (upper bound)** curve.

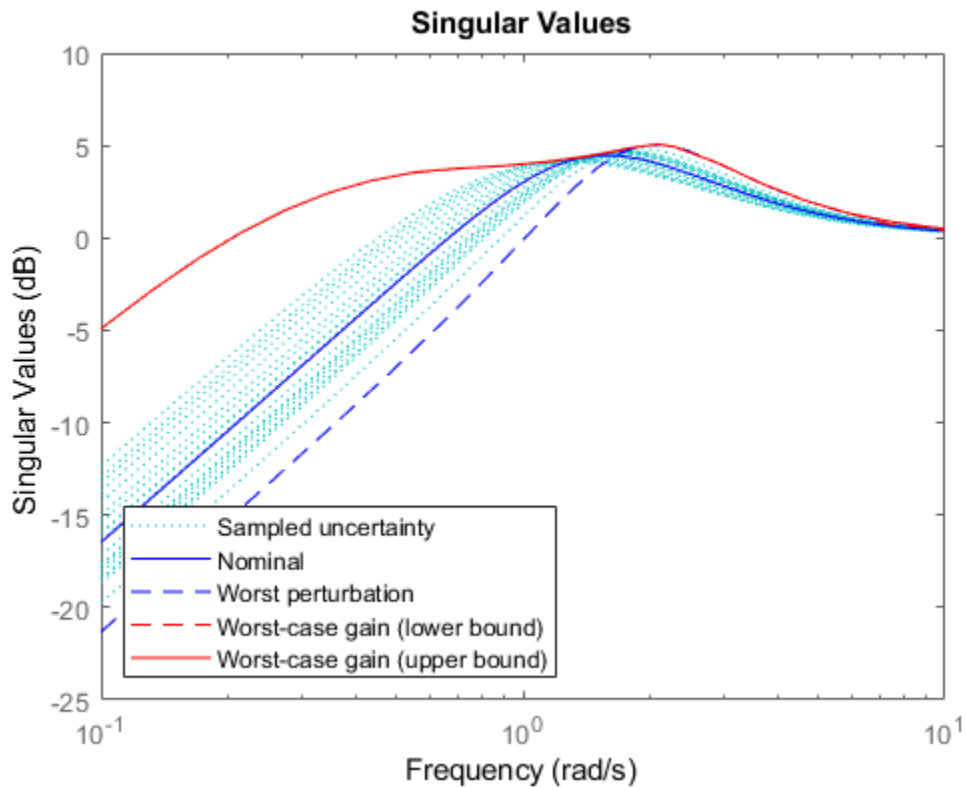
Focus the plot on the region between 0.1 and 10 rad/s.

```
w = {0.1 10};
wcsigma(usys,w)
```



Examine the effect on the worst-case response of increasing the uncertainty range. To do this without changing the uncertainty specified in `usys`, use the `ULevel` option of `wcOptions`. This option scales the normalized uncertainty by the factor you specify. For example, examine the worst-case response a 50% greater uncertainty range.

```
opts = wcOptions('ULevel',1.5);
wcsigma(usys,w,opts)
```



The plot shows that increasing the uncertainty range substantially increases the worst-case gain at low frequencies.

## Input Arguments

### **usys** — Dynamic system with uncertainty

uss | ufrd | genss | genfrd

Dynamic system with uncertainty, specified as a uss, ufrd, genss, or genfrd model that contains uncertain elements.



For `genss` or `genfrd` models, `wcsigma` uses the current value of any tunable blocks and folds them into the known (not uncertain) part of the model.

### **w — Frequencies**

`{wmin,wmax}` | vector

Frequencies at which to plot worst-case gains, specified as the cell array `{wmin,wmax}` or as a vector of frequency values.

- If `w` is a cell array of the form `{wmin,wmax}`, then the function plots the worst-case gains at frequencies ranging between `wmin` and `wmax`.
- If `w` is a vector of frequencies, then the function plots the worst-case gains at each specified frequency. For example, use `logspace` to generate a row vector with logarithmically spaced frequency values.

Specify frequencies in units of `rad/TimeUnit`, where `TimeUnit` is the `TimeUnit` property of the model.

### **opts — Options for worst-case gain computation**

`wcOptions` object

Options for worst-case computation, specified as an object you create with `wcOptions`. Setting certain options for `mussv` can improve the results of the worst-case calculation. See `wcOptions` for more information.

Example: `wcOptions('ULevel',2,'MussvOptions','m3')`

## **Algorithms**

`wcsigma` uses `wcgain` to compute the worst-case gains. Use the `opts` argument to set options for the `wcgain` algorithm.

`wcsigma` uses `usample` to compute the Sampled Uncertainty curves.

## **See Also**

`sigma` | `uss` | `wcOptions` | `wcgain`

## **Topics**

“Robustness and Worst-Case Analysis”

**Introduced in R2016b**

# Block Reference

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# MultiPlot Graph

Plot multiple signals



## Description

The MultiPlot Graph block displays signals in a MATLAB figure.

If the input signal is a vector, then each component of the vector is plotted in a separate axes. Lines are added to the axes in subsequent simulations. The most recent data is plotted in red. Older plots cycle through seven different colors. The block acts as a “hold-on, subplotter.”

There are two buttons in the toolbar menu. The eraser button clears the data from all axes. The export button saves all the visible plot data to the MATLAB workspace in a variable named by the dialog box entry **Variable for Export to Workspace**. The format is a struct array, following the behavior of a To Workspace block, using the “Structure, With Time” save format.

The MultiPlot Graph block can be used in conjunction with the Uncertain State Space block to visualize Monte Carlo and worst-case simulation time responses.

## Parameters

### **t-min, t-max**

The parameter entries t-min and t-max are the minimum and maximum x-axis limits. t-min and t-max may be vectors corresponding to each subplot.

## **y-min, y-max**

The parameter entries y-min and y-max are the minimum and maximum y-axis limits and similarly may be vector quantities.

## **Sample time**

Sample time corresponds to the sample time at which to collect points.

## **Title**

Specifies the title of the multiplot figure.

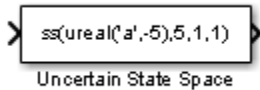
## **Variable for Export to Workspace**

Variable name of the MATLAB object to contain all the visible plot data exported to the MATLAB workspace. The format is a struct array, following the behavior of a To Workspace block, using the "Structure, With Time" save format.

## **Introduced in R2007a**

## Uncertain State Space

Specify uncertain system in Simulink



### Description

The Uncertain State Space block lets you model parametric and dynamic uncertainty in Simulink. The block accepts uncertain state space (uss) models or any model that can be converted to uss, such as `umat`, `ureal` and `ultidyn` objects.

### Parameters

#### Uncertain system variable (uss)

Linear state-space model with uncertainty (uss object). Specify an uss object using one of the following:

- Function or expression that evaluates to an uss object. For example:
  - `ss(ureal('a',-5),5,1,1)`
  - `wt*input_unc`, where `input_unc` is an `ultidyn` object and `wt` and `input_unc` are defined in the MATLAB workspace.
- Variable name, defined in the MATLAB workspace. For example, `unc_sys`, where you define `unc_sys = ss(ureal('a',-5),5,1,1)` in the workspace. This returns an uss object.
- Model type that can be converted to an uss object. For example:
  - LTI models (`tf`, `zpk` and `ss`)
  - Uncertain matrix (`umat`)

- Uncertain real parameters (`ureal`)
- Uncertain dynamics (`ultidyn`).

When the block is in a model with synchronous state control (see the State Control block), you must specify a discrete-time model.

## Uncertainty value (struct or [] to use nominal value)

Values of uncertain variables. The `uss` object that you enter in the **Uncertain system variable (uss)** field depends on uncertain variables (`ureal` or `ultidyn` object). Use this field to specify the values of these uncertain variables to use for simulation or linearization. Specify the value as one of the following:

| Value     | Description  |
|-----------|--|
| [ ]       | Use nominal values.  |
| Structure | Use user-defined values. For example, <code>struct('a',1)</code> specifies a value of 1 for the uncertain variable <code>a</code> .<br><br>Use <code>ufind</code> and <code>usample</code> to generate randomized values of uncertain variables for Monte Carlo simulation. For more information, see “Vary Uncertainty Values Using Individual Uncertain State Space Blocks” and “Vary Uncertainty Values Across Multiple Uncertain State Space Blocks” in the <i>Robust Control Toolbox User's Guide</i> . |

## Initial states (nominal dynamics)

If the nominal value of the uncertain state variable, `unc_sys.NominalValue` where `unc_sys` is the uncertain system variable specified in the **Uncertain system variable** field, has states, specify the initial condition for these states. The value defaults to zero.

## Initial states (uncertain dynamics)

If the uncertain system contains some dynamic uncertainty (`ultidyn`), specify the initial state of these dynamics. The value defaults to zero.

## **See Also**

ufind, usample, ulinearize, uss, umat, ureal, ultidyn

## **Tutorials**

Robustness Analysis in Simulink

Linearization of Simulink Models with Uncertainty

## **How To**

“Simulate Uncertainty Effects”

“Compute Uncertain State-Space Models from Simulink Models”

**Introduced in R2009b**



# USS System

Import uncertain systems into Simulink

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**Note** USS System block will be removed in a future release. Use Uncertain State Space block instead.

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## Description

The USS System block accepts USS and UMAT containing `ureal` and `ultidyn` uncertain objects, as well as `ureal` and `ultidyn` objects. An instance of the uncertain system is used in the simulation or linearization. Internally, USS models are converted to their state space equivalent for evaluation.

## Parameters

### USS system variable

The uncertain object (USS, UMAT, `ureal`, or `ultidyn`) is entered in the USS system variable.

### Initial states (nominal dynamics)

If the nominal value for the USS system variable has states, then the initial condition for these states is entered in `Initial states (nominal dynamics)`.

### Uncertainty value

The values for the uncertain elements are controlled by the `Uncertainty value` menu. If `Nominal` is selected, then the nominal value of the uncertain object is used. If you select `User defined`, then you must enter a MATLAB structure in the `User-defined uncertainty (struct)` dialog box. The field names of the structure should correspond

to the names of the uncertain atoms within the USS system variable, while the values of the fields are the values used for the uncertain objects (using the command `usubs`). If some of these values are SS objects, then these states are referred to as uncertainty states.

The order of the uncertainty states is determined by the order of atoms in the `Uncertainty` property of the USS system variable. The state dimension is determined by the actual data in the `User-defined uncertainty` structure. Any extra fields in the `User-defined uncertainty` structure are ignored.

### **User-defined uncertainty (struct)**

If `User defined` is selected from the `Uncertainty` value pop-up menu, then the structure data entered in `User-defined uncertainty (struct)` must contain fields corresponding to every uncertain atom of the USS system variable. Extra fields are ignored. `usimsamp` generates a random instance of each atom in a Simulink model. It returns a structure, suitable for entry in `User-defined uncertainty (struct)`.

### **Initial states (uncertain dynamics)**

The initial condition for the uncertainty states is entered in `Initial states (uncertain dynamics)`.

**Introduced in R2007a**